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Adaptive Observer for a Class of Output-Delayed Systems with Parameter Uncertainty - A PDE Based Approach

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Abstract: The problem of state observation is addressed for a class of systems subject to sensor delay and parameter uncertainty. The unknown parameter vector enters a finite-dimensional state equation through a possibly output-dependent regressor. The sensor delay effect is captured by a first-order hyperbolic PDE. Doing so, the system turns out to be an ODE-PDE association with a connection point not accessible to measurements. An adaptive observer is constructed by combining ideas from PDE-based and ODE-based design approaches. The observer provides estimates of the ODE subsystem states and parameters, on the one hand, and of the sensor states, on the other. Observer exponential convergence is established under an ad-hoc persistent excitation condition involving the regressor.

Keywords: Delayed systems, Hyperbolic PDE, Adaptive observer, Backstepping design.

1. INTRODUCTION

Time delay is a common property that characterizes several categories of real-life systems. It accounts for physical phenomena such as material transport, traffic flows, networked systems, chemical and biological reactors, and others. From theoretical viewpoints, time delay are infinite dimensional operators and may be source of instability. Therefore, it is natural that an intensive research activity has been devoted to various issues of control system design in presence of time delay, see e.g. (Richard, 2003; Krstic, 2009) and reference lists therein. In this respect, much attention has been paid, over the past three decades, to observability analysis and observer design. Earliest results have mainly concerned linear systems, see e.g. (Richard, 2003; Krstic, 2009; Bhat and Koivo,1976; Leyva-Ramos and Rearson, 1995; Pearson and Fiagbedzi, 1989; Trinh and Aldeen, 1997). Lately, observer designs for nonlinear delayed systems have been proposed, see e.g. (Watanabe, 1996; Hou and Patton, 2002; Germani et al., 2002; Cacace et al., 2002; Ahmed-Ali et al., 2013).

In this paper we are considering the problem of state observation of delayed systems which are further subject to model parameter uncertainty. We propose an exponentially convergent adaptive observer for a class of output-delayed systems with unknown parameters. The latter enter linearly the state equation and the associated regressor is any nonlinear time function, that is allowed to be output-dependent. Just as in (Krstic and Smyshlyaev, 2008), the time-delay effect is captured through a first-hyperbolic PDE and a backstepping-like design technique is used to design an adaptive observer that estimates the ODE state and parameter vectors as well as the sensor states which, in fact, coincide with the system future outputs. The observer exponential convergence is established under an ad-hoc persistent excitation condition involving the regressor. Although it does not follow mutatis-mutandis the design approach in (Krstic and Smyshlyaev, 2008), the new observer can be seen as an adaptive extension of the observer proposed there. Compared with classical delay-compensating observers (e.g. Germani et al., 2002; Cacace et al., 2002; Ahmed-Ali et al., 2013), our adaptive observer is full-order because it estimates both the system (finite-dimensional) state and the sensor (infinite-dimensional) state. A more exhaustive comparison can be found in (Krstic, 2009, ch. 3].

The paper is organised as follows: first, the observation problem under study is formulated in Section 2; then, the observer design and analysis are respectively dealt with in Sections 3 and 4; a conclusion and reference list end the paper. To alleviate the presentation, some technical proofs are appended.

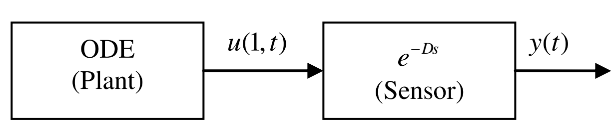

Fig. 1. System structure

2. PROBLEM FORMULATION

As this is depicted by Fig. 1, the system under study consists of a finite-dimensional nonlinear subsystem connected in series with a time delay. Analytically, the considered output-delayed system is described as follows:

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$$\dot{X}(t) = AX(t) + \phi(t)\theta, \quad t \geq 0, \tag{1a}$$
$$y(t) = CX(t - D) \quad \text{(output)} \tag{1b}$$

where $A \in \mathbf{R}^{n\times n}$ and $C \in \mathbf{R}^{1\times n}$ are known constant matrices and the pair (A, C) is observable; $\phi : C([0,\infty) : \mathbf{R}^{n\times m})$ is a known bounded continuous function; D denotes a known time delay which is just supposed to be nonnegative; the output $y(t)$ is accessible to measurements, but the state vector $X(t) \in \mathbf{R}^n$ is not.

Following the approach developed in (Ahmed-Ali et al., 2008), the output equation (1b) is represented by a first-order hyperbolic equation. Doing so, the system under study turns out to be modelled by the following state-space representation:

$$\dot{X}(t) = AX(t) + \phi(t)\theta, \quad t \geq 0 \tag{2a}$$
$$u(D,t) = CX(t) \tag{2b}$$
$$u_t(x,t) = u_x(x,t), \quad 0 \leq x \leq D \tag{2c}$$
$$y(t) = u(0,t) \tag{2d}$$

It is well known that the solution of (2b-c) is $u(x,t) = CX(t + x - D)$ (see e.g. Krstic (2009)). Therefore, the output equation (2d) gives the delayed output $y(t) = CX(t - D)$, which is identical to (1b).

The aim is to design an observer that provides accurate online estimates of the finite-dimensional state $X(t)$, the distributed state $u(x,t)$ $(0 \leq x \leq 1)$, and the unknown parameter vector θ. The observer must only make use of the system output $y(t)$.

Remark 1. The above observation problem extends a similar problem in (Krstic and Smyshlyaev, 2008), but no uncertain parameters were considered i.e. $\phi(t)\theta = 0$ in that work. In this regard, note that the vector $\phi(t)$ in (2a) is allowed to be output-dependent i.e. one can have $\phi(t) = \psi(t, y(t))$ for some continuous function ψ. In such a case, the dynamics of the ODE (2a) turns out to be nonlinear. On the other hand, the present setting is quite different from the one in (Ahmed-Ali et al., 2015) even though an ODE-PDE system structure is considered in both. Indeed, the ODE subsystem in (Ahmed-Ali et al., 2015) is more general (than the present one) because it includes a Lipschitz state function. But, in the same time, that system is less general (compared to the present one) because it is a triangular structure and involves no parameter uncertainty. Owing to the infinite-dimensional subsystem, it is a parabolic type in (Ahmed-Ali et al., 2015) while it is presently a hyperbolic nature □

3. ADAPTIVE OBSERVER DESIGN

A quite general observer structure is the following:

$$\dot{\hat{X}} = A\hat{X} + \phi(t)\hat{\theta} - K\tilde{u}(0,t) + v_0(t) \tag{3a}$$
$$\hat{u}_t(x,t) = \hat{u}_x(x,t) - k(x)\tilde{u}(0,t) + v_1(x,t) \tag{3b}$$
$$\hat{u}(D,t) = C\hat{X}(t) \tag{3c}$$

for all $t \geq 0$ and all $x \in [0, D]$, where $\tilde{u}(0,t) = \hat{u}(0,t) - u(0,t) = \hat{u}(0,t) - y(t)$. The vector and scalar gains, $K \in \mathbf{R}^n$ and $k(x) \in \mathbf{R}$, as well as the additional correction terms, $v_0(t), v_1(t) \in \mathbf{R}$, have yet to be defined. To this end, introduce the state and parameter estimation errors:

$$\tilde{X} = \hat{X} - X, \quad \tilde{u} = \hat{u} - u, \quad \tilde{\theta} = \hat{\theta} - \theta \tag{4}$$

From (2a-c) and (3a-c), it is readily seen that these errors undergo the following equations:

$$\dot{\tilde{X}} = A\tilde{X} + \phi(t)\tilde{\theta} - K\tilde{u}(0,t) + v_0(t) \tag{5a}$$
$$\tilde{u}_t(x,t) = \tilde{u}_x(x,t) - k(x)\tilde{u}(0,t) + v_1(x,t) \tag{5b}$$
$$\tilde{u}(D,t) = C\tilde{X}(t) \tag{5c}$$

Consider the following backstepping transformations, partly inspired by (Krstic M. and A. Smyshlyaev, 2008) and (Zhang, 2002):

$$Z(t) = \tilde{X}(t) - \lambda_0(t)\tilde{\theta}(t), \tag{6a}$$
$$\varepsilon(x,t) = \tilde{u}(x,t) - CM(x)\tilde{X}(t) - \lambda_1(x,t)\tilde{\theta}(t) \tag{6b}$$

where $M(x) \in \mathbf{R}^{n\times n}$, $\lambda_0(t) \in \mathbf{R}^{n\times m}$ and $\lambda_1(x,t) \in \mathbf{R}^{1\times m}$ are auxiliary functions yet to be defined. The error system (5a-c) rewrites in terms of the new coordinates Z and ε, as follows (see Appendix A):

$$\begin{aligned}\dot{Z}(t) &= [A - KCM(0)]Z(t) - K\varepsilon(0,t) + v_0(t) - \lambda_0(t)\dot{\tilde{\theta}}(t) \\ &\quad + \left([A - KCM(0)K]\lambda_0(t) + \phi(t) - K\lambda_1(0,t) - \dot{\lambda}_0(t)\right)\tilde{\theta}(t)\end{aligned} \tag{7a}$$
$$\begin{aligned}\varepsilon_t(x,t) &= \varepsilon_x(x,t) + \left(CM(x)K - k(x)\right)\tilde{u}(0,t) \\ &\quad + C\left(\frac{d^2M}{dx^2}(x) - M(x)A\right)\tilde{X}(t) \\ &\quad + \left(\lambda_{1,x}(x,t) - CM(x)\phi(t) - \lambda_{1,t}(x,t)\right)\tilde{\theta}(t) \\ &\quad + v_1(x,t) - \lambda_1(x,t)\dot{\tilde{\theta}}(t) - CM(x)v_0(t)\end{aligned} \tag{7b}$$
$$\varepsilon(D,t) = C\tilde{X}(t) - \lambda_1(D,t)\tilde{\theta}(t) - CM(D)\tilde{X}(t) \tag{7c}$$

We seek functions $k(x)$, $v_0(t)$, $v_1(x,t)$, $M(x)$, $\lambda_0(t)$ and $\lambda_1(x,t)$ that make the error system (7a-c) coincide with the following target system:

$$\dot{Z}(t) = [A - KCM(0)]Z(t) - K\varepsilon(0,t) \tag{8a}$$
$$\varepsilon_t(x,t) = \varepsilon_x(x,t) \tag{8b}$$
$$\varepsilon(D,t) = 0 \tag{8c}$$

for all $t \geq 0$ and all $x \in [0, D]$. The target system (8a-c) is motivated by the fact that, the subsystem (8b) (which represents a time delay) has the solution