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The Effect of Sensor Modality on Posterior Cramer-Rao Bounds for Simultaneous Localisation and Mapping ⋆

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Abstract: This paper applies Posterior Cramer-Rao Bound theory to the SLAM problem to measure the information supplied by different sensor modalities over time. Range-only, bearing-only and full range-bearing sensors were considered, as well as the gain in information achieved by using multiple sensors in centralized co-operative SLAM. An efficient recursive formula was used to compute the bound for a set of simulated scenarios, and its validity verified by comparing the bound with the second-order error performance of FastSLAM 2.0 and the EKF.

Keywords: SLAM, localisation, map-building, estimation theory, filtering, Cramer-Rao bounds, Bayesian bounds, centralized, multi-agent, co-operative, range-only, bearing-only

1. INTRODUCTION

The Simultaneous Localisation and Mapping (SLAM) problem requires a mobile robot to use measurements from an on-board sensor to construct a map of its environment as it moves through it, while simultaneously estimating its own location within the map. This is an essential capability for any robot to navigate and function truly autonomously in a unknown environment. While several approaches to SLAM exist (Thrun et al., 2005), developing efficient and robust SLAM algorithms for real-world applications continues to generate significant interest.

SLAM is fundamentally a dynamic state estimation problem. This study focuses on *feature-based SLAM*, which models the features of interest in the environment as discrete points called *landmarks*. The *map* is a vector containing the landmark co-ordinates. Feature-based SLAM is typically formulated as a discrete-time, non-linear filtering problem, in which the state vector consists of the robot's position and orientation (the *pose*), and the map. As the pose evolves stochastically with time, noisy measurements of the landmarks must be used to estimate the full state.

This paper applies a useful result from estimation theory to the SLAM problem. The *Cramer-Rao bound* is a lower bound on the mean square error (MSE) of an unbiased estimator. Since any SLAM algorithm is at its core, an estimator, the Cramer-Rao Bound provides a natural benchmark for assessing its second-order error performance.

The original Cramer-Rao Bound applies to the estimation of a deterministic but unknown parameter. A Bayesian version of the bound exists for estimating random parameters (Van Trees and Bell, 2013), which is often referred to as the *Posterior Cramer-Rao Bound* (PCRB) or *Bayesian Cramer-Rao Bound*. A Bayesian framework relaxes the assumption that the estimator is unbiased, and the bound can thus be safely applied to non-linear filtering problems. Tichavsky et al. (1998) used the PCRB to derive an efficient recursive formula that computes a lower bound on the MSE at each time-step. It is this recursive PCRB that we apply to SLAM.

The recursive PCRB has been used extensively in the target tracking literature as a benchmark that quantifies optimal filtering performance (Hernandez et al., in press). Although the bound is typically not achievable, a filter achieving accuracy close to the bound has little room for improvement.

The PCRB itself is a covariance matrix, the inverse of which is the *Bayesian Information Matrix* (BIM). The BIM intuitively measures the amount of information about the state that can be extracted from the observations and any prior knowledge, based on the assumed probability distributions (Bar-Shalom et al., 2004). It plays a similar role to the Fisher Information Matrix for a deterministic parameter. Thus the recursive PCRB represents the availability of information over time, with a low valued bound reflecting high information content. This information measure is independent of the filtering algorithm chosen, and limits the accuracy that can be achieved with the given hardware and sensors. We use the PCRB to investigate the flow of information provided by different sensor modalities, and the gains achieved by using multiple sensors.

While some sensors measure both range and bearing with relatively high accuracy (e.g. lidar), others have just one dominant mode of sensing. Typically active sensors will

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obtain accurate range measurements, but some (e.g. radar, sonar) can have a low resolution in bearing. Conversely, certain passive sensors (e.g. monocular vision cameras) will obtain accurate bearing measurements but cannot measure range. Economic or environmental constraints may necessitate the use of these types of sensors. We therefore consider the PCRB for range-only (RO), bearing-only (BO), and full range-bearing (RB) sensors, which covers a wide range of the available sensing modalities.

The use of multiple sensors via co-operative SLAM (C-SLAM) has gained a lot of attention, as it provides redundancy and the potential for faster exploration. Increasing the number of sensors should intuitively provide more information about the state, and this effect is investigated using the PCRB. We assumed a centralized architecture, in which all agents send their estimates to a fusion centre that performs the filtering.

The PCRB has not previously seen widespread use in the context of SLAM. Jiang et al. (2005) computed the PCRB for a single robot traversing a circular path, however their results showed the EKF MSE starting below the bound, which indicates improper initialization. Ahmad and Namerikawa (2011) used the PCRB to derive upper and lower bounds on the EKF error covariance for SLAM with intermittent measurements. Feder et al. (1999) used the BIM as a performance metric for *Active SLAM*, closing the loop by applying controls that maximise the information gain.

2. MULTI-AGENT SLAM FORMULATION

The SLAM problem is formulated below for an arbitrary number of agents, n. We adopt a centralized architecture which follows the formulation of Fenwick et al. (2002), except that we assume agents do not observe each other. This means that improvements in the bound are solely due to gaining more information about the map. It is assumed that the agents have access to the same reference frame. Since we seek a lower bound on MSE, which is an upper bound on accuracy, we also assume the best case scenario of perfect data association, implying that the agents can uniquely identify each landmark that is observed. We consider plane motion in a 2-D environment for simplicity.

Let $s^i[k] = [X^i[k] \quad Y^i[k] \quad \phi^i[k]]^T$ represent the pose of agent i at time-step k, where $(X^i[k], Y^i[k]) \in \mathbb{R}^2$ indicates position, and $\phi^i[k] \in \mathbb{R}$ is the angle at which the robot is oriented relative to the X-axis.

It is common practice in single-agent SLAM to define the initial pose of the robot as the origin of the global co-ordinate system. We define the origin as the initial pose of the first agent, so that $s^1[0] = [0\ 0\ 0]^T$, and we assume that all other agents have perfect knowledge of their initial pose with respect to this co-ordinate system.

Let $\ell_j = [X_j \quad Y_j]^T$ be the coordinates of landmark j, and define the map as $\mathbf{m} = \left[\ell_1^T, \ldots, \ell_m^T\right]^T$. Note that in general, superscripts will relate to agents and subscripts to landmarks, unless otherwise specified.

2.1 Dynamic Model

Define the state vector as $\mathbf{x}[k] = \left[s^1[k]^T \ \ldots \ s^n[k]^T \ \mathbf{m}^T\right]^T$, and let $u^i[k]$ be the control input to agent i. Let function $f_s^i(s^i[k], u^i[k])$ model the agent's dynamics so that the full state equation is given by

$$\begin{bmatrix} s^1[k] \\ \vdots \\ s^n[k] \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} f_s^1(s^1[k-1], u^1[k-1]) \\ \vdots \\ f_s^n(s^n[k-1], u^n[k-1]) \\ \mathbf{m} \end{bmatrix} + \begin{bmatrix} v^1[k-1] \\ \vdots \\ v^n[k-1] \\ 0 \end{bmatrix}, \quad (1)$$

where $v^i[k] \sim \mathcal{N}(0, Q^i[k])$ is a random variable that models uncertainty in the dynamics. This is called the process noise, and we assume it is independent of the state and uncorrelated in time. The state equation can be condensed to form

$$\mathbf{x}[k] = \mathbf{f}(\mathbf{x}[k-1], \mathbf{u}[k-1]) + \mathbf{v}[k-1], \quad (2)$$

$$\mathbf{v}[k-1] \sim \mathcal{N}\left(0, \mathbf{Q}[k-1]\right). \quad (3)$$

2.2 Observation Model

The measurement of landmark j by sensor i at time k is given by the observation model

$$z_j^i[k] = h(s^i[k], \ell_j) + w_j^i[k], \quad (4)$$

$$w_j^i[k] \sim \mathcal{N}(0, R^i[k]), \quad (5)$$

where $w_j^i[k]$ is the measurement noise, also uncorrelated in time and independent of the state.

The observation function for a typical range-bearing sensor is given by

$$h(s^i[k], \ell_j) = \begin{bmatrix} \sqrt{(X_j - X^i[k])^2 + (Y_j - Y^i[k])^2} \\ \operatorname{atan2}(Y_j - Y^i[k], X_j - X^i[k]) - \phi^i[k] \end{bmatrix}. \quad (6)$$

Define $\mathbf{z}[k]$ to be the vector containing all measurements from all sensors at time k. Let

$$\mathbf{h}_k(\mathbf{x}[k]) = \begin{bmatrix} h(s^{i_1}[k], \ell_{j_1}) \\ \vdots \\ h(s^{i_N}[k], \ell_{j_M}) \end{bmatrix}$$

select only the observations corresponding to the landmarks detected at time k and the agents that observed them, which causes the dimension of $\mathbf{z}[k]$ to change accordingly.

The observation model can then be written as

$$\mathbf{z}[k] = \mathbf{h}_k(\mathbf{x}[k]) + \mathbf{w}[k], \quad (7)$$

$$\mathbf{w}[k] \sim \mathcal{N}(0, \mathbf{R}[k]). \quad (8)$$

We assume $\mathbf{w}[k]$ and $\mathbf{v}[k]$ are independent of each other and the state.

It should be noted that this does not precisely model the limited detection range of the sensor. To do this more rigorously, $\mathbf{z}[k]$ could remain a constant dimension, but the $w_j^i[k]$ corresponding to landmarks outside the detection range be assigned an infinite covariance reflecting the lack of information supplied. This would make $\mathbf{R}[k]$ state-dependant, which is not permitted by the recursive PCRB equations that we use (refer to Section 3.1). The model (7) - (8) is therefore an approximation which has independent measurement noise.