Available online at www.sciencedirect.com **12th IFAC International Workshop on International Workshop on International Workshop on International Workshop Adaptation and Learning in Control and Signal Processing** Available online at w

IFAC-PapersOnLine 49-13 (2016) 152–157

Adaptive control of second-order plants in daptive control of second-order plants
the presence of unmodeled dynamics * \mathbf{A} Adaptive control of second-order plants in Adaptive control of second-order plants in
the presence of unmodeled dynamics \star

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loop adaptive system comprised of a general second-order plant, whose state variables are accessible, in the presence of a minimally restrictive class of unmodeled dynamics. Abstract: In this paper, we provide a sufficient condition for global boundedness of the closed-Abstract: In this paper, we provide a sufficient condition for global boundedness of the closed-Abstract: In this paper, we provide a sufficient condition for global boundedness of the closed-
loop adaptive system comprised of a general second-order plant, whose state variables are loop adaptive system comprised of a general second–order plant, whose state variables are second-order plant, whose state variables are α accessible, in the presence of a minimally restrictive class of unmodeled dynamics.

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Keywords: Adaptive Control, Robustness, Stability Criteria Keywords: Adaptive Control, Robustness, Stability Criteria

1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

The classical model reference adaptive control problem showed that a control law with a suitable parameter adjustment mechanism can be designed such that signal boundedness and asymptotic tracking is achieved for systems containing parametric uncertainties. It was shown (see, for example, Rohrs et al. (1985)) that the stability and performance properties of the adaptive system deteriorated with the introduction of nonparametric perturbations, thus invoking the body of work known as robust adaptive control. Despite tremendous advancements made in this field, obtaining quantifiable and practically meaningful robustness margins for adaptive systems has eluded researchers for the past three decades. The classical model reference adaptive control problem \mathbf{I} The classical model reference adaptive control problem The classical model reference adaptive control problem The classical model reference adaptive control problem showed that a control law with a suitable parameter showed that a control law with a suitable parameter showed that a control law with a suitable parameter adjustment mechanism can be designed such that signal adjustment mechanism can be designed such that signal adjustment mechanism can be designed such that signal boundedness and asymptotic tracking is achieved for sys-boundedness and asymptotic tracking is achieved for sys-boundedness and asymptotic tracking is achieved for systems containing parametric uncertainties. It was shown tems containing parametric uncertainties. It was shown tems containing parametric uncertainties. It was shown
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adaptive control. Despite tremendous advancements made in this field, obtaining quantifiable and practically mean-in this field, obtaining quantifiable and practically mean-in this field, obtaining quantifiable and practically meaningful robustness margins for adaptive systems has eluded ingful robustness margins for adaptive systems has eluded ingful robustness margins for adaptive systems has eluded and performance properties of the adaptive system dete-and performance properties of the adaptive system dete-

In this paper, we show that for a class of plants whose states are accessible, adaptive control in the presence of unmodeled dynamics can lead to global boundedness. Our recent work in Hussain et al. (2013a) showed the same result for scalar plants with a single unknown parameter. Hussain et al. $(2013b)$ extended this result to secondorder plants but with fairly restrictive assumptions on the parametric uncertainties. In this paper, we remove this restriction and demonstrate global boundedness for the case of zero reference input. The theoretical result is validated through numerical simulations in Hussain et al. (2016) where the Rohrs counterexample in Rohrs et al. (1985) is considered as the underlying unmodeled dynamics. Earlier work in this area has either shown semi-global boundedness Ioannou and Sun (1996); Narendra and Annaswamy (1989) or have considered fairly restrictive classes of unmodeled dynamics. This paper relaxes both of these components. ponents. ponents. In this paper, we show that for a class of plants whose T robust adaptive control problem is presented first in problem is presented first in problem in T In this paper, we show that for a class of plants whose In this paper, we show that for a class of plants whose In this paper, we show that for a class of plants whose states are accessible, adaptive control in the presence of states are accessible, adaptive control in the presence of states are accessible, adaptive control in the presence of unmodeled dynamics can lead to global boundedness. Our unmodeled dynamics can lead to global boundedness. Our unmodeled dynamics can lead to global boundedness. Our uninousted dynamics can read to grobal boundedness. Our recent work in riussam et al. (2010a) showed the same ter. Hussain et al. (2013b) extended this result to secondorder plants but with fairly restrictive assumptions on the parametric uncertainties. In this paper, we remove this re-parametric uncertainties. In this paper, we remove this re-parametric uncertainties. In this paper, we remove this restriction and demonstrate global boundedness for the case striction and demonstrate global boundedness for the case striction and demonstrate global boundedness for the case of zero reference input. The theoretical result is validated of zero reference input. The theoretical result is validated of zero reference input. The theoretical result is validated of zero reference input. The theoretical result is vanuated
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The robust adaptive control problem is presented first in First results dulper control problem is presented into in premised upon a nonsingular transformation of the system dynamics and analyzing the behavior of its corresponding solutions. The former is introduced in $\S3-\S5$ and the latter in §7 after identifying regions for which the underlying The robust adaptive control problem is presented first in The robust adaptive control problem is presented first in
So The robustic methodology employed in this present premised upon a nonsingular transformation of the system dynamics and analyzing the behavior of its corresponding ay namics and analyzing the behavior of its corresponding solutions. The former is introduced in g_0 - g_0 and the latter $\frac{1}{2}$ premised upon a nonsingular transformation of the system premised upon a nonsingular transformation of the system solutions. The former is introduced in §3-§5 and the latter solutions. The former is introduced in §3-§5 and the latter dynamics are stable in §6. Conditions for global boundeddynamics are stable in go. Conditions for global bounded-
ness of a second-order adaptive system in the presence of unmodeled dynamics is presented in \S , which is the main result of this paper. result of this paper. result of this paper. result of this paper. dynamics are stable in §6. Conditions for global boundeddynamics are stable in §6. Conditions for global boundeddynamics are stable in §6. Conditions for global bounded-dynamics are stable in §6. Conditions for global bounded-dynamics are stable in §6. Conditions for global boundedunmodeled dynamics is presented in §8, which is the main unmodeled dynamics is presented in §8, which is the main unmodeled dynamics is presented in §8, which is the main

2. THE PROBLEM STATEMENT 2. THE PROBLEM STATEMENT 2. THE PROBLEM STATEMENT 2. THE PROBLEM STATEMENT 2. THE PROBLEM STATEMENT

The problem we address in this paper is the adaptive control of a second–order plant whose state variables are accessible described by accessible described by accessible described by accessible described by The problem we address in this paper is the adaptive $x^2 + y^2 = 4$ The problem we address in this paper is the adaptive The problem we address in this paper is the adaptive The problem we address in this paper is the adaptive
control of a second-order plant whose state variables are

$$
\dot{x}_p(t) = A_p x_p(t) + b_p v(t)
$$
 (1)

where $A_p \in \mathbb{R}^{2 \times 2}$ is known and constant, $b_p \in \mathbb{R}^{2 \times 1}$ is known and constant, (A_p, b_p) controllable, and $v(t)$ is a
scalar input. It is assumed that Approximating an admissable scalar input. It is assumed that A_p contains an admissable
parametric uncertainty $|e| \le \bar{e}$ where \bar{e} is a lineur parametric uncertainty $|a_p| \leq \bar{a}$ where \bar{a} is a known positive constant. The unmodeled dynamics are unknown and defined as where $A_p \in \mathbb{R}^{2 \times 2}$ is known and constant, $b_p \in \mathbb{R}^{2 \times 1}$ is $x^2 + 4y^2 +$ where $A \in \mathbb{R}^{2 \times 2}$ is known and constant, $b \in \mathbb{R}^{2 \times 1}$ is where $A_p \in \mathbb{R}$ is known and constant, $v_p \in \mathbb{R}$ is a
leading and constant (A, b) controllable, and v(t) is a parametric uncertainty $|u_p| \geq u$ where u is a Known $\dot{x}(t) = A(x(t)) + b(y(t))$ $x_p(t) = A_p x_p(t) + b_p v(t)$ (1)
where $A_p \in \mathbb{R}^{2 \times 2}$ is known and constant, $b_p \in \mathbb{R}^{2 \times 1}$ is known and constant, (A_p, b_p) controllable, and $v(t)$ is a parametric uncertainty $|a_p| \leq \bar{a}$ where \bar{a} is a known parametric uncertainty $|a_p| \leq \bar{a}$ where \bar{a} is a known positive constant. The unmodeled dynamics are unknown where $A_p \in \mathbb{R}^{2 \times 2}$ is known and constant, $b_p \in \mathbb{R}^{2 \times 1}$ is

$$
\dot{x}_{\eta}(t) = A_{\eta} x_{\eta}(t) + b_{\eta} u(t)
$$
\n
$$
v(t) = c_{\eta} \tau x_{\eta}(t)
$$
\n
$$
(2)
$$

where $u(t)$ is the control input, $c_{\eta}^{\top} \in \mathbb{R}^{1 \times m}$, $x_{\eta} \in \mathbb{R}^{m \times 1}$, and $A_{\eta} \in \mathbb{R}^{m \times m}$ is Hurwitz with $G_{\eta}(s) \triangleq c_{\eta}^{\top}(sI_{m \times m} (A_{\eta})^{-1}b_{\eta}$, and $(c_{\eta}^{\top}, A_{\eta}, b_{\eta})$ is observable and controllable. The goal is to design the control input such that $x_p(t)$ follows $x_m(t)$ which is specified by the reference model where $u(t)$ is the control input, $c_{\eta} \in \mathbb{R}^{1 \times m}$, $x_{\eta} \in \mathbb{R}^{m \times 1}$, and $A_{\eta} \in \mathbb{R}^{m \times m}$ is Hurwitz with $G_{\eta}(s) \equiv c_{\eta}(sI_{m \times m} - A_{\eta})^{-1}$ where $u(t)$ is the control input, $c \stackrel{\top}{\leftarrow} \in \mathbb{R}^{1 \times m}$, $c \in \mathbb{R}^{m \times 1}$, where $u(t)$ is the control input, $c_{\eta} \in \mathbb{R}$, $x_{\eta} \in \mathbb{R}$,
and $A \in \mathbb{R}^{m \times m}$ is Hurwitz with $C(s) \triangleq c^{\top}(sI)$, \Box $A = \begin{bmatrix} 1 & 0 \\ 1 & -1 & 0 \end{bmatrix}$ $(A_{\eta})^{-1}b_{\eta}$, and $(c_{\eta}^{\top}, A_{\eta}, b_{\eta})$ is observable and controllable. The goal is to design the control input such that $x_p(t)$ The goal is to design the control input such that x_p
follows x_p (t) which is specified by the reference model where $u(t)$ is the control input, c_n ^T $\in \mathbb{R}^{1 \times m}$, $x_n \in \mathbb{R}^{m \times 1}$. where $u(t)$ is the control input, $c_{\eta}^{\perp} \in \mathbb{R}^{1 \times m}$, $x_{\eta} \in \mathbb{R}^{m \times 1}$,
and $A_n \in \mathbb{R}^{m \times m}$ is Hurwitz with $G_n(s) \triangleq c_n^{\top}(sI_{m \times m}$ follows $x_m(t)$ which is specified by the reference model and $A_{\eta} \in \mathbb{R}^{m \times m}$ is Hurwitz with $G_{\eta}(s) \triangleq c_{\eta}^{\dagger} (sI_{m \times m} -$

$$
\dot{x}_m(t) = A_m x_m(t) + b_m r(t)
$$
 (3)

where $A_m \in \mathbb{R}^{2 \times 2}$ is Hurwitz, $b_m \equiv b_p$, and $r(t)$ is a bounded reference input. It is important to note that the scope of this paper is limited to the case of zero reference scope of this paper is infinited to the case of zero reference
input. That is, $r(t) = 0$ for all time. It is further assumed where $A_m \in \mathbb{R}^{2 \times 2}$ is Hurwitz, $b_m \equiv b_p$, and $r(t)$ is a where $A_m \in \mathbb{R}^{2\times 2}$ is Hurwitz, $b_m \equiv b_p$, and $r(t)$ is a
hounded potences input. It is important to note that the bounded reference input. It is important to note that the bounded reference input. It is important to note that the

Fig. 1. Adaptive control with unmodeled dynamics. Fig. 1. Adaptive control with unmodeled dynamics.

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[∗] This work is supported by the Boeing Strategic University Initiative. tive. tive. ★ This work is supported by the Boeing Strategic University Initia-

that A_m is chosen such that there exists a $\theta^* = [\theta_0^* \ \theta_1^*]^\top$ satisfying

$$
A_p + b_p \theta^{\star \top} = A_m \tag{4}
$$

for the plant in (1).

A control law is chosen (see Fig. 1) as

$$
u(t) = \theta^{\top}(t)x_p(t)
$$
 (5)

where the parameter $\theta(t)=[\theta_0(t) \ \theta_1(t)]^{\top}$ is updated using a modified version of the standard adaptive law.

In this paper, we will extend the method in Hussain et al. (2013a) and Matsutani et al. (2013) and establish conditions under which global boundedness of the closed-loop adaptive system given by $(1)-(5)$ can be achieved.

3. A NONSINGULAR TRANSFORMATION

In order to properly identify the effect the perturbation due to the presence of unmodeled dynamics has on the adaptive system, nonsingular transformations of the adaptive parameter $\theta(t)$ and tracking error $e(t) = x_p(t)$ – $x_m(t)$ are performed. The will become clear in future deliberations (see §7.1 and §7.2).¹ We will refer to the i^{th} component of the transformed states as $\mathscr{E}_i(t)$ and $\vartheta_i(t)$ for $i = \{0, 1\}$. We define a transformed error $\mathcal{E}(t)$ and transformed parameter $\vartheta(t)$ by

$$
\mathcal{E}(t) \equiv Ce(t) \tag{6}
$$

$$
\mathcal{E}(t) = Me(t) \tag{7}
$$

$$
\vartheta(t) \equiv M\theta(t) \tag{7}
$$

where C and M are transformation matrices whose construction utilizes (3) and the solution $P = P^{\top} > 0$ of the Lyapunov Equation $A_m^{\top} P + P A_m = -Q$. The transformation matrices C and \tilde{M} are defined as

$$
C = \begin{bmatrix} c_0 & c_1 \end{bmatrix}^\top \tag{8}
$$

$$
M = p_{bb}CP^{-1} \tag{9}
$$

where $\sqrt{b_m^{\top}Pb_m}$, and the vectors $c_i \in \mathbb{R}^{2\times 1}$ are constructed such that

$$
c_0 = p_{bb}^{-1} P b_m \tag{10}
$$

$$
c_1 c_1^{\top} = P - c_0 c_0^{\top}.
$$
 (11)

Remark 1. Two important properties of the construction of C include $c_0^{\dagger} b_m = p_{bb}$ and $c_1^{\dagger} b_m = 0$. The latter can be shown from the definition of c_0 in (10) and by design since (11) can be rewritten as ²

$$
CP^{-1}C^{\top} = I.
$$
 (12)

Together, these two properties aid in capturing the dominant effect the perturbation has on the adaptive system.

4. AN ADAPTIVE LAW

The specific adaptive law we propose is an extension of the scalar form of the projection algorithm in Hussain et al. (2013a) to higher dimensional plants. We propose the adaptive law as

$$
\dot{\theta} = M^{-1}\dot{\vartheta} \tag{13}
$$

with M in (9) and

$$
\dot{\vartheta}_i = \text{Proj}\left(\{M\theta\}_i, -\{M\Gamma x_p b_m^\top P e\}_i\right) \tag{14}
$$

where the scalar projection algorithm can be defined as

$$
Proj(\Theta_i, y_i) = \begin{cases} \frac{\Theta_{i,\max}^2 - \Theta_i^2}{\Theta_{i,\max}^2 - \Theta_{i,\max}^2} y_i & \Theta_i \in \Omega_i \wedge \Theta_i y_i > 0\\ y_i & \text{otherwise} \end{cases}
$$
(15)

where

$$
\overline{\Omega}_{i} = \{ \Theta_{i} \in \mathbb{R}^{1} \mid \Theta'_{i, \max} \leq \Theta_{i} \leq \Theta_{i, \max} \}
$$

$$
\underline{\Omega}_{i} = \{ \Theta_{i} \in \mathbb{R}^{1} \mid -\Theta_{i, \max} \leq \Theta_{i} \leq -\Theta'_{i, \max} \}
$$
(16)

$$
\Omega_{i} = \overline{\Omega}_{i} \cup \underline{\Omega}_{i}
$$

with positive constants $\Theta_{i,\text{max}} > \Theta'_{i,\text{max}}$. It can be shown in Lavretsky and Gibson (2011) that the projection algorithm $\dot{\Theta}_i = \text{Proj}(\Theta_i, y_i)$ ensures that $|\Theta_i(t)| \leq \Theta_{i, \text{max}}$ for all $t \geq t_0$. That is, the projection algorithm guarantees boundedness of the parameter $\Theta_i(t)$ independent of the system dynamics.

5. PROPERTIES OF THE REFERENCE MODEL

We will now use the reference model in (3) and C in (8) to define the $n \times n$ matrix

$$
\mathscr{A}_m = CA_m P^{-1} C^{\top}.
$$
 (17)

Furthermore, we define the scalars
\n
$$
\alpha_{ij} = c_i^{\top} A_m P^{-1} c_j, \quad \forall i, j = \{0, 1\}
$$
\n(18)

and partition \mathscr{A}_m as

$$
\mathscr{A}_m = \begin{bmatrix} \alpha_{00} & a_1 \\ a_0 & \mathscr{A}'_m \end{bmatrix} . \tag{19}
$$

It can be shown that both \mathscr{A}_m and \mathscr{A}'_m are Hurwitz. The former holds since $\mathscr{A}_m = CA_m C^{-1}$ which implies that the eigenvalues of \mathscr{A}_m are equal to those of A_m . For the latter, it can be shown using a suitable permutation similarity of $\mathscr{A}_{m}^{\top} + \mathscr{A}_{m}$ and Sylvester's Minorant Criterion that $\mathscr{A}_{m}^{\prime \top} + \mathscr{A}_{m}^{\prime}$ is negative definite, proving \mathscr{A}_{m}^{\prime} is Hurwitz.²

5.1 Preliminaries

Before proceeding to the main result, we will introduce notation and mathematical preliminaries used throughout the paper. For a matrix $A \in \mathbb{R}^{n \times n}$, we define

$$
\frac{\lambda_A \triangleq \min_i |\Re(\lambda_i(A))|}{\overline{\lambda}_A \triangleq \max_i |\Re(\lambda_i(A))|}
$$

where λ_i is the ith eigenvalue of A and $\Re(\lambda_i)$ denotes its real part.

For any vector $x \in \mathbb{R}^{n \times 1}$, we refer to the *i*th component as x_i for each $i = \{0, \ldots, n-1\}$ and define $\bar{x} \triangleq \max_t ||x(t)||$ where $\|\cdot\| = \|\cdot\|_2$ represents the Euclidean norm. Furthermore, the $n - 1$ subvector of x shall be defined as $x' \equiv [x_1 \quad x_2 \cdots x_{n-1}]^\top$ and the subregion \mathfrak{X}' defined as $\mathfrak{X}' \equiv \cup_{i=1}^{n-1} \mathfrak{X}_i$ where $\mathfrak{X}_i \in \mathbb{R}^{2n}$. Let $\mu(\bullet)$ denote the measure of a set.

Lemma 1. (Barbălat's Lemma). If $f : \mathbb{R}^+ \to \mathbb{R}$ is uniformly continuous for $t \geq 0$, and $\lim_{t \to \infty} \int_0^t |f(\tau)| d\tau$ exists and is finite, then $\lim_{t\to\infty} f(t) = 0$.

Corollary 1. If $g \in \mathscr{L}_2 \cap \mathscr{L}_{\infty}$, and $\dot{g} \in \mathscr{L}_{\infty}$, then $\lim_{t\to\infty} g(t) = 0.$

Proof. Choose $f(t) = q^2(t)$. Then the conditions of Barbalat's Lemma (see Narendra and Annaswamy (1989)) are satisfied. \square

¹ This methodology was originally proposed in Matsutani et al. (2013) for general n^{th} order plants in the presence of time delay.
² We refer the reader to Hussain (2016) for further details

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