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Consensus Problem in General Linear Multiagent Systems Under Stochastic Topologies

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Abstract: Consensus control of a class of multiagent systems with general linear dynamics is studied. Based on solution of some linear matrix inequalities, a protocol is obtained which guarantees achieving consensus among agents in the presences of stochastically switching topologies. By invoking the concept super-martingales, it is shown that if the probability of the network connectivity is not zero, the agents reach almost sure consensus upon their state vectors. Despite existing results in the literature for consensus control of general linear multiagent systems under stochastic networks, the proposed consensus protocol in this paper requires no knowledge on the set of feasible topologies, and this issue significantly decreases the design computational costs. Simulation results for a diving consensus problem among a team of autonomous underwater vehicles validate the proposed consensus protocol.

Keywords: Almost sure consensus, general linear dynamics, multiagent systems, stochastic networks.

1. INTRODUCTION

In each distributed cooperative mission, there are some quantities of interests should be identical for all agents. These quantities can be determined by a supervisor when the agents are commanded via a central controller. However, in decentralized autonomous multiagent systems (MASs), the agents should reach consensus upon these quantities via an interaction protocol. Hence, the consensus problem is one of interesting topics in MASs decentralized control with a broad range of applications from formation and flocking control of multivehicle systems to data fusion in sensory networks [Ren (2007); Rezaee and Abdollahi (2015b, 2014); Olfati-Saber (2007)].

The consensus problem first was studied for networks of first-order kinematics from various perspectives in which velocities were the control commands of the agents [Olfati-Saber and Murray (2004); Olfati-Saber et al. (2007); Ren et al. (2007)]. However, in practice, a large number of systems and vehicles are actuated by forces and torques. In this condition, it is necessary to consider second-order dynamics to describe the agents behaviors, and therefore the consensus problem in second-order dynamics was widely investigated [Ren (2007); Cheng et al. (2011); Qin et al. (2012)]. Furthermore, in some systems and vehicles such as unmanned helicopters and autonomous underwater vehicles (AUVs) [Kaloust et al. (1997); Saboori and Khorasani (2014)], the agents just can be described by higher-order differential equations. Accordingly, the consensus problem in high-order MASs has been a more challenging problem studied in the literature as well [He and Cao (2011); Rezaee and Abdollahi (2015a, 2016); Saboori and Khorasani (2014)].

One of main problems in consensus control of networked systems is reaching consensus in the presences of stochastic topologies. For instance, in a rendezvous problem among a team of mobile agents, due to environmental obstacles or stochastic properties of electronic, power, and communication devices, some links may fail and rebuild stochastically. Hence, the communication topology of the MAS is stochastically switching, and in this condition, the existing protocols for deterministic networks cannot guarantee achieving consensus in the MAS. Therefore, some studies have been dedicated to the consensus problem in MASs under stochastic topologies. Most of those studies have been devoted to MASs modeled by simple first-order kinematics. For instance, in [Hatano and Mesbahi (2005)], achieving consensus under undirected networks was studied. Those results were extended to directed networks in [Wu (2006)] and [Fagnani and Zampieri (2008)] and to weighted directed networks in [Porfiri and Stilwell (2007)] and [Porfiri et al. (2008)] as well, and necessary and sufficient conditions for achieving consensus in terms of ergodicity of networks topologies were studied in [Tahbaz-Salehi and Jadbabaie (2008)] and [Tahbaz-Salehi and Jadbabaie (2010)]. In addition to the aforementioned approaches dedicated to first-order stochastic networks, a few studies have been devoted to stochastic networks with higher-order models as well. For instance, in [Li et al. (2015)], by considering some restrictive conditions on the average of switching

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topologies, the consensus problem over second-order MASs with Lipschitz nonlinearities was addressed. The consensus problem over high-order MASs with general linear models under stochastic networks was studied in [Vengertsev et al. (2015)]. However, in that approach, the eigenvalues of the coupling matrices of all feasible topologies should be available to design the gains of a consensus protocol. It is clear that increasing the number of agents led to an increase in the number of feasible topologies. For instance, the number of feasible topologies for networks of four and six agents may be 2^6 and 2^{15}, respectively. In that condition, to design a consensus protocol, in order to analyze the eigenvalues of the coupling matrices, a huge computation was required. A similar problem existed with the consensus protocol proposed in [Zhou and Xiao (2013)]. Moreover, in [Sun et al. (2012)], a criterion for stochastic cooperative convergence of second- and high-order stochastic networks was proposed. However, no strategy to guarantee achieving consensus was proposed in that paper.

Considering the above-mentioned issues, because of using the information of the set of feasible topologies in designing consensus protocols, the existing results for consensus control of stochastic networks with high-order dynamical models were not easily scalable. Considering this problem, in this paper, a consensus protocol for a network of general linear dynamics is studied. The network topology is considered dynamically changing and existence of communication links among the agents is considered stochastic and stochastically independent. Based on the concept of super-martingales, we will show that if the probability of the network connectivity is not zero, almost sure consensus can be guaranteed, while the network topology may not be connected constantly. Despite the existing results in the literature, the consensus protocol proposed in this paper can be designed independent of the information of the set of the network feasible topologies.

The paper organization is as follows. Preliminaries are provided in the next section. The main results are presented in Section 3. Simulation results are given in Section 4, and Section 5 concludes the paper.

2. PRELIMINARIES

In this section, notations and some concepts and definitions on graph theory and stochastic processes are presented.

2.1 Notations

$\mathbb{R}$ and $\mathbb{R}_+$ express the set of real and positive real numbers, respectively. I_n is an $n \times n$ identity matrix. $\mathbf{1}_n$ expresses an $n \times 1$ vector of ones. $\mathbf{0}_{n\times m}$ denotes an $n \times m$ matrix of zeros. lim stands for limit. $\otimes$ stands for the Kronecker product. Let $M > 0$ and $M \geq 0$ if M is symmetric positive definite and symmetric positive semi-definite, respectively. Accordingly, M is symmetric negative (semi-)definite if $-M$ is symmetric positive (semi-)definite. $v(p)$ denotes a stochastic switching parameter where $p \in \{1, 2, \dots, n_v\}$ is the index associated with the switching set (with n_v members) and changes stochastically over time. $\|\cdot\|$ denotes the standard Euclidian norm. $\mathbb{E}\{\cdot\}$ and $\mathbb{P}\{\cdot\}$ are the expected value and probability of a stochastic variable, respectively. $\mathbb{E}\{X|H\}$ is the conditional expected value of X given an event H. rank$(\cdot)$ denotes the rank. nullity$(\cdot)$ denotes the dimension of the null space, and 'a.s.' represents almost surely.

2.2 Graph Theory

The MAS network topology is expressed by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, 2, \dots, N\}, N > 1$, is the set of N nodes or agents, $\mathcal{E} = \{(i,j) \big| i \neq j, i, j \in \mathcal{V}\}$ expresses the set of edges or links which an edge (i, j) means that the ith agent exchanges information with the jth one, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix associated with $\mathcal{G}$ where $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathcal{E}$ and $a_{ij} = a_{ji} = 0$ otherwise. Moreover, an undirected graph is connected if all of its pair nodes are connected via a sequence of edges called a path.

Now, the Laplacian matrix $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{N\times N}$ associated with $\mathcal{G}$ can be stated as

$$\ell_{ij} = \begin{cases} \sum_{i=1, i\neq j}^{N} a_{ij} & i = j, \\ -a_{ij} & i \neq j \end{cases}$$

which has rows/columns with zero entries summation. It can be said that in the case of connected graph, $\mathcal{L}$ has a zero eigenvalue and other eigenvalues of $\mathcal{L}$ are in the open right half plane. Furthermore, the right and left eigenvectors associated with this zero eigenvalue are $\mathbf{1}_N/\sqrt{N}$ [Li et al. (2013)].

2.3 Stochastic Processes

We model a stochastic process with the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ where Ω denotes the space of events, $\mathcal{F}$ is a σ-algebra on Ω defined as subsets of Ω closed under union and complement operations, and $\mathbb{P}$ denotes a probability measure on $(\Omega, \mathcal{F})$ where $0 \leq \mathbb{P}\{\cdot\} \leq 1$ and $\mathbb{P}\{\Omega\} = 1$ [Williams (1991)].

A filtration $\{\mathcal{F}_t, t \geq 0\}$ on $(\Omega, \mathcal{F}, \mathbb{P})$ can be defined as a family of sub σ-algebras of $\mathcal{F}$ where

$$\mathcal{F}_s \subset \mathcal{F}_t, s < t.$$

In this condition, a stochastic process $X = \{X(t), t \geq 0\}$ is called to be adapted to the filtration $\{\mathcal{F}_t\}$ if $X(t)$ is $\mathcal{F}_t$-measurable for each $t \geq 0$, i.e., $X(t)$ only depends on $\{\mathcal{F}_s, 0 \leq s \leq t\}$. Now, a process X is a called a super-martingale relative to $\{\mathcal{F}_t\}$ and $\mathbb{P}$ if (Mahmoud et al., 2003, Sec. 4.4.1),(Williams, 1991, Chap. 10)

i) X is adapted to the filtration $\{\mathcal{F}_t\}$,
ii) $\mathbb{E}\{|X(t)|\} < \infty, \forall t$,
iii) $\mathbb{E}\{X(t)|\mathcal{F}_s\} \leq X(s), t > s$.

Now, we introduce two well known criteria for convergence of stochastic variables [Tempo et al. (2013)]:

- A stochastic variable $X(t)$ converges to X_∞ in the sense of probability if for any $\varepsilon \in \mathbb{R}_+$,
$$\lim_{t\to\infty} \mathbb{P}\{|X(t) - X_\infty| \geq \varepsilon\} = 0.$$
- A stochastic variable $X(t)$ almost surely converges to X_∞ if
$$\mathbb{P}\{\lim_{t\to\infty} X(t) = X_\infty\} = 1.$$