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A Dynamic Threshold Based Algorithm for Change Detection in Autonomous Systems

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Abstract: The paper deals with the detection of abrupt changes in dynamical systems in the presence of external noise. The system is considered as a black box. That is, it is supposed that nothing is known about a structure and relations within the system, and we can just obtain the values of some parameters of the system. A new algorithm is proposed in the paper to deal with such highly uncertainty. At first, we use moving average with adaptive window size, then GLR (Generalized Likelihood Ratio) is used to detect abrupt changes in the obtained data using an adaptive threshold. This approach is applied to slowdown detection of a small autonomous car with only accelerometer on the board.

Keywords: Uncertain systems; Fault detection; Estimation

1. INTRODUCTION

The problem of change detection is to determine changes in the characteristics of a dynamical system during time. Often, such characteristics can be variation in mean value and variance of the distribution of some parameters of the system. This study is close in meaning to fault detection. But in case of fault detection a fault occurs only inside the system, while in case of change detection there are no difference what is a source of change.

The problem of abrupt changes detection is highly important, because an abrupt change may be a result of some underlying or external problem. This problem is well studied, and many approaches are proposed (see Akimov and Matasov (2015), Basseville and Nikiforov (1998), Blanke et al. (2006)). Some of them suppose that probability density function depends upon a scalar parameter, while others suppose multidimensional probability density function. In this paper we consider the first one - a case of scalar parameter.

In this paper it is assumed that the dynamical system is considered as a black box. That is, we do not know how the parameters of the system are related to each other, and we can only measure some of these parameters. Also it is assumed that measurable data contains Gaussian noise, which is a practical and reasonable assumption. In this conditions it is impossible to apply such well known approaches as Kalman filter, because the system is just a black box. Therefore, one of the possible way could be to use moving average to filter noise firstly. For this purpose we use spatial adaptive estimation of nonparametric regression (see Goldenshluger and Nemirovski (1997), Lepskii (1990)), which proposes how to select adaptive window size. After we got filtered data, we are interested in the two following things - a method for detection an abrupt change and selection of a threshold. There are many ways to detect an abrupt change on the filtered data (see Basseville and Nikiforov (1998)). We have selected GLR, because it shows good properties when the actual value of a parameter after change is unknown (it is supposed that it is from some predefined set). Finally, after we detect a change on the filtered observations, a question about threshold in the presence of external noise arises. That is, some constant threshold is used in GLR, but we remember that there is a noise initially. So we need to choose a new dynamic threshold, such that it would be no worse than constant one in the presence of noise.

The idea of reconstruction of a constant threshold to the dynamic one is not new. Such thresholds can be used to reduce the delays associated with the constant threshold method (see Perhinschi et al. (2006), Verdier et al. (2008), Shu et al. (2008)) and for more accurate estimation in the presence of noise (see Davis et al. (2006)).

The main contribution of the paper lies in the combining of Generalized Likelihood Ratio algorithm with an idea of dynamic threshold obtained by moving average with

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adaptive window size to create more accurate and efficient method for abrupt changes detection.

We have made an experiment to show how this approach works in the real world. We consider a small autonomous car, that detects its own slowdown. The car is based on STM32F3-Discovery with accelerometer inside. It moves along a smooth road and at some point faces with a piece of a foam rubber. It does not stop, but slows down. We detect this moment of time and stop the motor.

The article is organized as follows. Section 2 presents the problem statement. In sections 3 and 4 an overview of used methods is provided. Section 5 presents the main result. The results of numerical experiments are shown in Section 6, followed by Conclusions and Future work discussion.

2. PROBLEM STATEMENT

Let us consider the dynamical system with the model of observations. Because we consider the system as a black box, we can only deal with measurable part. So let us assume that we have a sequence of independent random variables Y_t, which can be measured at each time instance.

Our goal is to determine a fault in the observable variables Y_t. Each of variables $Y^i = (y_1, y_2, ..., y_m)$ have the probability density function p^i_θ depending upon only one scalar parameter $\theta \in \mathbb{R}$. Before the unknown moment of time t_0, the parameter θ is equal to θ_0. The problem is to detect a change of the parameter θ.

We solve this problem in two steps:

(1) Filter a noise out of Y_t.
(2) Apply Generalized Likelihood Ratio to the obtained data.

3. SPATIAL ADAPTIVE ESTIMATION OF NONPARAMETRIC REGRESSION

Let us consider the following problem. We want to restore a signal function from noisy observations. Suppose there are noisy observations $y(x)$ of a signal function $f(x)\colon [0,1] \to \mathbb{R}$ – along the regular grid $\Gamma_n = \{i/n, i = 0, ..., n\}$:

$$y(x) = f(x) + \xi(x), \tag{1}$$

where $\{\xi(x)\}_{x\in\Gamma_n}$ is a sequence of independent $\mathcal{N}(0,1)$ random variables defined on the underlying probability space (Ω, A, P).

In Goldenshluger and Nemirovski (1997) the estimates by the least square method of $f(x)$ at a given point $u \in [0,1]$ are considered. Suppose that the degree of estimate is 1. So we get the following approximation at a given point x_0:

$$\hat{f}_\Delta(x_0) = \frac{1}{N_\Delta} \sum_{x\in M_\Delta} y(x), \tag{2}$$

where $\Delta \in [0,1]$ is some segment $[x_0 - \delta, x_0 + \delta]$ centered at x_0 and containing at least one observation point, M_Δ is the set of observation points in Δ, N_Δ is the cardinality of M_Δ.

The problem is to select "the best" window when no a priori information on f is available. Let us introduce the following estimation:

$$|\hat{f}(x_0) - f(x_0)| \le \omega_f(x_0, \delta) + N_\Delta^{-1/2}|\zeta(\Delta)|, \tag{3}$$

where $\zeta(\Delta) = \frac{1}{N_\Delta}\sum_{x\in M_\Delta} \xi(x)$, $\omega_f(x,\delta) = \sup\limits_{x\in\Delta} |f(x) - f(x_0)|$. The right hand side is comprised of two terms – deterministic $\omega_f(x,\delta)$ and stochastic error $N_\Delta^{-1/2}|\zeta(\Delta)|$. Since $\zeta(\Delta)$ is $\mathcal{N}(0,1)$, the stochastic error typically is of order of $(n\delta)^{-1/2}$:

$$P\{N_\Delta^{-1/2}|\zeta(\Delta)| > \kappa(n\delta)^{-1/2}\} \le exp\{-c\kappa^2\}, \tag{4}$$

with certain absolute constant $c > 0$. Now, there are no more than n essentially different (resulting in different sets M_Δ) choices of Δ. Let these choices be

$$\Delta_1 \subset \Delta_2 \subset ... \subset \Delta_N,$$

and let $2\delta_1, 2\delta_2, ..., 2\delta_N$ be the length of the windows $\Delta_1, \Delta_2, ..., \Delta_N$. We obtain

$$\Omega_\kappa = \{\omega \in \Omega \mid N_{\Delta_i}^{-1/2}|\zeta(\Delta_i)| \le \kappa(n\delta_i)^{-1/2}, \quad where\ i = 1, ..., N\}. \tag{5}$$

Assuming that $\omega \in \Omega_k$, (3) can be strengthen as

$$|\hat{f}_{\Delta_i}(x_0) - f(x_0)| \le \omega_f(x_0, \delta_i) + \kappa(n\delta_i)^{-1/2}, \tag{6}$$

Notice that as i grows, then the first term in the right hand side increases, and the second term decreases; therefore a reasonable choice of the window to be used is that one which balances both the terms, say, the one related to the largest i with $\omega_f(x_0, \delta_i) \le \kappa(n\delta_i)^{-1/2}$.

4. GENERALIZED LIKELIHOOD RATIO

This method is based on the likelihood ratio test (see Granichin et al. (2015)). The main reason to use it is that the parameter θ_1 is unknown after change. Let us introduce log-likelihood ratio for the observations from time j up to time k is

$$S_k^j = \sum_{i=j}^{k} \ln \frac{p_{\theta_1}(y_i)}{p_{\theta_0}(y_i)} \tag{7}$$

In the present case, θ_1 is unknown; therefore, this ratio is a function of two unknown independent parameters : the change time and the value of the parameter after change. The standard statistical approach is to use the maximum likelihood estimates of these two parameters, and thus the double maximization:

$$g_k = max_{1\le j\le k} \sup_{|\theta_1-\theta_0|\ge\nu>0} S_k^j(\theta_1) \tag{8}$$

and the following stopping rule:

$$t_a = \min\{k : \sum_{i=0}^{N-1} I_{\{g_{k-i}\ge h\}} \ge \eta\}, \tag{9}$$

where I is an indicator function, h is a threshold for the derivative, and η is a threshold for the number of crossings of h, and ν is a known minimum magnitude.

5. THE ESTIMATE OF A SIGNAL

In this section we will introduce a dynamic threshold and show how it depends on a constant threshold in the presence of standard Gaussian noise. To do this we apply (6) for the estimation f of the input signal (2). And then we will use this estimation in Generalized Likelihood Ratio.