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Improved Transients in Multiple Frequencies Estimation via Dynamic Regressor Extension and Mixing

Stanislav Aranovskiy [*,**] **Alexey Bobtsov** [**,***]
Romeo Ortega [****] **Anton Pyrkin** [**]

[*] *NON-A team, INRIA-LNE, Parc Scientifique de la Haute Borne 40, avenue Halley Bat.A, Park Plaza, 59650 Villeneuve d'Ascq, France*
[**] *Department of Control Systems and Informatics, ITMO University, St. Petersburg, Russia*
[***] *Laboratory "Control of Complex Systems", Institute for Problems of Mechanical Engineering, St. Petersburg, Russia*
[****] *Laboratoire des Signaux et Systèmes, CNRS-CentraleSupélec, Plateau du Moulon, 91192, Gif-sur-Yvette, France*

Abstract: The problem of performance enhancement for multiple frequencies estimation is studied. First, we consider a basic gradient-based estimation approach with global exponential convergence. Next, we apply the dynamic regressor extension and mixing technique to improve transient performance of the basic approach and ensure non-strict monotonicity of estimation errors. Simulation results illustrate benefits of the proposed solution.

Keywords: estimation, performance, frequency identification, gradient methods

1. INTRODUCTION

The problem of frequencies estimation for sinusoidal signals attracts researchers' attention both in control and signal processing communities due to its practical importance. Indeed, frequency identification methods are widely used in fault detection systems (Goupil (2010)), to attenuate periodic disturbances (Landau et al. (2011); Bobtsov et al. (2011)), in naval applications (Belleter et al. (2013)) and so on.

Many online frequency estimation methods are currently available in literature, e.g. a phase-locked loop (PLL) proposed in Wu and Bodson (2003), adaptive notch filters (Regalia (1991); Mojiri and Bakhshai (2004)). Another popular approach is to find a parametrization yielding a linear regression model, which parameters are further identified with pertinent estimation techniques, see Xia (2002); Chen et al. (2014); Fedele and Ferrise (2014). However, the most of online methods are focused on stability studies and local or global convergence analysis; transients performance is not usually considered and is only demonstrated with simulations. On the other hand, it is well-known that many gradient-based estimation methods can exhibit poor transients even for a relatively small number of estimated parameters: the transients can oscillate or provide a peaking phenomena. A method to increase frequency estimation performance with adaptive band-pass filters was proposed in Aranovskiy et al. (2015b) but for a single frequency case only. Therefore, the problem of performance improvement for multiple frequencies estimation remains open.

A novel way to improve transient performance for linear regression parameters estimation was proposed in Aranovskiy et al. (2015a); the approach is based on extension and mixing of the original vector regression in order to obtain a set of scalar equations. In this paper we apply this approach to the problem of multiple frequencies estimation. It is shown that under some reasonable assumptions and neglecting fast-decaying terms we can ensure *non-strict monotonicty* of estimates of parameters avoiding any oscillatory or peaking behavior.

The paper is organized as follows. First, in Section 2 a multiple frequencies estimation problem is stated. A basic method to solve the problem is presented in Section 3. Next, in Section 4 we consider dynamic regressor extension and mixing (DREM) procedure and apply it to the previously proposed method. Illustrative results are given in Section 5 and the paper is wrapped up with Conclusion.

2. PROBLEM STATEMENT

Consider the measured scalar signal

$$u(t) = \sum_{i=1}^{N} A_i \sin(\omega_i t + \varphi_i), \tag{1}$$

where $t \geq 0$ is time, $A_i > 0$, $\varphi_i \in [0, 2\pi)$, and $\omega_i > 0$ are the unknown amplitudes, phases, and frequencies, respectively, $i \in \overline{N} := \{1, 2, \dots N\}$, N is the number of the frequencies in the signal.

Assumption 1. All the frequencies ω_i, $i \in \overline{N}$, are distinguished, i.e.

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$$\omega_i \neq \omega_j \; \forall i \neq j, \; i, j \in \overline{N}.$$

Remark 1. The signal (1) can be seen as an output of a marginally stable linear signal generator

$$\dot{z}(t) = \Gamma z(t), \quad z(0) = z_0 \in \mathrm{R}^{2N},$$
$$u(t) = Hz,$$

where $\Gamma \in \mathrm{R}^{2N\times 2N}$ and $H \in \mathrm{R}^{1\times 2N}$. The characteristic polynomial of the matrix Γ is given by

$$P_\theta(s) := s^{2N} + \theta_1 s^{2N-2} + \ldots \theta_{N-1}s^2 + \theta_N,$$

where the parameters θ_i are such that the roots of the polynomial $P_\theta(s)$ are $\pm \mathrm{i}\omega_i$, where $\mathrm{i} := \sqrt{-1}$, $i \in \overline{N}$. Obviously, given a vector $\theta := \mathrm{col}(\theta_i) \in \mathrm{R}^N$, the frequencies can be uniquely (up to numerical procedures accuracy) defined, and *vice versa*. Thus, in many multiple frequency estimation methods the vector θ is identified instead of separate frequencies values. In our paper we follow this approach and assume that the frequencies are estimated if the vector θ is obtained. The problem of *direct* frequency identification is considered, for example, in Pin et al. (2015).

Frequencies Estimation Problem. The goal is to find mappings $\Psi : \mathrm{R}^l \times \mathrm{R} \mapsto \mathrm{R}^l$ and $\Theta : \mathrm{R}^l \mapsto \mathrm{R}^N$, such that the following estimator

$$\begin{aligned} \dot{\chi}(t) &= \Psi(\chi(t), u(t)), \\ \hat{\theta}(t) &= \Theta(\chi(t)). \end{aligned} \tag{2}$$

ensures

$$\lim_{t\to\infty} |\hat{\theta}(t) - \theta| = 0. \tag{3}$$

3. A BASIC FREQUENCIES IDENTIFICATION METHOD

In this section we consider a multiple frequencies estimation method, proposed in Aranovskiy et al. (2010) and further extended in Bobtsov et al. (2012); Pyrkin et al. (2015). This method is based on State-Variable Filter (SVF) approach, see Young (1981); Garnier et al. (2003).

Lemma 1. Consider the following SVF

$$\dot{\xi}(t) = A\xi(t) + Bu(t), \tag{4}$$

where $\xi := [\xi_1(t), \; \xi_2(t), \; \ldots \; \xi_{2N}(t)]^\top$,

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ -a_0 & -a_1 & -a_2 & \ldots & -a_{2N-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ a_0 \end{bmatrix},$$

a_i, $i \in \{0, 2, \ldots, 2N-1\}$, are the coefficients of the Hurwitz polynomial

$$a(s) = s^{2N} + a_{2N-1}s^{2N-1} + \ldots a_1 s + a_0.$$

Define

$$y(t) := -\dot{\xi}_{2N}(t) = \sum_{i=1}^{2N} a_{i-1}\xi_i(t) - a_0 u(t). \tag{5}$$

Then the following holds:

$$y(t) = \phi^\top(t)\theta + \varepsilon(t), \tag{6}$$

where

$$\phi(t) := [\xi_{2N-1}(t), \; \xi_{2N-3}(t), \; \ldots \; \xi_3(t) \; \xi_1(t)]^\top, \tag{7}$$

θ is defined in Remark 1, and $\varepsilon(t)$ is an exponentially decaying term. □

The proof is straightforward and follows the proof presented in Pyrkin et al. (2015).

Using Lemma 1 we can propose a multiple frequencies estimator.

Proposition 2. Consider the signal (1) satisfying Assumption 1, the SVF (4), and the signals $y(t)$ and $\phi(t)$, defined by (5) and (7), respectively. Then the estimator

$$\dot{\hat{\theta}}(t) = K_\theta \phi(t) \left(y(t) - \phi^\top(t)\hat{\theta}(t) \right), \tag{8}$$

where $K_\theta \in \mathrm{R}^{N\times N}$, $K_\theta > 0$, ensures the goal (3). Moreover, the estimation error $\tilde{\theta}(t) := \hat{\theta}(t) - \theta$ converges to zero exponentially fast. □

The proof follows the one given in Pyrkin et al. (2015)

Remark 2. The proposed estimator can be also written as (2) (the argument of time is omitted):

$$\begin{aligned} \chi &:= \mathrm{col}(\xi, \hat{\theta}), \\ \Psi(\chi, u) &:= \begin{bmatrix} A\xi + Bu \\ K_\theta \phi(y - \phi^\top \hat{\theta}) \end{bmatrix}, \\ \Theta(\chi) &:= \hat{\theta}. \end{aligned}$$

The estimation algorithm (8) ensures global exponential convergence of $\tilde{\theta}(t)$, but do not guarantee performance of the transients. It is known from practice that for $N \geq 2$ behavior of the estimator (8) becomes oscillatory and can exhibit peaking phenomena. However, these limitations can be overcome with the DREM technique presented in the next section.

4. ENHANCING THE BASIC ALGORITHM VIA DREM PROCEDURE

In this section we first present the DREM procedure proposed in Aranovskiy et al. (2015a), and then apply it to the basic frequencies estimation algorithm described in Section 3.

4.1 Dynamic Regressor Extension and Mixing

Consider the basic linear regression

$$\rho(t) = m^\top(t) r, \tag{9}$$

where $\rho \in \mathrm{R}$ and $m \in \mathrm{R}^q$ are measurable bounded signals and $r \in \mathrm{R}^q$ is the vector of unknown constant parameters to be estimated. The standard gradient estimator, equivalent to (8),

$$\dot{\hat{r}}(t) = K_r m(t) \left(\rho(t) - m^\top(t)\hat{r}(t) \right),$$

with a positive definite adaptation gain $K_r \in \mathrm{R}^{q\times q}$ yields the error equation

$$\dot{\tilde{r}}(t) = -K_r m(t) m^\top(t) \tilde{r}(t), \tag{10}$$

where $\tilde{r}(t) := \hat{r}(t) - r$ is the parameters estimation error.

We propose the following dynamic regressor extension and mixing procedure. The first step in DREM is to introduce $q-1$ *linear*, $\mathcal{L}_\infty$*-stable* operators $H_i : \mathcal{L}_\infty \to \mathcal{L}_\infty$, $i \in \{1, 2, \ldots, q-1\}$, whose output, for any bounded input, may be decomposed as

$$(\cdot)_{f_i}(t) := [H_i(\cdot)](t) + \epsilon_t, \tag{11}$$