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Faster computation of closed loop transfers with frequency response data for multivariable loopshaping

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Abstract: For industrial applications containing high-precision motion systems, feedback controllers are typically designed via loopshaping using measured frequency response data. Often, these positioning systems have multiple degrees of freedom including cross-coupling. If the cross-coupling is significantly large, then it should be taken into account carefully. This, to be able to guarantee stability, robustness, and performance. For multiple frequency responses with multiple degrees of freedom, the required calculation time with a ”standard” implementation slows down the loopshaping process and hampers an automated implementation. Moreover, a high frequency grid density is needed for, often weakly damped, motion systems which further complicates the computational process. In this paper, a number of methods are proposed which significantly reduce the calculation time to compute relevant closed loop frequency responses for multivariable control systems. This enables that loopshaping for a set of multivariable frequency responses can still be carried out without too much time lag.

Keywords: Feedback control, MIMO, signal processing algorithms, computational methods, frequency responses, multivariable control systems

1. INTRODUCTION

High-precision motion systems, such as wafer scanners (Butler (2011)), are typically designed such that they exhibit linear behavior as much as possible. Furthermore, servo performance heavily depends on the design of both feedforward and feedback controllers. Often, the feedforward controller is tuned machine specifically by using an iterative time-domain optimization (see e.g. Heertjes et al. (2010)). On the other hand, the feedback controller is typically a fixed-structure controller (e.g a PID controller with a number of notches and low pass filters in series) where parameters are tuned for a population of machines for robustness. For each machine, usually, multiple frequency responses are used to ensure robustness for position dependency. Also, a high frequency density is used to capture weakly-damped resonances well. Furthermore, high-precision motion systems are often multivariable systems having multiple inputs and outputs. The large set of frequency responses, a high frequency grid density, and multiple degrees of freedom, all contribute to an increased calculation time to compute all relevant closed loop frequency responses. As a result, the tuning process of controller parameters is being slowed down.

The growing amount of frequency responses can be dealt with using model-based control, where most effort is spent on system identification of both a nominal and an uncertainty model (see e.g. Oomen et al. (2014)). However, if a fixed-structure controller is desired, the controller design problem becomes non-convex which is a hard problem to solve. Model-based fixed-structure control (Gahinet and Apkarian (2012)) is a way to optimize a fixed-structure controller, however, the difficult system identification step is still needed.

In this paper, the focus will lie on fixed-structure controller design using only non-parametric frequency response data instead of parametric models. This prevents the need for the difficult parametric modeling step. The resulting non-convex problem is optimized using loopshaping. With loopshaping is meant, a manual (Doyle and Stein (1981)) or optimization-based (Henke et al. (2015)) way of tuning controller parameters. Relevant frequency responses such as the open loop and the sensitivity are being shaped.

For a large set of multivariable systems, the calculation time of relevant closed loop frequency responses becomes too high for convenient loopshaping. Think of multiple seconds or even minutes for one evaluation. The contribution of this paper is the derivation of a couple of methods which speed up the calculation of 1) the determinant of the inverse sensitivity, 2) closed loop frequency responses, and 3) equivalent frequency responses. The key ingredients are the Matrix Determinant Lemma (Brookes (2005)):

$$\det(A + uv^T) = (1 + v^T A^{-1} u)\det(A) \tag{1}$$

and the Sherman-Morrison formula (Sherman and Morrison (1950)):

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1} u} \tag{2}$$

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First, in Section 2, a short introduction is given on important aspects for multivariable feedback controller design (Skogestad and Postlethwaite (2005), Doyle and Stein (1981)) when using a loopshaping approach in combination with frequency responses data. Next, in Section 4, 3, and 5, ways to speed up the relevant computations are derived. A calculation time comparison is made in Section 6 and conclusions are drawn in Section 7.

2. MULTIVARIABLE FEEDBACK CONTROL SYSTEMS

2.1 Multivariable Case

In Fig. 1, a basic control scheme is shown with feedback controller $C^{[n\times m]}$, plant $P^{[m\times n]}$, reference $r^{[m\times 1]}$, input disturbance $d^{[n\times 1]}$, output disturbance $w^{[m\times 1]}$, feedforward signal $f^{[n\times 1]}$, servo error $e^{[m\times 1]}$, control signal $u^{[n\times 1]}$, and measurement signal $y^{[m\times 1]}$. The multivariable control systems contains n plant inputs and m plant outputs. Although C is typically diagonal and square for high-precision motion systems, the results in this paper are generically applicable to full-MIMO non-square C and P.

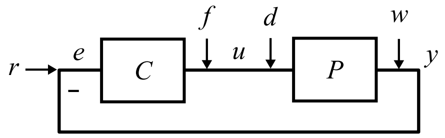

Fig. 1. Control scheme

An example plant and 2 controllers are shown in Fig. 2 and Fig. 3, respectively. They will be used in this section to explain the relevance of a number of frequency responses for multivariable loopshaping. The plant is a 2x2 system with decoupled rigid body modes and coupled non-rigid body modes. For this case, the mode shape of the first resonance at 300 Hz is chosen such that it can be best actuated in the second direction, while it can be best observed in the first direction. This is clearly visible in Fig. 2, where the top-right figure is the only figure with a large 300 Hz resonance.

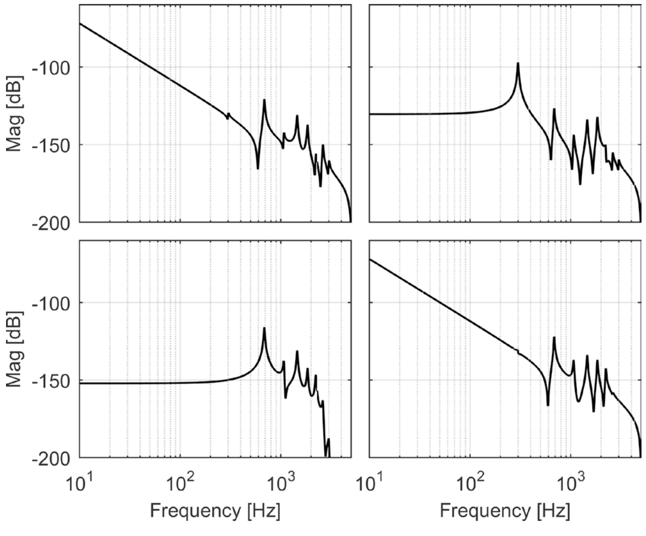

Fig. 2. Plant dynamics of a 2x2 system, decoupled rigid body modes, coupled non-rigid body modes

The designed controllers as shown in Fig. 3 consist of 2 rigid body controllers (PID + low pass filter + notch filter). The second controller ("MIMO controller") also contains an off-diagonal controller (low pass filter + high pass filter) with the goal to add closed loop damping to the 300 Hz mode. The controllers have been designed using an optimization-based loopshaping procedure which exploits the computational enhancements as proposed in this paper.

For the MIMO controller, the open loop and equivalent open loop are shown in Fig. 4. For each controller, the equivalent open loop is defined as the resulting SISO open loop while all other control loops are closed (as if they are absorbed in the plant). The equivalent open loop for the off-diagonal controller clearly shows a magnitude above 0 dB around 300 Hz resulting in closed-loop damping. The increased damping is also visible in the equivalent open loops of the diagonal controller elements (damped resonance near 300 Hz).

In Fig. 5, the process sensitivity $S_p = (I+PC)^{-1}P$ impulse responses are shown with the designed MIMO controller compared to the diagonal controller. The increased closed loop damping due to the off-diagonal controller is clearly visible in the top-right figure.

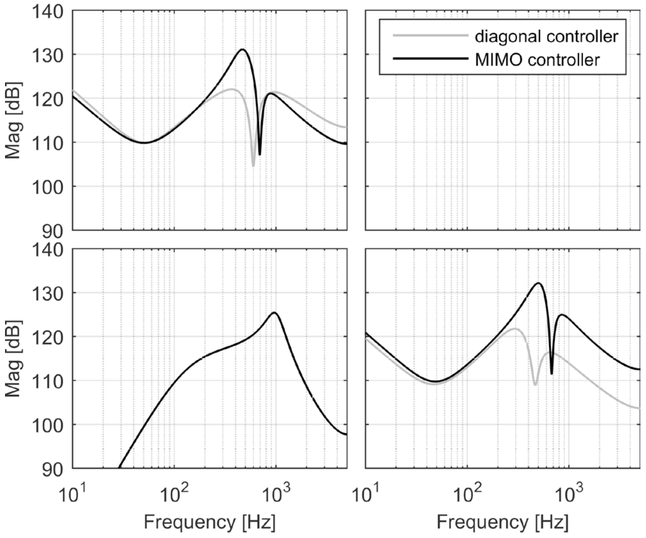

Fig. 3. MIMO controller design with one off-diagonal term to damp the 300 Hz mode, diagonal controller design to show the impact of the off-diagonal term

2.2 Controller Design Criteria

For any controlled system, there are three main aspects that should be taken care of:

(1) *Stability*
(2) *Robustness*
(3) *Performance*

In view of multivariable loopshaping using measured frequency responses, these aspects will be briefly discussed next.

(1) *Stability*