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Robust adaptive control with disturbances compensation[1]

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Abstract: The paper is concerned with design of the robust adaptive algorithm with disturbance compensation under parametric uncertainties and external disturbances. The designed control algorithm provides compensation of disturbances and tracking of reference signal with high accuracy in a finite time. Computer simulations prove the efficiency of the proposed algorithm.

Keywords: synchronization, uncertain dynamic systems, bounded disturbances, adaptive control, robust control.

1. INTRODUCTION

The problem of accuracy improvement in dynamic tracking has always been a significant issue for the systems, which have the need for high precision in functioning. These days such systems are widely spread in biology, robotics, telecommunications, etc. From theoretical point of view, adaptive control seems to be a good solution to the control problem. For example, in papers (Bazylev et al., 2013, Bazylev et al., 2014a, Bazylev et al., 2014b) the authors consider the application of the adaptive control for mechanical systems. However, the question of quality and reliability of adaptive schemes performance under disturbances in practice arose in 1970s. Poor results motivated many researchers to study mechanisms of disturbances and ways to counteract them (Ioannou et al., 2012), i.e. to make systems robust.

External disturbances, varying parameters, unmodeled dynamics and other complications of practical use of the automatic control schemes highlight the significance of robustness of control systems, which grows with increasing complexity of systems and quality requirements to their functioning. Consequently, robust control as a type of algorithm, which specifically insures this property in systems, has spread.

The issues of adaptive and robust control schemes were studied in (Gutman, 2001). According to the author, the strength of robust control lies in good transient characteristics. As for the adaptive control, its advantage lies in its ability to achieve tighter specifications in steady state and solve the control problem for a larger uncertainty set than robust control. It is suggested to always design the adaptive control on top of the robust one to unite their advantages.

Currently, there are many papers dedicated to robust adaptive control. Theoretical papers mainly concentrate on nonlinear systems control, while varying in specific issues of the performance of the considered systems such as saturation (Wen et al., 2011), deadzone (Jasim, 2013) and others. The usual design scheme is as follows: the original adaptive control is developed to counteract specific issues which cause disturbances, i.e. made robust to them. Rather exhaustive information on such control laws is given in (Ioannou et al., 2012). Robust adaptive control laws have wide aria of application which is apparent from topics considered in papers with practical applications: controlling motors (Cunha et al., 2005) and power systems (Wan et al., 2014), spacecraft cooperating rendezvous and docking (Sun et al., 2013), administration of medical drugs (Malagutti, 2014) and so on.

The present paper considers development of robust adaptive control by building the adaptive control on top of the robust control, initially designed in (Tsykunov, 2007). The advantage of the original robust control law is in its uniformity for a wide range of practical situations because of its robustness to external disturbances and model uncertainties. By implementing estimation of disturbances and their consequent compensation, the author manages to achieve good accuracy, but as it was mentioned in the beginning: there is always room for improvement. Therefore, in this paper we suggest to improve the performance of robust auxiliary loop control method by adding adaptive variable structure control law with approximated sign function.

The specific advantage of the sign adaptive control is its ability to achieve convergence in a finite time (Emelyanov et al., 1997) even in presence of small bounded disturbances. By building it on top of robust control law, we solve the problem of its transient performance in presence of significant disturbances, while achieving improvement of

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tracking accuracy in comparison to (Tsykunov, 2007) method performance. To simplify the proof of main results, sign function is approximated by smooth function of hyperbolic tangent similar to approximation used in (Wen et al., 2011) to approximate saturation function.

The organization of the paper is as follows. Section 2 states the description of the considered system and the control goal. In Section 3 the developed control algorithm is presented. Section 4 provides simulation results that illustrate the performance of the algorithm. Concluding remarks are drawn in Section 5. Appendix contains the proof of main results.

2. PROBLEM STATEMENT

Consider a dynamic system described by nth order differential equation as

$$\dot{x}(t) = Ax(t) + Bu(t) + Df(t), x(0) = x_0, \qquad (1)$$

where $x(t) \in R^n$ is a state vector, $x_0 \in R^n$ is an initial condition, $u(t) \in R$ is an input signal, $f(t) \in R$ is an external disturbance, $A \in R^{n\times n}$, $B \in R^n$, $D \in R^n$.

Reference model is defined as

$$\dot{x}_r(t) = A_r x_r(t) + B_r r(t), \; x_r(0) = x_{0r}, \qquad (2)$$

where $x_r(t) \in R^n$ is a state space vector, x_{0r} is an initial conditions vector, $r(t) \in R$ is piecewise continuous reference signal, matrix $A_r \in R^{n\times n}$ is Hurwitz, $B_r = [b_{r1}, b_{r2}, \ldots, b_{rn}]^{\mathrm{T}}$.

The control goal is to synthesize the control law such that:

$$|x(t) - x_r(t)| < \delta \text{ for } t > T, \qquad (3)$$

where $T > 0$ is a transient time, $\delta > 0$ is a required accuracy. The considered problem is solved under the following assumptions.

Assumptions

1) The pair of matrices (A, B) is controllable.
2) The elements of matrix A and the coefficients of vectors B, D and x_0 are unknown but belong to the known bounded set Ξ.
3) Following conditions are satisfied: $A = A_r + B_r c^{\mathrm{T}}$, $B = B_r + B_r k_1$, $D = B_r k_2$, where $c \in R^n$, $k_1 \in R$, $k_2 \in R$ are an unknown vector and numbers accordingly.
4) The reference signal and disturbance are bounded functions, i.e. $|r(t)| \le \bar{r}$, $|f(t)| \le \bar{f}$.

3. MAIN RESULTS

Taking into account Assumption 3, equation (1) is as follows

$$\dot{x}(t) = A_r x(t) + B_r u(t) + B_r\left(c^{\mathrm{T}} x(t) + k_1 u(t) + k_2 f(t)\right).$$

Then, the dynamics of the tracking error $\varepsilon(t) = x(t) - x_r(t)$ can be defined as

$$\dot{\varepsilon}(t) = A_r \varepsilon(t) + B_r u(t) + B_r \varphi(t), \qquad (4)$$

where $\varphi(t) = c^{\mathrm{T}} x(t) + k_1 u(t) + k_2 f(t) - r(t)$, $\varphi(t) \in R$ is the signal depending on the parametric and external disturbances of the system (1).

Let us use the auxiliary loop method (Tsykunov, 2007) to extract signal $\varphi(t)$ from (4). According to this method, we introduce the auxiliary loop in the following form

$$\dot{\varepsilon}_a = A_r \varepsilon_a + B_r u(t), \; \varepsilon_a(0) = \varepsilon_{a0}. \qquad (5)$$

Taking into account (4) and (5) the mismatch function $\zeta(t) = \varepsilon(t) - \varepsilon_a(t)$ is described as follows

$$\dot{\zeta}(t) = A_r \zeta(t) + B_r \varphi(t). \qquad (6)$$

Let $b_{rm} \neq 0$ in B_r, where $1 \le m \le n$. Selecting mth equation of the system (6), we get

$$\dot{\zeta}_m(t) = a_{rm}^{\mathrm{T}} \zeta(t) + b_{rm} \varphi(t),$$

where a_{rm}^{T} is the mth row of the matrix A_r. Then, the signal $\varphi(t)$ can be expressed as follows

$$\varphi(t) = \frac{1}{b_{rm}}\left(\dot{\zeta}_m(t) - a_{rm}^{\mathrm{T}} \zeta(t)\right). \qquad (7)$$

Substituting (7) into (4), we get the following expression for error dynamics

$$\begin{aligned}\dot{\varepsilon}(t) &= A_r \varepsilon(t) + B_r u(t) + B_r \frac{1}{b_{rm}}\left(\hat{\zeta}_m(t) - a_{rm}^{\mathrm{T}} \zeta(t)\right) \\ &+ B_r \frac{1}{b_{rm}}\left(\dot{\zeta}_m(t) - \hat{\zeta}_m(t)\right),\end{aligned} \qquad (8)$$

where $\hat{\zeta}_m(t)$ is an estimate of the unmeasured signal $\dot{\zeta}_m(t)$ which is obtained by means of the observer

$$\hat{\zeta}_m(t) = \frac{p}{\mu p + 1} \zeta_m(t), \qquad (9)$$

where $p = d/dt$, $\mu > 0$ is a small number.

The control signal $u(t)$ is introduced in the following form

$$\begin{aligned} u(t) &= u_1(t) + u_2(t), \\ u_1(t) &= -1/b_{rm}\left(\hat{\zeta}_m(t) - a_{rm}^{\mathrm{T}} \zeta(t)\right), \\ u_2(t) &= -\kappa(t)\tanh\left(\alpha B_r^{\mathrm{T}} P \varepsilon(t)\right). \end{aligned} \qquad (10)$$

In (10) $u_1(t)$ is the robust control law, and $u_2(t)$ is the adaptive control law, $\alpha > 0$, the matrix $P = P^{\mathrm{T}} > 0$ is the solution of the equation

$$A_r^{\mathrm{T}} P + P A_r = -Q, \; Q = Q^{\mathrm{T}} > 0. \qquad (11)$$

The adaptation algorithm is defined as

$$\dot{\kappa}(t) = -\beta \kappa(t) + \gamma \varepsilon^{\mathrm{T}}(t) \varepsilon(t), \qquad (12)$$

where $\gamma > 0$, $\beta > 0$ and β is a small number.

In (10) the coefficient $\alpha > 0$ is large enough so that the hyperbolic tangents $\tanh(\alpha \upsilon)$ is the approximation of the sign function (Fig. 1)

$$\operatorname{sgn}(\upsilon) = \begin{cases} 1, \upsilon > 0, \\ 0, \upsilon = 0, \\ -1, \upsilon < 0. \end{cases}$$