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Adaptive H_∞ Consensus Control of Multi-Agent Systems on Directed Graph by Utilizing Neural Network Approximators

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Abstract: Design methods of adaptive H_∞ consensus control of multi-agent systems composed of the first-order and the second-order regression models on directed network graphs and with nonlinear terms by utilizing neural network approximators, are presented in this paper. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters and approximate and algorithmic errors in neural network estimation schemes are regarded as external disturbances to the process. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable consensus tracking is achieved approximately via adaptation schemes and L_2-gain design parameters.

1. INTRODUCTION

Among plenty of cooperative control problems of multi-agent systems, distributed consensus tracking with limited communication networks, has been a basic and important topic, and various research results have been reported for various processes and under various conditions (for example, Kingston et al. (2005), Olfati-Saber et al. (2007), Ren et al. (2007), Cao and Ren (2011)). Adaptive control or sliding mode control methodologies were also proposed in order to deal with uncertainties of agents, and stability of control systems was assured via Lyapunov function analysis. Furthermore, robustness properties of the control schemes were also discussed. However, most of those results are restricted to simple linear processes, and so much attention does not have been paid on control performance such as optimal property or transient performance. Furthermore, the case of multi-agent systems with unknown and different system parameters on directed information network graphs, does not have been discussed in detail (Mei et al. (2014a), Mei et al. (2014b), Li and Ding (2015)).

The purpose of the paper is to present design methods of adaptive H_∞ consensus control of multi-agent systems composed of the first-order and the second-order regression models on directed information network graphs, and with nonlinear terms based on the notion of inverse optimality. This is an extension of the work (Miyasato (2014)) to the case of directed information networks, and also an extension of the one (Miyasato (2015)) to nonlinear regression models, where neural network approximators are introduced to estimate nonlinear parametric elements. The proposed control schemes are derived as solutions of certain H_∞ control problems, where estimation errors of tuning parameters and approximate and algorithmic errors in neural network estimation schemes are regarded as external disturbances to the process. By choosing that control strategy, the present work provides an extended version of consensus control of nonlinear systems on directed graphs with consideration of control performance.

2. MULTI-AGENT SYSTEM AND INFORMATION NETWORK

Mathematical preliminaries on information network graph of multi-agent systems are summarized (Ren and Cao (2011), Mei et al. (2014a), Mei et al. (2014b)). As a model of interaction among agents, a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is considered, where $\mathcal{V} = \{1, \cdots, N\}$ is a node set, which corresponds to a set of agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set. An edge $(i, j) \in \mathcal{E}$ indicates that agent j can obtain information from i, but not necessarily vice versa. In the edge (i, j), i is called as a parent node and j is called as a child node, and the in-degree of the node i is the number of its parents, and the out-degree of i is the number of its children. Especially, an agent which has no parent (or with the in-degree 0), is called as a root. A directed path is a sequence of edges in the form (i_1, i_2), (i_2, i_3), $\cdots$ $(\in \mathcal{E})$, where $i_j \in \mathcal{V}$. The directed graph is called strongly connected, if there is always a directed path between every pair of distinct nodes. A directed tree is a directed graph in which every node has exactly one parent except for a unique root, and the root has directed paths to all other node. An directed spanning tree $\mathcal{G}_S = (\mathcal{V}_S, \mathcal{E}_S)$ of the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a subgraph of $\mathcal{G}$ such that $\mathcal{G}_S$ is a directed tree and $\mathcal{V}_S = \mathcal{V}$.

Associated with the edge set $\mathcal{E}$, a weighted adjacency matrix $A = [a_{ij}] \in \mathbf{R}^{N\times N}$ is introduced, and the entry a_{ij} of it is defined such as $a_{ij} > 0$: when $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$: otherwise. For the adjacency matrix $A = [a_{ij}]$, the Laplacian matrix $L = [l_{ij}] \in \mathbf{R}^{N\times N}$ is defined by $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}, \quad (i \neq j)$.It is known that Laplacian matrix has at least one zero

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eigenvalue and all nonzero eigenvalues have positive real parts. Especially, the Laplacian matrix has a simple 0 eigenvalue with the associated eigenvector $\mathbf{1} = [1 \cdots 1]^T$, and all other eigenvalues have positive real parts, if and only if the corresponding directed graph has a directed spanning tree.

In this manuscript, a consensus control problem of leader-follower type is considered. Let x_0 be a leader which each agent $i \in \mathcal{V}$ should follow (i is called a follower). Then, a_{i0} is defined such as $a_{i0} > 0$: when leader's information is available to follower i, and $a_{i0} = 0$: otherwise, and from a_{i0} and L, the matrix $M \in \mathbf{R}^{N \times N}$ is defined by $M = L + \mathrm{diag}\,(a_{10} \cdots a_{N0})$. It is shown that $-M$ is a Hurwitz matrix, if and only if 1. at least one a_{i0} $(1 \le i \le N)$ is positive, and 2. the graph $\mathcal{G}$ has a directed spanning tree with the root $i = 0$.

Hereafter, it is assumed that 1. The graph has a directed spanning tree with the root $i = 0$, 2. At least one a_{i0} $(1 \le i \le N)$ is positive, that is, the information of the leader x_0 $(\dot{x}_0, \ddot{x}_o)$, is available to at least one follower i, 3. For the leader, $x_0, \dot{x}_0, \ddot{x}_0$ are uniformly bounded.

In the manuscript, two adjacency matrices $A = [a_{ij}]$, $C = [c_{ij}] \in \mathbf{R}^{N \times N}$ are considered for a directed graph $\mathcal{G}$, and the corresponding matrices are denoted as L_a, L_c (Laplacian matrices), and M_a, M_c, respectively.

3. ADAPTIVE H_∞ CONSENSUS CONTOL FOR FIRST-ORDER MODELS

3.1 Problem Statement

We consider a multi-agent system composed of the first-order regression models on directed graphs with nonlinear terms described as follows $(i = 1, \cdots, N)$:

$$\dot{x}_i(t) = X_i(t)\theta_i + F_i(x_i(t)) + B_i u_i(t), \tag{1}$$

where $x_i \in \mathbf{R}^n$ is a state, $u_i \in \mathbf{R}^n$ is an input, $\theta_i \in \mathbf{R}^l$ is an unknown parameter vector, and $X_i \in \mathbf{R}^{n \times l}$ is a regressor matrix composed of x_i and its structure is known a priori. It is assumed that X_i is bounded for bounded x_i. $F_i(x_i) \in \mathbf{R}^n$ is an unknown nonlinear term, and $B_i \in \mathbf{R}^{n \times n}$ is an unknown matrix of the form $B_i = \mathrm{diag}\,(b_{i1}, \cdots, b_{in})$, and the sign of b_{ij} is known a priori. Hereafter, it is assumed that $b_{ij} > 0$ without loss of generality. The communication structure among agents and a leader is prescribed by the information network graph $\mathcal{G}$ with the associated adjacency matrices A and C, the Laplacian matrices L_a and L_c, and the matries M_a and M_c. The control objective is to achieve consensus tracking of the leader-follower type such as $x_i \to x_j$, $x_i \to x_0$ $(i, j = 1, \cdots, N)$.

3.2 Representation of Nonlinear Term

In this paper, it is assumed that $F_i(x_i)$ is approximated by a three-layered neural network (a nonlinear parametric model) as follows:

$$F_i(x_i) = \begin{bmatrix} W_{i1}^{\mathsf{T}} S(V_{i1}^{\mathsf{T}} \bar{x}_i) + \mu_{i11}(x_i) \\ \vdots \\ W_{in}^{\mathsf{T}} S(V_{in}^{\mathsf{T}} \bar{x}_i) + \mu_{i1n}(x_i) \end{bmatrix}$$

$$\equiv W_i^{\mathsf{T}} S(V_i^{\mathsf{T}} \bar{x}_i) + \mu_{i1}(x_i) \ \in \mathbf{R}^n, \tag{2}$$

$$\bar{x}_i = [x_i^{\mathsf{T}},\, 1]^{\mathsf{T}} \in \mathbf{R}^{n+1}, \tag{3}$$

$$W_{ij} = [w_{ij1}, \cdots, w_{ijm}]^{\mathsf{T}} \in \mathbf{R}^m, \quad (1 \le j \le n), \tag{4}$$

$$V_{ij} = [v_{ij1}, \cdots, v_{ijm}] \in \mathbf{R}^{(n+1) \times m},$$

$$v_{ijk} \in \mathbf{R}^{n+1}, \quad (1 \le j \le n,\ 1 \le k \le m), \tag{5}$$

$$S(V_{ij}^{\mathsf{T}} \bar{x}_i) = [s(v_{ij1}^{\mathsf{T}} \bar{x}_i), \cdots, s(v_{ijm}^{\mathsf{T}} \bar{x}_i)]^{\mathsf{T}} \in \mathbf{R}^m, \tag{6}$$

$$s(v^{\mathsf{T}} \bar{x}) = \frac{1}{1 + \exp\{-\gamma^* (v^{\mathsf{T}} \bar{x})\}}, \quad (\gamma^* > 0), \tag{7}$$

$$W_i = \begin{bmatrix} W_{i1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & W_{in} \end{bmatrix} \in \mathbf{R}^{mn \times n}, \tag{8}$$

$$S(V_i^{\mathsf{T}} \bar{x}_i) = [S(V_{i1}^{\mathsf{T}} \bar{x}_i)^{\mathsf{T}}, \cdots, S(V_{in}^{\mathsf{T}} \bar{x}_i)^{\mathsf{T}}]^{\mathsf{T}} \in \mathbf{R}^{mn}, \tag{9}$$

$$\mu_{i1}(x_i) = [\mu_{i11}(x_i), \cdots, \mu_{i1n}(x_i)]^{\mathsf{T}} \in \mathbf{R}^n, \tag{10}$$

where V_{ij} and W_{ij} are layer weights of the j-th neural network for the i-th agent, and m is a number of cells of each neural network. $s(v^{\mathsf{T}} \bar{z})$ is a sigmoid function, and $\mu_{i1}(x_i)$ is a vector of an approximation error for $F_i(x_i)$.

3.3 Neural Network Approximator

Based on the fact that any continuous function over a compact set can be approximated by a three-layered neural network with an arbitrary small approximate error (Funahashi (1989)), the following assumption is introduced.

Assumption 1. There exist layer weights V_{ij} and W_{ij} satisfying the following relations.

$$|\mu_{i1j}(x_i)| \le d_{i1j} \psi_{ij}(x_i), \quad (1 \le j \le n), \tag{11}$$

where d_{i1j} are unknown positive constants, and $\psi_{ij}(x_i)$ are known positive functions.

The estimates of the layer weights V_{ij} and W_{ij} are denoted by $\hat{V}_{ij}$ and $\hat{W}_{ij}$, respectively. Then, the neural network estimation error $\hat{W}_{ij}^T S(\hat{V}_{ij}^T \bar{x}_i) - W_{ij}^T S(V_{ij}^T \bar{x}_i)$ is evaluated in the following lemma (Zhang et al. (1999)).

Lemma 2. For the three-layered neural network, the estimation error is evaluated as follows:

$$\hat{W}_{ij}^{\mathsf{T}} S(\hat{V}_{ij}^{\mathsf{T}} \bar{x}_i) - W_{ij}^{\mathsf{T}} S(V_{ij}^{\mathsf{T}} \bar{x}_i)$$

$$= \tilde{W}_{ij}^{\mathsf{T}} (\hat{S}_{ij} - \hat{S}_{ij}' \hat{V}_{ij}^{\mathsf{T}} \bar{x}_i) + \hat{W}_{ij}^{\mathsf{T}} \hat{S}_{ij}' \tilde{V}_{ij}^{\mathsf{T}} \bar{x}_i + \mu_{i2j}, \tag{12}$$

$$|\mu_{i2j}| \le \|V_{ij}\| \cdot \|\bar{x}_i \hat{W}_{ij}^{\mathsf{T}} \hat{S}_{ij}'\| + \|W_{ij}\| \cdot \|\hat{S}_{ij}' \hat{V}_{ij}^{\mathsf{T}} \bar{x}_i\| + |W_{ij}|_1, \tag{13}$$

$$\tilde{W}_{ij} = \hat{W}_{ij} - W_{ij}, \quad \tilde{V}_{ij} = \hat{V}_{ij} - V_{ij}, \tag{14}$$

$$\hat{S}_{ij} = S(\hat{V}_{ij}^{\mathsf{T}} \bar{x}_i), \tag{15}$$

$$\hat{S}_{ij}' = \mathrm{diag}(\hat{s}_{ij1}', \cdots, \hat{s}_{ijm}'), \tag{16}$$

$$\hat{s}_{ijk}' = s'(\hat{v}_{ijk}^{\mathsf{T}} \bar{x}_i) = \left[\frac{ds(z)}{dz}\right]_{z = \hat{v}_{ijk}^{\mathsf{T}} \bar{x}_i}. \tag{17}$$

For convenience' sake, $\tilde{W}_{ij}^{\mathsf{T}} (\hat{S}_{ij} - \hat{S}_{ij}' \hat{V}_{ij}^{\mathsf{T}} \bar{x}_i)$ and $\hat{W}_{ij}^{\mathsf{T}} \hat{S}_{ij}' \tilde{V}_{ij}^{\mathsf{T}} \bar{x}_i$ in (12) are rewritten into the following regression forms.

$$\tilde{W}_{ij}^{\mathsf{T}} (\hat{S}_{ij} - \hat{S}_{ij}' \hat{V}_{ij}^{\mathsf{T}} \bar{x}_i) = \tilde{W}_{ij}^{\mathsf{T}} \omega_{ij0}, \tag{18}$$

$$\hat{W}_{ij}^{\mathsf{T}} \hat{S}_{ij}' \tilde{V}_{ij}^{\mathsf{T}} \bar{x}_i = \sum_{k=1}^{m} \hat{w}_{ijk} \hat{s}_{ijk}' \tilde{v}_{ijk}^{\mathsf{T}} \bar{x}_i = \sum_{k=1}^{m} \tilde{v}_{ijk}^{\mathsf{T}} \omega_{ijk}, \tag{19}$$