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Adaptive Multisinusoidal Signal Tracking System with Input Delay ⋆

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Abstract:
In this paper the problem of adaptive tracking of unknown multisinusoidal signal is addressed. The control input is characterized by the known time delay. It is assumed that all the parameters of the plant are known. In order to demonstrate efficiency of the proposed approach it is implemented to the robotic application. Detailed description of the experimental results are presented in the paper. In addition to the latter comparison between the presented algorithm and the proportional controller is performed.

Keywords: Adaptive control, tracking system, time delay, state observer, robotic manipulator.

1. INTRODUCTION

Control of systems with delay is very important and actual problem even today. It attracts an attention of many researchers. A delay in automatic control systems can occur for various reasons Richard [2003]. For example, it can be caused by a limited performance of system components, inertia of involved processes, remoteness of a plant and etc. Sometimes these factors can be neglected, but this often leads to deterioration of a system or its instability in some cases. The mentioned problem becomes more actual because of digital technologies which are widespread nowadays. Microcontrollers need some time to compute control signals. Limited channel capacity leads to increasing of time delay.

Among meaningful results in the domain of time-delayed control it is worth noting the approach proposed by Smith [1959], which is known as Smith predictor. According to this method stability and transient properties of constructed systems are not affected by delays. This approach is implementable to asymptotically stable plants with parametric uncertainties. Later its different modifications were developed including the adaptive version Niculescu and Annaswamy [2003] for plant which relative degree is less or equal to two. Moreover, note the results for plants with parametric uncertainties Mirkin [2004], Furtat and Tsykunov [2007].

In addition to delay this work focuses on a multisinusoidal reference signal tracking Hu and Tomizuka [1993], Fedele et al. [2013], Moon et al. [1998], Graham et al. [2006]. The relative problem is compensation of an unknown multisinusoidal disturbance. In the papers Pyrkin et al. [2010, 2013] the stabilization algorithms for unstable linear systems with input delay Krstic and Smyshlyaev [2008], Krstic [2010] were modified for the case of unknown external disturbances compensation.

Developing the results Nikiforov [1998], Bobtsov and Pyrkin [2012], Pyrkin et al. [2010] in this paper the problem of a multisinusoidal signal tracking under condition of delay is addressed. In order to demonstrate the efficiency of the proposed approach it is successfully implemented to robotic application. The corresponding experimental results are provided in the paper.

2. PROBLEM FORMULATION

Consider the problem of an unknown multisinusoidal signal tracking for a linear system under conditions of input delay

$$\dot{x}(t) = Ax(t) + Bu(t-h), \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

$$e(t) = g(t) - y(t), \quad (3)$$

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where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input signal, h is the known constant delay, $y \in \mathbb{R}$ is the output of the system, $g \in \mathbb{R}$ is the reference signal as the desired output of the system, $e \in \mathbb{R}$ is the error of reference signal tracking, $A_{n\times n}$ is the state matrix, $B_{n\times 1}$ is the matrix of the control inputs, $C_{1\times n}$ is the matrix of the output.

For the control signal $u(t-h) = 0$ holds for $t < h$.

Biased multisinusoidal signal represents the reference signal σ^g

$$g(t) = \sigma^g + \sum_{j=1}^{l_2} \mu_j^g \sin(\omega_j^g t) + \nu_j^g \cos(\omega_j^g t), \tag{4}$$

consisting of l_2 harmonics with unknown amplitudes μ_j^g and ν_j^g, and frequencies ω_j^g, $j = \overline{1, l_2}$ is the harmonic number.

The purpose is to perform the reference signal tracking. In order to do achieve this goal we need to design the control law u ensuring the asymptotic stability of the tracking error (3)

$$\lim_{t\to\infty} |e(t)| = 0, \tag{5}$$

taking into account the following assumptions

Assumption 1. All the parameters of the system (1)-(2) are known.

Assumption 2. The matrix triple (A, B, C) is completely controllable and observable.

Assumption 3. The lower boundary of the frequencies ω_0 is known for the reference signal $g(t)$

$$\omega_i^\delta \geq \omega_0, \quad i = \overline{1, l_1},$$
$$\omega_j^g \geq \omega_0, \quad j = \overline{1, l_2}.$$

Assumption 4. All the frequencies ω_i^δ, $i = \overline{1, l_1}$ ω_j^g, $j = \overline{1, l_2}$ are different.

3. PROBLEM OF ADAPTIVE TRACKING OF MULTISINUSOIDAL SIGNAL

Consider the tracking problem with the stable state matrix A. Introduce the observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-h), \tag{6}$$
$$\hat{y}(t) = C\hat{x}(t), \tag{7}$$
$$z(t) = \hat{y}(t) + e(t), \tag{8}$$

where $\hat{x} \in \mathbb{R}^n$ is the state vector of the observer, $\hat{y} \in \mathbb{R}$ is the output of the observer, $z \in \mathbb{R}$ is the estimate of the reference signal.

Note that the observer model is not able to change the convergence rate which is not always acceptable. An approach devoid of this disadvantage is considered in the next subsection.

The error $\tilde{x}(t) = x(t) - \hat{x}(t)$ converges to zero due to Hurwitz matrix A

$$\begin{aligned}\dot{\tilde{x}}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\ &= Ax(t) + Bu(t-h) - A\hat{x}(t) - Bu(t-h) \\ &= A\tilde{x}(t).\end{aligned} \tag{9}$$

With (9) and the expressions (3), (8), it is easy to show, that $z(t)$ converges to the reference signal $g(t)$

$$\begin{aligned}g(t) - z(t) &= g(t) - \hat{y}(t) - e(t) \\ &= g(t) - \hat{y}(t) - g(t) + y(t) \\ &= \tilde{y}(t) = C\tilde{x}(t) = \varepsilon(t),\end{aligned} \tag{10}$$

where $\varepsilon(t)$ is the exponentially decaying function with exponentially decaying derivatives.

In order to solve the given problem we need to design the control input $u(t)$ such that the response $y_u(t)$ of the system (1)-(3) is equal to $z(t)$ excluding the exponential component $\varepsilon(t)$.

Choose the control in the form

$$\begin{aligned}u(t-h) =& \hat{\sigma}^u(t) + \sum_{j=1}^{l_2} \hat{\mu}_j^u(t) \sin\big(\hat{\omega}_j^g(t)t\big) \\ &+ \hat{\nu}_j^u(t) \cos\big(\hat{\omega}_j^g(t)t\big),\end{aligned} \tag{11}$$

where $\hat{\omega}_j^g(t)$, $\hat{\sigma}^u(t)$, $\hat{\mu}_j^u(t)$, $\hat{\nu}_j^u(t)$ are the estimates for ω_j^g, σ^u, μ_j^u, ν_j^u respectively. With the known reference signal $g(t)$ the values of the parameters σ^u, μ_j^u, ν_j^u ensuring equality $y(t) = g(t)$ can be calculated as

$$\sigma^u = \frac{1}{L_0}\sigma^g, \; \mu_j^u = \frac{1}{L_j}\mu_j^g, \; \nu_j^u = \frac{1}{L_j}\nu_j^g, \tag{12}$$

where L_0 and L_j are the transfer coefficients for the constant and harmonic signals with frequency ω_j^g

$$L_0 = \Big|W(j_c\omega)\Big|_{\omega=0}, \tag{13}$$
$$L_j = \Big|W(j_c\omega)\Big|_{\omega=\omega_j^g}, \tag{14}$$

which can be calculated using the complex transfer function from $u(t)$ to $y(t)$ of the system (1)-(2)

$$W(j_c\omega) = C(j_c\omega I - A)^{-1}B. \tag{15}$$

In order to get the estimate $\hat{\omega}_j^g(t)$ introduce the linear filter with the transfer function $F(s)$ and relative degree of not less than $2l_2$. Let us send the signal $z(t)$ to its input. Further, use the outputs of the linear filter in the estimation scheme.

Proposition 1. The update alghorithm

$$\hat{\omega}_j = \sqrt{\Big|\hat{\hat{\theta}}_j\Big|},$$

where estimates θ_j calculated using $\hat{\hat{\theta}}_j$, that are elements of a vector $\hat{\hat{\Theta}}^T = \Big[\hat{\hat{\theta}}_1 \; \dots \; \hat{\hat{\theta}}_{l-1} \; \hat{\hat{\theta}}_l\Big]$

$$\hat{\hat{\Theta}}(t) = \Upsilon(t) + K\Omega(t)\xi^{(2l)}(t),$$
$$\dot{\Upsilon}(t) = -K\Omega(t)\Omega^T(t)\hat{\hat{\Theta}}(t) - K\dot{\Omega}(t)\xi^{(2l)}(t),$$

where $K = \mathrm{diag}\{k_j > 0, j = \overline{1, l}\}$ guarantees that the estimation error $\tilde{\omega}_j(t) = \omega_j - \hat{\omega}_j(t)$ exponentially converges to zero:

$$|\tilde{\omega}_j(t)| \leq \rho_1 e^{-\beta_1 t}, \quad \rho_1, \beta_1 > 0, \quad \forall t \geq 0.$$

To get the estimates $\hat{\sigma}^u(t)$, $\hat{\mu}_j^u(t)$, $\hat{\nu}_j^u(t)$ let's take the auxiliary filter in the form

$$\vartheta(s) = W(s)F(s)\Delta(s), \tag{16}$$

where $W(s)$ is the transfer function of the system (1)-(2), $\Delta(s) = \mathcal{L}^{-1}\{\Delta(t)\}$, the input of this filter is defined as $\Delta(t) = \sum_{j=0}^{l} \sin(\hat{\omega}_j(t)t)$.