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Adaptive control of second-order plants in the presence of unmodeled dynamics ⋆

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Abstract: In this paper, we provide a sufficient condition for global boundedness of the closed-loop adaptive system comprised of a general second–order plant, whose state variables are accessible, in the presence of a minimally restrictive class of unmodeled dynamics.

Keywords: Adaptive Control, Robustness, Stability Criteria

1. INTRODUCTION

The classical model reference adaptive control problem showed that a control law with a suitable parameter adjustment mechanism can be designed such that signal boundedness and asymptotic tracking is achieved for systems containing parametric uncertainties. It was shown (see, for example, Rohrs et al. (1985)) that the stability and performance properties of the adaptive system deteriorated with the introduction of nonparametric perturbations, thus invoking the body of work known as robust adaptive control. Despite tremendous advancements made in this field, obtaining quantifiable and practically meaningful robustness margins for adaptive systems has eluded researchers for the past three decades.

In this paper, we show that for a class of plants whose states are accessible, adaptive control in the presence of unmodeled dynamics can lead to global boundedness. Our recent work in Hussain et al. (2013a) showed the same result for scalar plants with a single unknown parameter. Hussain et al. (2013b) extended this result to second-order plants but with fairly restrictive assumptions on the parametric uncertainties. In this paper, we remove this restriction and demonstrate global boundedness for the case of zero reference input. The theoretical result is validated through numerical simulations in Hussain et al. (2016) where the Rohrs counterexample in Rohrs et al. (1985) is considered as the underlying unmodeled dynamics. Earlier work in this area has either shown semi-global boundedness Ioannou and Sun (1996); Narendra and Annaswamy (1989) or have considered fairly restrictive classes of unmodeled dynamics. This paper relaxes both of these components.

The robust adaptive control problem is presented first in §2. The analytic methodology employed in this paper is premised upon a nonsingular transformation of the system dynamics and analyzing the behavior of its corresponding solutions. The former is introduced in §3-§5 and the latter in §7 after identifying regions for which the underlying dynamics are stable in §6. Conditions for global boundedness of a second-order adaptive system in the presence of unmodeled dynamics is presented in §8, which is the main result of this paper.

2. THE PROBLEM STATEMENT

The problem we address in this paper is the adaptive control of a second–order plant whose state variables are accessible described by

$$\dot{x}_p(t) = A_p x_p(t) + b_p v(t) \tag{1}$$

where $A_p \in \mathbb{R}^{2\times 2}$ is known and constant, $b_p \in \mathbb{R}^{2\times 1}$ is known and constant, (A_p, b_p) controllable, and $v(t)$ is a scalar input. It is assumed that A_p contains an admissable parametric uncertainty $|a_p| \leq \bar{a}$ where $\bar{a}$ is a known positive constant. The unmodeled dynamics are unknown and defined as

$$\begin{aligned}\dot{x}_\eta(t) &= A_\eta x_\eta(t) + b_\eta u(t)\\ v(t) &= {c_\eta}^\top x_\eta(t)\end{aligned} \tag{2}$$

where $u(t)$ is the control input, ${c_\eta}^\top \in \mathbb{R}^{1\times m}$, $x_\eta \in \mathbb{R}^{m\times 1}$, and $A_\eta \in \mathbb{R}^{m\times m}$ is Hurwitz with $G_\eta(s) \triangleq c_\eta^\top(sI_{mxm} - A_\eta)^{-1}b_\eta$, and $(c_\eta^\top, A_\eta, b_\eta)$ is observable and controllable. The goal is to design the control input such that $x_p(t)$ follows $x_m(t)$ which is specified by the reference model

$$\dot{x}_m(t) = A_m x_m(t) + b_m r(t) \tag{3}$$

where $A_m \in \mathbb{R}^{2\times 2}$ is Hurwitz, $b_m \equiv b_p$, and $r(t)$ is a bounded reference input. It is important to note that the scope of this paper is limited to the case of zero reference input. That is, $r(t) = 0$ for all time. It is further assumed

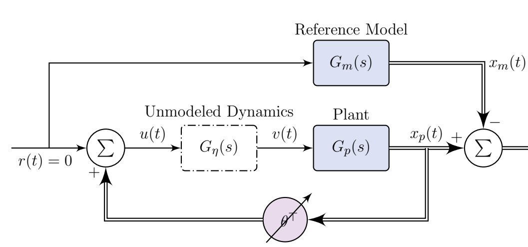

Fig. 1. Adaptive control with unmodeled dynamics.

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that A_m is chosen such that there exists a $\theta^\star = [\theta_0^\star \;\; \theta_1^\star]^\top$ satisfying

$$A_p + b_p \theta^{\star\top} = A_m \tag{4}$$

for the plant in (1).

A control law is chosen (see Fig. 1) as

$$u(t) = \theta^\top(t) x_p(t) \tag{5}$$

where the parameter $\theta(t) = [\theta_0(t) \;\; \theta_1(t)]^\top$ is updated using a modified version of the standard adaptive law.

In this paper, we will extend the method in Hussain et al. (2013a) and Matsutani et al. (2013) and establish conditions under which global boundedness of the closed-loop adaptive system given by (1)-(5) can be achieved.

3. A NONSINGULAR TRANSFORMATION

In order to properly identify the effect the perturbation due to the presence of unmodeled dynamics has on the adaptive system, nonsingular transformations of the adaptive parameter $\theta(t)$ and tracking error $e(t) = x_p(t) - x_m(t)$ are performed. The will become clear in future deliberations (see §7.1 and §7.2).[1] We will refer to the i^{th} component of the transformed states as $\mathscr{E}_i(t)$ and $\vartheta_i(t)$ for $i = \{0, 1\}$. We define a transformed error $\mathscr{E}(t)$ and transformed parameter $\vartheta(t)$ by

$$\mathscr{E}(t) \equiv Ce(t) \tag{6}$$

$$\vartheta(t) \equiv M\theta(t) \tag{7}$$

where C and M are transformation matrices whose construction utilizes (3) and the solution $P = P^\top > 0$ of the Lyapunov Equation $A_m^\top P + PA_m = -Q$. The transformation matrices C and M are defined as

$$C = \begin{bmatrix} c_0 \;\; c_1 \end{bmatrix}^\top \tag{8}$$

$$M = p_{bb} C P^{-1} \tag{9}$$

where $\sqrt{b_m^\top P b_m}$, and the vectors $c_i \in \mathbb{R}^{2\times 1}$ are constructed such that

$$c_0 = p_{bb}^{-1} P b_m \tag{10}$$

$$c_1 c_1^\top = P - c_0 c_0^\top. \tag{11}$$

Remark 1. Two important properties of the construction of C include $c_0^\top b_m = p_{bb}$ and $c_1^\top b_m = 0$. The latter can be shown from the definition of c_0 in (10) and by design since (11) can be rewritten as[2]

$$CP^{-1}C^\top = I. \tag{12}$$

Together, these two properties aid in capturing the dominant effect the perturbation has on the adaptive system.

4. AN ADAPTIVE LAW

The specific adaptive law we propose is an extension of the scalar form of the projection algorithm in Hussain et al. (2013a) to higher dimensional plants. We propose the adaptive law as

$$\dot\theta = M^{-1}\dot\vartheta \tag{13}$$

with M in (9) and

$$\dot\vartheta_i = \text{Proj}\left(\{M\theta\}_i, -\{M\Gamma x_p b_m^\top P e\}_i\right) \tag{14}$$

[1] This methodology was originally proposed in Matsutani et al. (2013) for general n^{th} order plants in the presence of time delay.

[2] We refer the reader to Hussain (2016) for further details

where the scalar projection algorithm can be defined as

$$\text{Proj}(\Theta_i, y_i) = \begin{cases} \dfrac{\Theta_{i,\max}^2 - \Theta_i^2}{\Theta_{i,\max}^2 - \Theta_{i,\max}'^2} y_i & \Theta_i \in \Omega_i \wedge \Theta_i y_i > 0 \\ y_i & \text{otherwise} \end{cases} \tag{15}$$

where

$$\begin{aligned} \overline{\Omega}_i &= \{\Theta_i \in \mathbb{R}^1 \mid \quad \Theta_{i,\max}' \le \Theta_i \le \Theta_{i,\max}\} \\ \underline{\Omega}_i &= \{\Theta_i \in \mathbb{R}^1 \mid -\Theta_{i,\max} \le \Theta_i \le -\Theta_{i,\max}'\} \\ \Omega_i &= \overline{\Omega}_i \cup \underline{\Omega}_i \end{aligned} \tag{16}$$

with positive constants $\Theta_{i,\max} > \Theta_{i,\max}'$. It can be shown in Lavretsky and Gibson (2011) that the projection algorithm $\dot\Theta_i = \text{Proj}(\Theta_i, y_i)$ ensures that $|\Theta_i(t)| \le \Theta_{i,\max}$ for all $t \ge t_0$. That is, the projection algorithm guarantees boundedness of the parameter $\Theta_i(t)$ independent of the system dynamics.

5. PROPERTIES OF THE REFERENCE MODEL

We will now use the reference model in (3) and C in (8) to define the $n \times n$ matrix

$$\mathscr{A}_m = CA_m P^{-1} C^\top. \tag{17}$$

Furthermore, we define the scalars

$$\alpha_{ij} = c_i^\top A_m P^{-1} c_j, \quad \forall i, j = \{0, 1\} \tag{18}$$

and partition $\mathscr{A}_m$ as

$$\mathscr{A}_m = \begin{bmatrix} \alpha_{00} & a_1 \\ a_0 & \mathscr{A}_m' \end{bmatrix}. \tag{19}$$

It can be shown that both $\mathscr{A}_m$ and $\mathscr{A}_m'$ are Hurwitz. The former holds since $\mathscr{A}_m = CA_mC^{-1}$ which implies that the eigenvalues of $\mathscr{A}_m$ are equal to those of A_m. For the latter, it can be shown using a suitable permutation similarity of $\mathscr{A}_m^\top + \mathscr{A}_m$ and Sylvester's Minorant Criterion that $\mathscr{A}_m'^\top + \mathscr{A}_m'$ is negative definite, proving $\mathscr{A}_m'$ is Hurwitz.[2]

5.1 Preliminaries

Before proceeding to the main result, we will introduce notation and mathematical preliminaries used throughout the paper. For a matrix $A \in \mathbb{R}^{n\times n}$, we define

$$\underline{\lambda}_A \triangleq \min_i |\Re\left(\lambda_i(A)\right)|$$

$$\overline{\lambda}_A \triangleq \max_i |\Re(\lambda_i(A))|$$

where λ_i is the i^{th} eigenvalue of A and $\Re(\lambda_i)$ denotes its real part.

For any vector $x \in \mathbb{R}^{n\times 1}$, we refer to the i^{th} component as x_i for each $i = \{0, \ldots, n-1\}$ and define $\bar{x} \triangleq \max_t \|x(t)\|$ where $\|\cdot\| = \|\cdot\|_2$ represents the Euclidean norm. Furthermore, the $n-1$ subvector of x shall be defined as $x' \equiv [x_1 \quad x_2 \cdots x_{n-1}]^\top$ and the subregion $\mathfrak{X}'$ defined as $\mathfrak{X}' \equiv \cup_{i=1}^{n-1} \mathfrak{X}_i$ where $\mathfrak{X}_i \in \mathbb{R}^{2n}$. Let $\mu(\bullet)$ denote the measure of a set.

Lemma 1. (Barbălat's Lemma). If $f : \mathbb{R}^+ \to \mathbb{R}$ is uniformly continuous for $t \ge 0$, and $\lim_{t\to\infty} \int_0^t |f(\tau)|\, d\tau$ exists and is finite, then $\lim_{t\to\infty} f(t) = 0$.

Corollary 1. If $g \in \mathscr{L}_2 \cap \mathscr{L}_\infty$, and $\dot g \in \mathscr{L}_\infty$, then $\lim_{t\to\infty} g(t) = 0$.

Proof. Choose $f(t) = g^2(t)$. Then the conditions of Barbalat's Lemma (see Narendra and Annaswamy (1989)) are satisfied. □