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IFAC-PapersOnLine 49-15 (2016) 199–204

Some remarks on wheeled autonomous vehicles and the evolution of their control design ⋆

Brigitte d'ANDRÉA-NOVEL * **Lghani MENHOUR** ** **Michel FLIESS** ***,† **Hugues MOUNIER** ****

* *Centre de Robotique, MINES ParisTech & PSL Research University, 60 boulevard Saint-Michel, 75272 Paris cedex 06, France. (e-mail: brigitte.dandrea-novel@mines-paristech.fr)*

** *Université de Reims Champagne-Ardenne, IUT de Troyes, 9 rue du Québec, 10026 Troyes, France. (e-mail: lghani.menhour@univ-reims.fr)*

*** *LIX (CNRS, UMR 7161), École polytechnique, 91128 Palaiseau, France. (e-mail: Michel.Fliess@polytechnique.edu)*

**** *L2S (UMR 8506), CNRS – Supélec – Université Paris-Sud, 3 rue Joliot-Curie, 91192 Gif-sur-Yvette, France. (e-mail: hugues.mounier@lss.supelec.fr)*

† *AL.I.E.N. (ALgèbre pour Identification et Estimation Numériques), 24-30 rue Lionnois, BP 60120, 54003 Nancy, France. (e-mail: michel.fliess@alien-sas.com)*

Abstract: Recent investigations on the longitudinal and lateral control of wheeled autonomous vehicles are reported. Flatness-based techniques are first introduced via a simplified model. It depends on some physical parameters, like cornering stiffness coefficients of the tires, friction coefficient of the road, ..., which are notoriously difficult to identify. Then a model-free control strategy, which exploits the flat outputs, is proposed. Those outputs also depend on physical parameters which are poorly known, *i.e.*, the vehicle mass and inertia and the position of the center of gravity. A totally model-free control law is therefore adopted. It employs natural output variables, namely the longitudinal velocity and the lateral deviation of the vehicle. This last method, which is easily understandable and implementable, ensures a robust trajectory tracking problem in both longitudinal and lateral dynamics. Several convincing computer simulations are displayed.

Keywords: Autonomous vehicles, wheeled vehicles, longitudinal control, lateral control, flatness-based control, model-free control, intelligent controllers, algebraic estimation.

1. INTRODUCTION

The lateral and longitudinal control of wheeled autonomous vehicles is an important topic which has already attracted many promising studies (see, *e.g*, Ackermann *et al.* [1995], d'Andréa-Novel *et al.* [2001], Antonov *et al.* [2008], Attia *et al.* [2014], Choi *et al.* [2009], Cerone *et al.* [2009], Chou *et al.* [2005], Fuchshumer *et al.* [2005], Marino *et al.* [2005], Martinez *et al.* [2007], Menhour *et al.* [2011], Nouvelière [2002], Poussot-Vassal *et al.* [2011], Rajamani *et al.* [2000], Villagra *et al.* [2009, 2011, 2012], Zheng *et al.* [2006], ...), where various advanced theoretical tools are utilized. This short communication does not permit unfortunately to summarize them. Let us nevertheless notice that most of them are model-based. The aim of this presentation is to explain and justify the evolution of our viewpoint which started with a flatness-based setting (Menhour *et al.* [2011]), *i.e.*, a model-based approach. It is now adopting a fully model-free standpoint (see Menhour *et al.* [2013] and Menhour *et al.* [2015]). As a matter of fact severe difficulties are encountered to

- write a mathematical model which takes into account all the numerous complex phenomena,
- calibrate the existing model in various changing situations like cornering stiffness coefficients of the tires, and friction coefficient of the road.

Our paper is organized as follows. Section 2 presents a nonlinear longitudinal and lateral flatness-based control. In Section 3 the two different model-free control strategies are developed. Simulation results with noisy data and suitable reference trajectories acquired on a track race, are displayed in Section 4. Let us emphasize that the second model-free control is quite robust. Concluding remarks may be found in Section 5.

⋆ Work partially supported by the French national project INOVE/ANR 2010 BLANC 308.

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.732

2. LONGITUDINAL AND LATERAL FLATNESS-BASED CONTROL

2.1 3DoF NonLinear Two Wheels Vehicle Model

The 3DoF-NLTWVM in Figure 1, which is used to design the combined control law, provides an interesting approximation of the longitudinal and lateral dynamics of the vehicle in normal driving situations. See Table 1 for the notations.

Table 1.

Symbol	Variable name
V_x	longitudinal speed $[km.h]$
V_y	lateral speed $[km.h]$
a_x	longitudinal acceleration $[m/s^2]$
a_y	lateral acceleration $[m/s^2]$
$\dot{\psi}$	yaw rate $[rad/s]$
ψ	yaw angle $[rad]$
β	sideslip angle $[rad]$
$\alpha_{f,\,r}$	front and rear tire slip angles $[rad]$
ω_i	wheel angular speed of the wheel i $[rad/s]$
T_ω	wheel torque $[Nm]$
δ	wheel steer angle $[deg]$
C_f, C_r	front and rear cornering stiffnesses $[N.rad\ \ ^1]$
F_{xi}	longitudinal forces in vehicle coordinate $[N]$
F_{yi}	lateral forces in vehicle coordinate $[N]$
F_{xf}	front longitudinal forces in tire coordinate $[N]$
F_{yf}	front lateral forces in tire coordinate $[N]$
R	tire radius $[m]$
g	acceleration due to gravity $[m/s^2]$
L_f	distances from the CoG to the front axles $[m]$
L_r	distances from the CoG to the rear axles $[m]$
I_z	yaw moment of inertia $[kgm^2]$
I_r	wheel moment of inertia $[kgm^2]$
m	vehicle mass $[kgm^2]$
M_z	yaw moment $[Nm]$

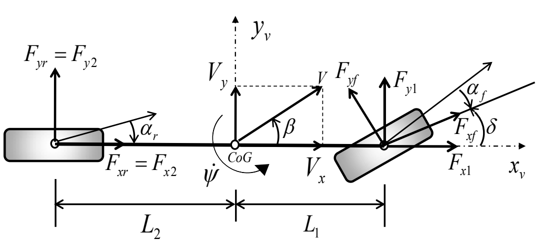

Fig. 1. Nonlinear two wheels vehicle control model

The corresponding dynamical equations read:

$$\begin{cases} ma_x = m(\dot{V}_x \quad \dot{\psi}V_y) = (F_{x1} + F_{x2}) \\ ma_y = m(\dot{V}_y + \dot{\psi}V_x) = (F_{y1} + F_{y2}) \\ I_z\ddot{\psi} = M_{z1} + M_{z2} \end{cases}$$

If a linear tire model with small slip angles is assumed (see Menhour *et al.* [2011] for more details), C_r (resp. C_f) denoting the cornering stiffness coefficient of the rear (resp. front) wheel, and ω_r (resp. ω_f) being the angular velocity of the rear (resp. front) wheel, the previous system can be rewritten:

$$\dot{x} = f(x,t) + g(x,t)u \tag{1}$$

with

$$f(x,t) = \begin{bmatrix} \dot{\psi}V_y - \frac{I_r}{mR}(\dot{\omega}_r + \dot{\omega}_f) \\ -\dot{\psi}V_x + \frac{1}{m}\left(-C_f\left(\frac{V_y + L_f\dot{\psi}}{V_x}\right) - C_r\left(\frac{V_y - L_r\dot{\psi}}{V_x}\right)\right) \\ \frac{1}{I_z}\left(-L_fC_f\left(\frac{V_y + L_f\dot{\psi}}{V_x}\right) + L_rC_r\left(\frac{V_y - L_r\dot{\psi}}{V_x}\right)\right) \end{bmatrix}$$

$$g(x,t) = \begin{bmatrix} \frac{1}{mR} & \frac{C_f}{m}\left(\frac{V_y + L_f\dot{\psi}}{V_x}\right) \\ 0 & (C_fR - I_r\dot{\omega}_f)/mR \\ 0 & (L_fC_fR - L_fI_r\dot{\omega}_f)/I_zR \end{bmatrix}, \ x = \begin{bmatrix} V_x \\ V_y \\ \dot{\psi} \end{bmatrix}$$

$u = [\, u_1 = T_\omega, \ u_2 = \delta \,]^T$.

2.2 Flatness property

The system $\dot{x} = f(x,u)$, where $x = (x, \cdots, x_n) \in \mathbb{R}^n$ and $u = (u, \cdots, u_m) \in \mathbb{R}^m$, is said to be *differentially flat* (see Fliess *et al.* [1995, 1999], and Åström *et al.* [2008], Lévine [2009], Sira-Ramírez *et al.* [2004]) if, and only if,

- there exists a vector-valued function h such that
$$y = h(x, u, \dot{u}, \cdots, u^{(r)}) \tag{2}$$
where $y = (y, \cdots, y_m) \in \mathbb{R}^m$, $r \in \mathbb{N}$;
- the components of $x = (x, \cdots, x_n)$ and $u = (u, \cdots, u_m)$ may be expressed as
$$x = A(y, \dot{y}, \cdots, y^{(r_x)}), \quad r_x \in \mathbb{N} \tag{3}$$
$$u = B(y, \dot{y}, \cdots, y^{(r_u)}), \quad r_u \in \mathbb{N} \tag{4}$$

Remember that y in Equation (2) is called a *flat output.*

2.3 Flatness-based longitudinal and lateral control

Problem 1. Introduce the outputs:

$$\begin{cases} y_1 = V_x \\ y_2 = L_fmV_y - I_z\dot{\psi} \end{cases} \tag{5}$$

We want to show that the longitudinal speed y_1 and the angular momentum y_2 of a point on the axis between the centers of the front and rear axles are flat outputs.

Proof 1. Some algebraic manipulations (see Menhour *et al.* [2011] for more details) yield:

$$x = [\, V_x \ V_y \ \dot{\psi} \,]^T = A(y_1, y_2, \dot{y}_2) =$$

$$\begin{bmatrix} y_1 \\ \frac{y_2}{L_fm} \quad \frac{I_z}{L_fm}\left(\frac{L_fmy_1\dot{y}_2 + C_r(L_f + L_r)y_2}{C_r(L_f + L_r)(I_z \quad L_rL_fm) + (L_fmy_1)^2}\right) \\ \left(\frac{L_fmy_1\dot{y}_2 + C_r(L_f + L_r)y_2}{C_r(L_f + L_r)(I_z \quad L_rL_fm) + (L_fmy_1)^2}\right) \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \Delta(y_1, y_2, \dot{y}_2)\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \Phi(y_1, y_2, \dot{y}_2) \tag{7}$$

The flatness property holds if the matrix $\Delta(y_1, y_2, \dot{y}_2)$ is invertible:

$$\det(\Delta(y_1, y_2, \dot{y}_2)) = \Delta_{11}\Delta_{22} - \Delta_{21}\Delta_{12} =$$
$$\frac{(I_\omega\dot{\omega}_f - C_fR)\left(L_f^2y_1^2m^2 - Cr(L_f + L_r)L_rL_fm + C_rI_zL\right)}{I_zR^2y_1m^2} \neq 0 \tag{8}$$

This determinant, which only depends on the longitudinal speed $y_1 = V_x$, is indeed nonzero: