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Control of Linear Systems Subjected to Exogenous Disturbances: Combined Feedback

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Abstract: A new approach to rejection of exogenous disturbances in linear control systems via static combined feedback is proposed. Namely, in addition to static linear state feedback, we introduce a linear feedback from the components of exogenous disturbances whose instantaneous values are assumed to be known. The control design is based on the technique of linear matrix inequalities, and the proposed simple approach leads to a convex minimization problem. Moreover, a new approach to the design of sparse combined feedback in linear control systems is proposed; it can be interpreted as reduction of the control resource required to handling the system.

Keywords: linear system, exogenous disturbances, LMI, invariant ellipsoid, combined feedback.

1. INTRODUCTION

The problem of rejection the exogenous disturbances is one of the key problems in classical control theory; due to diverse origins, it is studied in various sections of control theory. There exists a lot of statements, as well as approaches, see Boyd et al. (1994); Blanchini and Miani (2008); Vidyasagar (1986); Hao and Wang (2007); Dahleh and Diaz-Bobillo (1995); Abedor et al. (1996); Polyak et al. (2014) and references therein.

Consider the linear control system

$$\begin{aligned} \dot{x} &= Ax + B_1 u + Dw, \quad x(0) = x_0, \\ z &= Cx + B_2 u, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times p}$, $B_2 \in \mathbb{R}^{l\times p}$, $D \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ are fixed known matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $z(t) \in \mathbb{R}^l$ is the system output, $u(t) \in \mathbb{R}^p$ is the control input, $w(t) \in \mathbb{R}^m$ is the exogenous disturbance satisfying the Euclidean norm constraints

$$w^\top(t)w(t) \le 1 \quad \forall t \ge 0. \tag{2}$$

It is assumed that the pair (A, B_1) is controllable.

In Nazin et al. (2007); Khlebnikov et al. (2011), the linear static state feedback

$$u = Kx, \quad K \in \mathbb{R}^{p\times n} \tag{3}$$

was used for the rejection of l_∞-bounded exogenous disturbances for system (1), (2).

Remark 1. The presence of the control component $B_2 u$ in the output of the system (1) allows for a more general statement of the problem where, along with the minimization of the system output, we try to avoid large values of the control signal.

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Note that the only available information about disturbances is their boundedness. However, sometimes the instantaneous values of certain components of the disturbances are known. For example, such situation appears at electric motor control systems when the deviation of the load torque is considered as an exogenous disturbance.

It is natural to exploit the additional information about the exogenous disturbances for control design. Namely, besides the linear static state feedback, we introduce the linear feedback via exogenous disturbances (or, by a part of its accessible components):

$$u = Kx + K_1 w, \quad K_1 \in \mathbb{R}^{p\times m}. \tag{4}$$

As a result, we obtain the so-called *combined feedback*.

The following remark is due at this point. Often, exogenous disturbances and the control input are applied to the plant "at the same point"; in other words, in this case we have

$$B_1 = D$$

for system (1). It is well-known (see, e.g., Polyak et al. (2014)) that in such case it is possible to construct the gain matrix (3) which guarantees the convergence of the system trajectories to the arbitrarily small tube. However it leads to the unconstrained growth of the gain matrix, and, consequently, to huge values of the control input.

Besides, closed-loop system (1) embraced with the combined feedback (4) with

$$K_1 = -I,$$

takes the form

$$\dot{x} = (A + B_1 K)x, \quad x(0) = x_0.$$

Its trajectories asymptotically tend to zero for any stabilizing controller K, and for *any* exogenous disturbances $w(t)$ (not necessarily satisfying condition (2)); therefore, the disturbances can be rejected completely.

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2. THE INVARIANT ELLIPSOID APPROACH

We recall essentials of the invariant ellipsoid approach. In the simplest statement, the goal is to describe the reachable set $\mathcal{R}$ for the dynamic system given by

$$\begin{aligned} \dot{x} &= Ax + Dw, \quad x(0) = 0, \\ z &= Cx, \end{aligned} \tag{5}$$

where $A \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{l\times n}$ are fixed known matrices, $x(t) \in \mathbb{R}^n$ is the state vector, $z(t) \in \mathbb{R}^l$ is the system output, $w(t) \in \mathbb{R}^m$ is the exogenous disturbance satisfying constraints (2). It is assumed that system (5) is stable, the pair (A, D) is controllable, and C is a full-rank matrix.

Since in the general case, the characterization of the reachable set $\mathcal{R}$ of system (1) and the respective set for the output variable is not doable in closed form for exogenous disturbances (2), outer approximations are often exploited, see Blanchini and Miani (2008). One of the popular, simple, and transparent methods is based on ellipsoidal approximations of reachable sets, see Schweppe (1973); Kurzhanski and Valyi (1997). We follow the invariant ellipsoid approach, which is based on the construction of a (common) quadratic Lyapunov function for the closed-loop system, see Boyd et al. (1994); Abedor et al. (1996); Polyak et al. (2014), etc. The technique of linear matrix inequalities is a natural and convenient tool for realization of such an approach.

The notion of invariant ellipsoid is central to all constructions in the paper.

Definition 2. The ellipsoid

$$\mathcal{E}_x \doteq \mathcal{E}_x(P) = \big\{x \in \mathbb{R}^n\colon\ x^\top P^{-1} x \le 1\big\}, \quad P \succ 0, \tag{6}$$

is said to be *invariant* for system (5), (2), if the condition $x(0) \in \mathcal{E}_x$ implies $x(t) \in \mathcal{E}_x$ for all $t \ge 0$.

The following result holds.

Theorem 3. (Boyd et al. (1994)). Ellipsoid (6) is invariant for system (5), (2), if and only if its matrix P satisfies the LMIs

$$AP + PA^\top + \alpha P + \frac{1}{\alpha} DD^\top \preceq 0, \qquad P \succ 0, \tag{7}$$

for some $\alpha > 0$.

Every invariant ellipsoid contains the reachable set of system (5) (which is the smallest possible invariant set), leading to an outer bound for $\mathcal{R}$. Therefore, it is natural to seek for the smallest invariant ellipsoid. Among the variety of criteria of minimality we adopt the trace criterion because of its linearity and transparent physical meaning (the sum of the squared semiaxis of the ellipsoid).

Usually, the goal is to characterize the magnitude of the output rather than that of the state. In that respect, it is seen that, associated with the invariant ellipsoid (6) is the *bounding* ellipsoid for the output variable z, specified by

$$\mathcal{E}_z = \big\{z \in \mathbb{R}^m\colon \quad z^\top (CPC^\top)^{-1} z \le 1\big\}$$

where P is the matrix of the state-invariant ellipsoid. Hence, the goal is to find the bounding ellipsoid which attains the minimum to the function

$$f(P) = \mathrm{tr}[CPC^\top].$$

This function is linear in P; hence, for α fixed, the minimization of $f(P)$ subject to the LMI constraints (7) is a semidefinite program, SDP.

This approach was extended to system control design via linear state feedback in Nazin et al. (2007) and via linear output feedback in Polyak and Topunov (2008b), to filtering problems, see Polyak and Topunov (2008a), to various robust statements, see Khlebnikov (2009), etc. A systematical exposition of the consistent technique was given in Polyak et al. (2014).

3. STATEMENT OF THE PROBLEM AND THE MAIN RESULT

Now we turn back to system (1) subjected to exogenous disturbances (2), while the instantaneous values of the components

$$w_i(t), \quad i \in \mathcal{I} \subseteq \{1, \dots, m\},$$

are known at any time instant t.

The goal is to find a combined feedback (4) with the indices $i \notin \mathcal{I}$ of the zero columns in the matrix $K_1 \in \mathbb{R}^{p\times m}$, which stabilizes the closed-loop system (1), (2) and minimizes the trace of the bounding ellipsoid for the system output z.

Remark 4. Embracing system (1) with combined feedback (4), it is natural to put only the state feedback component in the entry B_2u of the system output z, see Remark 1.

We now formulate the main result of the paper.

Theorem 5. Let $\widehat{P}$, $\widehat{Y}$, $\widehat{K}_1$, $\widehat{H}$ be the solution of the minimization problem

$$\min \mathrm{tr}\, H \tag{8}$$

subject to the constraints

$$\begin{pmatrix} AP + PA^\top + B_1Y + Y^\top B_1^\top + \alpha P & D + B_1K_1 \\ D^\top + K_1^\top B_1^\top & -\alpha I \end{pmatrix} \preceq 0, \tag{9}$$

$$\begin{pmatrix} CPC^\top - H + B_2YC^\top + CY^\top B_2^\top & B_2Y \\ Y^\top B_2^\top & -P \end{pmatrix} \preceq 0 \tag{10}$$

with respect to the matrix variables $P \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{p\times n}$, $H \in \mathbb{R}^{l\times l}$, $K_1 \in \mathbb{R}^{p\times m}$ (with indices $i \notin \mathcal{I}$ of the zero columns) and the scalar parameter $\alpha > 0$.

Then, the combined feedback

$$u = \widehat{Y}\widehat{P}^{-1}x + \widehat{K}_1 w$$

stabilizes system (1), (2), and the matrix

$$R_{\mathcal{I}} = \widehat{H}$$

defines the associated bounding ellipsoid for the closed-loop system with $x_0 = 0$.

Proof. Taking into account Remark 4 and using combined feedback (4), the closed-loop system (1) takes the form

$$\begin{aligned} \dot{x} &= (A + B_1K)x + (D + B_1K_1)w, \quad x(0) = 0, \\ z &= (C + B_2K)x. \end{aligned} \tag{11}$$

By Theorem 3, the matrix of invariant ellipsoid of system (11) satisfies the LMIs