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Adaptive Control LMI-based design for descriptor systems rational in the uncertainties ⋆

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Abstract: Uncertain systems are considered. They are represented in a descriptor form, where the matrices have an affine dependence on the uncertain parameter. S-variable approach for the design of a robust adaptive control feedback loop is presented. The only requirement to build such an adaptive law is robust stability of the closed-loop system by a static gain. No assumption about passivity of the system is made. Asymptotic stability of the given adaptive control is proved using Lyapunov arguments, and gain adaptation parameters are tunable by linear matrix inequality based convex optimization. An application to the attitude control of a microsatellite of the CNES Myriade series illustrates the results.

Keywords: Robust adaptive control design; Convex optimization; Descriptor systems.

1. INTRODUCTION

The parameters of a system cannot be well known and might be subject to important variations. Adaptive control theory proposes to deal with this issue by making the gains of its law time-varying, depending on the real time measurements. But can robustness be proved? For example using robust control methods?
They are two types of adaptive control approaches: in an indirect adaptive scheme, the gains of the controller evolve with relation to an estimation of the parameters of the system. The idea of such an estimator has been introduced in Kalman (1958). But the indirect scheme does not fit well with uncertain systems and its implementation is complex, as highlighted in Rohrs et al. (1985). For these reasons, we choose to use the direct adaptive scheme, where the gains are directly modified according to the measured outputs, making its implementation very simple. The counterpart is that it is based on strong hypothesis, as the passivity of the system to be controlled (Fradkov (1974)). Moreover, noise on the measurements tends to push the gains of the controller to infinity. To tackle this issue, Ioannou and Kokotović (1983) and Kaufman et al. (1994) propose the so-called $\sigma-$modification to achieve changes in the dynamics based on the measured outputs.
In robust control community, the effectiveness of LMI-based methods has been widely proved (Boyd et al. (1994)), but only a few works use them in the context of adaptive control of uncertain systems. In Luzi et al. (2014), only certain systems are treated, whereas in Lu and Xia (2013) and Zhu et al. (2011), some assumptions are made about the uncertainties, but they are not always verifiable; Ben Yamin et al. (2007) designs a simple adaptive controller, which does not require the knowledge of the system dynamics.
In this paper, we deal with direct adaptive control of uncertain systems, and the controllers are designed using LMI-based methods. The paper has three main contributions: First, the passivity of the system is not required. Second, we use the recent results of descriptor systems, that applies for systems rational in the uncertainties (Watanabe et al. (2013) and Ebihara et al. (2015)). The third major contribution of this paper is the establishment of new results proving that adaptive law has improved (at least no worse) robustness, compared to a given static feedback controller.
The paper is organized as follows: First, we justify our choice to use a descriptor representation. In section III, we design adaptive controllers, with no worse and improved robustness respectively. An application is given in section IV. Finally, we give some conclusions and outlooks for future work.

Notation. I stands for the identity matrix. $\{1;V\}$ is the set of all the integers between 1 and V. A^T is the transpose of the matrix A. $A^{\mathcal{S}}$ stands for the symmetric matrix $A+A^T$. $A(\preceq)\prec B$ is the matrix inequality stating that $A-B$ is negative (semi-)definite. If $A\in\mathbb{R}^{n\times m}$ and rank$A=r$, $A^{\perp}$ is a full rank matrix such that $A^{\perp}\in\mathbb{R}^{(n-r)\times n}$ and $A^{\perp}A=0$. A° is a full rank matrix such that $A^{\circ}\in\mathbb{R}^{m\times r}$ and AA° is full rank.

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2. PRELIMINARIES ABOUT DESCRIPTOR SYSTEMS

A system can be represented with the following descriptor form

$$E_{xx}\dot{x}(t) + E_{x\pi}\pi(t) = Ax(t) + Bu(t)\ ,\quad y(t) = Cx(t) \quad (1)$$

where $x \in \mathbb{R}^{n_x}$ is the state of the system, $u \in \mathbb{R}^{n_u}$ is the control input, $\pi \in \mathbb{R}^{n_\pi}$ is an auxiliary signal (see 4.). $E_{xx} \in \mathbb{R}^{n\times n_x}$, $E_{x\pi} \in \mathbb{R}^{n\times n_\pi}$, $A \in \mathbb{R}^{n\times n_x}$ and $B \in \mathbb{R}^{n\times n_u}$define the system.

One of the main advantages of descriptor systems is that they derive directly from physical representations. Moreover they happen to be well suited for dealing with uncertainties. The following result is stated in Masubuchi et al. (2003) and generalized in Ebihara et al. (2015):

Theorem 1. Assume a parameter-dependent descriptor model

$$\begin{array}{l}\bar{E}_{xx}(\delta)\dot{x}(t) + \bar{E}_{x\pi}(\delta)\bar{\pi}(t) = \bar{A}(\delta)x(t) + \bar{B}(\delta)u(t)\\ y(t) = Cx(t),\quad \delta \in \Delta^V\end{array} \quad (2)$$

where $\Delta^V := \left\{\delta \in \mathbb{R}^V : \delta \geq 0, \mathbf{1}^T\delta = 1\right\}$ and the δ-dependent matrices are rational with respect to the components of the uncertain vector δ. Then, there always exists another parameter-dependent descriptor model

$$\begin{array}{l}E_{xx}(\delta)\dot{x}(t) + E_{x\pi}(\delta)\pi(t) = A(\delta)x(t) + B(\delta)u(t)\\ y(t) = Cx(t),\quad \delta \in \Delta^V\end{array} \quad (3)$$

in which the δ-dependent matrices are affine functions of δ, that is $E_{xx}(\delta) = \sum_{v=1}^{V}\delta_v E_{xx}^{[v]}$, $E_{x\pi}(\delta) = \sum_{v=1}^{V}\delta_v E_{x\pi}^{[v]}$, $A(\delta) = \sum_{v=1}^{V}\delta_v A^{[v]}$ and $B(\delta) = \sum_{v=1}^{V}\delta_v B^{[v]}$, $E_{xx}^{[v]}$, $E_{x\pi}^{[v]}$, $A^{[v]}$ and $B^{[v]}$ being the values of the matrices of the system on the V vertices of δ. Descriptor representations allow to handle rational systems as if affine in the uncertainties, which is a key point.

In all the following, we consider that the matrices which describe the system are affine functions of the uncertain parameter δ. In order to get a condition of stability for systems of the form of (3), we suppose the following assumption holds:

Assumption 1: It is assumed that

$$[E_{xx}(\delta)\ \ E_{x\pi}(\delta)] = E_1(\delta)\left[E_{2xx}\ \ E_{2x\pi}\right] \quad (4)$$

where $E_1(\delta) = \sum_{v=1}^{V}\delta_v E_1^{[v]}$ is full column rank for all $\delta \in \Delta^V$.

Assumption 1 means that the potential impulsive and non dynamic modes of system (3) do not depend on the uncertainty δ.

We can now recall the result of Ebihara et al. (2015) for uncertain descriptor systems:

Theorem 2. Under assumption 1, let $E_2 = E_{2x\pi}^{\perp}E_{2xx}$. The system (3) is robustly stable if there exist matrices $\hat{P}^{[v]} = \hat{P}^{[v]^T}$, $\hat{Y}^{[v]}$ and $\hat{S}$ such that the following conditions hold for all $v \in \{1;V\}$:

$$(E_2E_2^{\circ})^T\hat{P}^{[v]}(E_2E_2^{\circ}) \succ 0 \quad (5)$$

$$\begin{bmatrix} 0 & \hat{P}_e^{[v]^T}\\ \hat{P}_e^{[v]} & 0\end{bmatrix} + \left\{\hat{S}\left[E_1^{[v]}\ \ -A^{[v]}\right]\right\}^{\mathcal{S}} \prec 0 \quad (6)$$

where $\hat{P}_e^{[v]} = (E_2^T\hat{P}^{[v]} + \hat{Y}^{[v]^T}E_2^{\perp})E_{2x\pi}^{\perp}$.
By stability, we mean boundedness and convergence of E_2x and the absence of impulsive modes, see Ebihara et al. (2015) for details.

Remark: Condition of Theorem 2 only requires that (6) is satisfied for all $v \in \{1;V\}$. By convexity, it implies that it holds for all $\delta \in \Delta^V$ with parameter dependent matrices $\hat{P}(\delta) = \sum_{v=1}^{V}\delta_v\hat{P}^{[v]}$ and $\hat{Y}(\delta) = \sum_{v=1}^{V}\delta_v\hat{Y}^{[v]}$. $\hat{P}(\delta)$ defines a parameter-dependent quadratic Lyapunov function for the plant.

3. LMI-BASED ROBUST ADAPTIVE CONTROL DESIGN

The main result of this paper aims at designing an adaptive law which stabilizes the system (3) for every value of the uncertain vector δ, under the following assumption:

Assumption 2: Under assumption 1, let $u(t) = K_0y(t)$ be a static output feedback. It is assumed that there exist matrices $\hat{P}^{[v]} = \hat{P}^{[v]^T}$, $\hat{Y}^{[v]}$ and $\hat{S}$ such that for all $v \in \{1;V\}$, conditions of Theorem 2 hold for the closed-loop system.

The proposed adaptive law consists in replacing the static feedback by a structured time-varying control

$$u(t) = (K_0 + LK(t)R)y(t) \quad (7)$$

where L and R are partitioned with appropriate dimensions such that $LKR = \sum_{k=1}^{\bar{k}}L_kK_kR_k$. $K(t) = \mathrm{diag}\left(K_1(t), K_2(t), ...\right)$, $L = [L_1\ L_2\ ...]$, $R^T = \left[R_1^T\ R_2^T\ ...\right]$ and the adaptation is driven by

$$\begin{array}{l}\dot{K}_k(t) = \mathrm{Proj}_{D_k}\left(K_k(t),\ W_k(t)\right)\\ W_k(t) = \gamma_k(-G_ky(t)(R_ky(t))^T - \sigma_kK_k(t)).\end{array} \quad (8)$$

where D_k defines an ellipsoidal set $\mathcal{E}_k$:

$$K_k \in \mathcal{E}_k \Leftrightarrow \mathrm{Tr}(K_k^TD_kK_k) \leq 1 \quad (9)$$

and Proj_{D_k} is the operator defined as in Praly (1992). When the gain K_k is inside the set, the operator outputs $\dot{K}_k = W_k$, and when K_k is at the border of the set, the operator aims at pushing it inside the set, so that the gains cannot exit the set:

$$\mathrm{Proj}_{D_k}\left(K_k,\ W_k\right) = W_k - H_k$$

where H_k is such that

$$\begin{array}{l}H_k = 0 \text{ if } K_k \in \mathcal{E}_k\\ \text{else s.t.} \begin{cases}\mathrm{Tr}(\dot{K}_k^TD_kK_k) \leq 0\\ \mathrm{Tr}((K_k - F_k)^TH_k) \geq 0\ \ \forall F_k \in \mathcal{E}_k\end{cases}\end{array} \quad (10)$$

The definition of the operator guarantees that K_k remains bounded, with a bound inversely proportional to the square-root of $\|D_k\|$. Notice that if the gains are scalar, (8) can be implemented as a saturated integrator.