

Robust Algorithm Using Delay for Multi-Agent Systems¹

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Abstract: The paper describes the robust control algorithm for linear multi-agent systems under parametric and structural uncertainties and external unmeasured disturbances. The proposed algorithm is based on left hand side differences for estimation of the derivatives. The resulting algorithm ensures required accuracy of difference between the plant output and the reference signal. The modeling results illustrate the effectiveness of the algorithm.

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1. INTRODUCTION

Design of simple control systems under parametric uncertainties and external disturbances when only plant output is available for measurement is important problem of control theory and practice. To construct such control schemes many solutions have been proposed in this regard. If the plant relative degree exceeds one then implementation of adaptive and robust control systems requires the estimation of the derivatives of the plant input and output.

The Luenberger observer is widely used to estimate the plant model state vector with known parameters (Luenberger, 1966). The Kalman filter allows to estimate the state vector of a dynamical system under disturbances and noises (Kalman, 1960). Under parametric uncertainty and external disturbances in the paper (Esfandiary and Khalil, 1992) so-called high-gain observer is proposed. Other form of an observer with a high gain is considered in (Slotine et al., 1987). In (Utkin, 1992, Han, 1995) the robust sliding-mode observer is proposed. In (Wang and Gao, 2003) the authors design a nonlinear extended state observer based on a generalization of a high-gain observer and a sliding-mode observer. In (Veluvolu et al., 2011) the observers (Luenberger, 1966, Esfandiary and Khalil, 1992, Utkin, 1992, Han, 1995, Wang and Gao, 2003) are investigated for second order dynamical systems. In paper (Veluvolu et al., 2011) a comparative analysis for each observer under different kinds of parametric uncertainties, external disturbances, and noises are given. Robust observers have found many applications in the synthesis of control systems under uncertainties. In (Atassi and Khalil, 1999) the stabilization of nonlinear dynamical plants is considered by using the control law that depends on estimates of plant output derivatives obtained

with a high-gain dynamical observer. The papers (Bazylev, Pyrkin, 2013, Bazylev, Zimenko, Margun et al., 2014, Bazylev, Margun, Zimenko, 2014, Pyrkin, Bobtsov, Kolyubin et al., 2013, Furtat, 2013, Furtat and Putov, 2013, Furtat, 2014,) are used the adaptive control with high gain observers.

Analysis of the above results shows that the designed algorithms use integrators for estimation of the derivatives. Unlike these results we design the observer using left-hand side differences. This observer allows designing the robust control system without using integrators but using delays. For algorithm synthesis the results of (Furtat, 2015) will be used.

The paper is organized as follows. The problem statement is presented in Section 2. In Section 3 we design the control algorithm for multi-agent systems with known relative degree of each agent. In Section 3 we propose the control algorithm for multi-agent systems with unknown relative degree of each agent. In Section 5 we consider simulation results and discuss an efficiency of the proposed scheme. Concluding remarks are given in Section 6. Appendix A gives the proof of the control system.

2. PROBLEM STATEMENT

Let each agent S_i of the multi-agent system S be described by the following equation

$$Q_i(p)y_i(t) = k_i R_i(p)u_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N Y_{ij}(p)y_j(t) + f_i(t), \quad (1)$$

$$p^{k-1}y(0) = y_{0k}, \quad k = 1, \dots, n, \quad i = 1, \dots, N,$$

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where $y_i(t) \in R$ are local outputs, $u_i(t) \in R$ are local inputs, $f_i(t) \in R$ are local uncontrollable bounded disturbances, $Q_i(p)$, $R_i(p)$ and $Y_{ij}(p)$ are linear differential operators with unknown coefficients, $\deg Q_i(p) = n_i$, $\deg R_i(p) = m_i$, $\deg Y_{ij}(p) = n_{ij}$, $n_i > n_{ij}$, $k_i > 0$, $i, j = 1, \dots, N$, y_{0k} , $k = 1, \dots, n$ are unknown initial conditions, $p = d/dt$.

Assume that the coefficients of $Q_i(p)$, $R_i(p)$, $Y_{ij}(p)$, k_i belong to a known compact set Ξ and the polynomials $R_i(\lambda)$ are Hurwitz, where $i, j = 1, \dots, N$, λ is a complex variable.

The problem is to design the control system such that the following condition holds

$$|y_i(t) - y_{mi}(t)| < \delta \text{ for } t > T, \quad (2)$$

where $i = 1, \dots, N$, $\delta > 0$ is an accuracy, $T > 0$ is a transient time.

3. ALGORITHM FOR STRUCTURALLY CERTAINTY AGENTS

First, consider the case where orders of the operators $Q_i(p)$, $R_i(p)$ and $Y_{ij}(p)$, $i, j = 1, \dots, N$ are known. Taking into account equations (1) and (2) rewrite the tracking error $e_i(t) = y_i(t) - y_{mi}(t)$ in the form

$$Q_i(p)e_i(t) = k_i R_i(p)u_i(t) + \sum_{\substack{j=1, \\ j \neq i}}^N Y_{ij}(p)y_j(t) + f_i(t) - Q_i(p)y_{mi}(t), \quad i = 1, \dots, N. \quad (3)$$

Since the derivatives of $y_i(t)$ and $u_i(t)$ are not available, then we introduce the control law in the form

$$u_i(t) = -\alpha_i \sum_{v=0}^{\gamma_i} d_{vi} \bar{e}_i^{(v)}(t), \quad i = 1, \dots, N, \quad (4)$$

where $\alpha_i > 0$ are design parameters, the coefficients $d_{0i}, d_{1i}, \dots, d_{\gamma_i, i}$ are chosen such that the polynomial

$D_i(\lambda) = d_{\gamma_i, i} \lambda^{\gamma_i} + d_{\gamma_i-1, i} \lambda^{\gamma_i-1} + \dots + d_{1i} \lambda + d_{0i}$ will be Hurwitz, $\gamma_i = n_i - m_i \geq 1$, γ_i is a relative degree of i th agent, $\bar{e}_i^{(v)}(t)$ is an estimate of the v th derivative of the signal $e_i(t)$.

Substituting (4) into (3), rewrite (3) as follows

$$F_i(p)e_i(t) = \psi_i(t) + \sum_{\substack{j=1, \\ j \neq i}}^N Y_{ij}(p)y_j(t), \quad i = 1, \dots, N, \quad (5)$$

where $F_i(p) = Q_i(p) + \alpha_i k_i R_i(p) D_i(p)$,

$$\psi_i(t) = f_i(t) - Q_i(p)y_{mi}(t) + \alpha_i k_i R_i(p) \left(D_i(p)e_i(t) - \sum_{v=0}^{\gamma_i} d_{vi} \bar{e}_i^{(v)}(t) \right).$$

Since we know the compact set Ξ , then there exist coefficients α_i and polynomials $D_i(\lambda)$ such that the polynomials $F_i(\lambda)$ are Hurwitz, $i = 1, \dots, N$.

For implementation of control law (4) we use the following algorithm

$$\begin{aligned} \bar{e}_i(t) &= e_i(t), \bar{e}_i^{(1)}(t) = \frac{\bar{e}_i(t) - \bar{e}_i(t-h)}{h}, \\ \bar{e}_i^{(2)}(t) &= \frac{\bar{e}_i^{(1)}(t) - \bar{e}_i^{(1)}(t-h)}{h}, \\ &\vdots \\ \bar{e}_i^{(\gamma_i)}(t) &= \frac{\bar{e}_i^{(\gamma_i-1)}(t) - \bar{e}_i^{(\gamma_i-1)}(t-h)}{h}, \quad i = 1, \dots, N. \end{aligned} \quad (6)$$

Substituting (6) into (4), rewrite control law (4) in the form

$$u_i(t) = -\alpha_i \sum_{v=0}^{\gamma_i} \left[\frac{d_{vi}}{h^v} \sum_{j=0}^v (-1)^j C_v^j e(t-jh) \right], \quad i = 1, \dots, N, \quad (7)$$

where $C_v^j = \frac{v!}{j!(v-j)!}$.

Rewrite the function $\psi_i(t)$ as the sum $\psi_i(t) = \alpha_i k_i R_i(p)g_i(t) + \varphi_i(t)$, where

$$\begin{aligned} \varphi_i(t) &= f_i(t) - Q_i(p)y_{mi}(t) + \sum_{\substack{j=1, \\ j \neq i}}^N Y_{ij}(p)y_{mj}(t), \\ g_i(t) &= D_i(p)e_i(t) - \sum_{v=0}^{\gamma_i} d_{vi} \bar{e}_i^{(v)}(t). \end{aligned}$$

Then equation (5) can be written as

$$\begin{aligned} \dot{\varepsilon}_i(t) &= A_i \varepsilon_i(t) + \alpha_i k_i B_{1i} g_i(t) + B_{2i} \varphi_i(t) \\ &+ \sum_{\substack{j=1, \\ j \neq i}}^N B_{3ij} \varepsilon_j(t), \quad e_i(t) = L_i \varepsilon_i(t), \quad i = 1, \dots, N, \end{aligned} \quad (8)$$

where $A_i, B_{1i}, B_{2i}, B_{3ij}$ and $L_i = [1 \ 0 \ \dots \ 0]$ are matrices obtained at the transition from (5) to (8).

Taking into account control law (7) and the structure of the function $g_i(t)$, rewrite the first equation of system (8) in the form

$$\begin{aligned} \dot{\varepsilon}_i(t) &= A_i \varepsilon_i(t) + \alpha_i k_i B_{1i} \rho_i^T \varepsilon_i(t) - \alpha_i k_i B_{1i} \\ &\times \left[\sum_{v=0}^{\gamma_i} \frac{d_{vi}}{h^v} L_i \varepsilon_i(t) + \sum_{v=1}^{\gamma_i} \sum_{j=1}^v \frac{d_{vi}}{h^v} (-1)^j C_v^j L_i \varepsilon_i(t-jh) \right] \\ &+ \sum_{\substack{j=1, \\ j \neq i}}^N B_{3ij} \varepsilon_j(t) + B_{2i} \varphi_i(t), \quad i = 1, \dots, N, \end{aligned} \quad (9)$$

where the vectors ρ_i are composed of coefficients of the operator $D_i(p)$.

Let us denote

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