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IFAC-PapersOnLine 49-13 (2016) 164–168

ADAPTIVE STABILIZATION OF LINEAR SYSTEMS THROUGH A TWO-WAY CHANNEL WITH LIMITED CAPACITY

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Abstract:
The paper is devoted to adaptive feedback stabilization for linear time-invariant control systems with saturated quantized measurements and control. The quantization occurs due to the finite capacity of the discrete-time two-way channel connecting the plant and the controller. An adaptive controller based on the Yakubovich's recursive goal inequalities method is designed. The bound for the minimum channel capacity sufficient to stabilize any stabilizable system with unknown parameters is evaluated for any prespecified compact set of unknown parameters.

Keywords: Adaptive control, finite-converging algorithms, quantization of measurements and control, communication channel of finite capacity.

1. INTRODUCTION

Control under communication constraints attracts attention of the control theorists for more than a decade, see surveys Baillieul & Antsaklis (2007); Nair *et al* (2007); Andrievsky *et al* (2010), monograph Matveev & Savkin (2009) and references therein. For linear systems the conditions for stabilizability by a linear feedback via communication channel with limited capacity are provided by the seminal data rate theorem. It was proposed independently by several authors, see Nair & Evans (2003); Tatikonda & Mitter (2004); Matveev & Savkin (2004) for its different formulations. For nonlinear and adaptive systems the situation is more complicated. The existing results for nonlinear synchronization and control are either local Nair *et al* (2004); Savkin (2006) or provide only sufficient conditions with different degree of conservativeness — Liberzon & Hespanha (2003); De Persis & Nesi (2008); Fradkov *et al.* (2015). The situation for adaptive and robust systems is even more difficult due to uncertainty Fradkov *et al.* (2008).

A new approach to treating control under communication constraints was proposed by Bondarko (2014) based on the so called *method of recursive goal inequalities (RGI)*. The method of RGI was proposed by Yakubovich (1966) and later was successfully applied to a variety of estimation and control problems Yakubovich (1968, 1969); Fomin *et al.* (1981); Gusev (1989); Bondarko *et al.* (1991); Bondarko & Yakubovich (1992); Bondarko (2015). Since the RGI method is based on special type of quantization it is perfectly suited for dealing with coding-decoding procedures.

In this paper the approach of Bondarko (2014) is extended to adaptive stabilization of discrete LTI system.

2. PROBLEM STATEMENT

Let control system consist of a linear time-invariant SISO (single input – single output) plant, a digital two-way discrete-time communication channel, and remote adaptive feedback controller. Any digital channel introduces a distortion into transmitted signals depending on the channel word length and saturation level. Given a fixed word length we will try to design a control law to stabilize the closed loop system. This control law should include a description of the controller and a rule to choose a saturation level. Finally, we should specify a word length sufficient to yield stability.

This problem statement is close to the concept of measurements quantization Brockett & Liberzon (2000). However, we additionally consider quantization of control input, too. Besides that, we will assume that we don't know the accurate values of plant parameters. Namely, plant is described by the following equation:

$$\alpha(q)y_k = q\beta(q)\tilde{u}_k, \tag{1}$$

where $qy_k = y_{k-1}$, $k = 0, 1, \ldots$, $y_{-1}, y_{-2}, \ldots y_{-n}$ $\tilde{u}_{-1}, \tilde{u}_{-2}, \ldots \tilde{u}_{-n}$ are initial data of the system, $\tilde{u}_k$ and y_k denotes control input and observable output respectively,

$$\alpha(\lambda) = 1 - \alpha_1\lambda - \cdots - \alpha_n\lambda^n,$$
$$\beta(\lambda) = \beta_0 + \beta_1\lambda + \cdots + \beta_{n-1}\lambda^{n-1},\ \beta_0 \neq 0.$$

⋆ The work was supported by the SPbSU grant 6.38.230.2015. Principal auxiliary result (Lemma 3) is performed in IPME RAS and supported solely by RSF, grant 14-29-00142.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.945

It is assumed that closed convex set $T = \{\tau\}$ containing a vector

$$\tau = (\alpha_1, \alpha_2, \dots, \alpha_n, \beta_1, \dots, \beta_n)' \quad (2)$$

of unknown true plant parameters is known. In other words T can be used for control design, but τ is not available.

Our main assumption is that polynomials $\alpha(\lambda)$ and $\beta(\lambda)$ are coprime for all $\tau \in T$. This assumption seems to be not too restrictive. Indeed, if the common polynomial factor of $\alpha(\lambda)$ and $\beta(\lambda)$ is stable, then it may be reduced, otherwise stabilizaton of the system (1) is impossible.

Communication channel between plant and remote controller has limited capacity ν bits per step of discrete time k in both direction. Hence the channel can transmit integer numbers $\hat{u}_k$ and $\hat{y}_k$ in the interval $[-M, M]$ only, where $M = 2^{\nu-1}$.

They depend on true signals u_k and y_k according to the following coder algorithm:

$$\hat{u}_k = Q_{\Delta_k}(u_k),\ \hat{y}_k = Q_{\Delta_k}(y_k), \quad (3)$$

where

$$Q_\Delta(x) = \begin{cases} -M & \text{if } \dfrac{x}{\Delta} \leqslant -M - 1/2, \\ \left\lfloor \dfrac{x}{\Delta} + \dfrac{1}{2} \right\rfloor, & \text{if } \dfrac{x}{\Delta} \in (-M - 1/2, M + 1/2], \\ M & \text{if } \dfrac{x}{\Delta} > M + 1/2, \end{cases}$$

$\lfloor x \rfloor = \max\{k \in \mathbb{Z} : k < x\}$, $\mathbb{Z}$ denotes the set of integer numbers. An update rule for Δ_k will be described later. Furthermore, having quantized signals $\hat{y}_k$ and $\hat{u}_k$ one can use

$$\tilde{y}_k = \Delta_k \hat{y}_k,$$
$$\tilde{u}_k = \begin{cases} \hat{u}_k \Delta_k, \text{ if } |\hat{u}_k| < M, \\ 0, \text{ if } |\hat{u}_k| = M, \end{cases} \quad (4)$$

to approximate true values of y_k and u_k. Error $e = \hat{x} - x$ between original value x and its approximation $\hat{x}$ depends on the band where x lies. If $|x| \leqslant \Delta(M + 1/2)$, then $|e|$ is bounded by $\Delta/2$. Otherwise $|e|$ may be arbitrarily large. For this reason the strategy of control will alternate two modes. Namely, we will have to zoom out, i.e., increase Δ until the output of the system can be adequately measured, and control value can be adequately transmitted. And then we will zoom in, i.e., decrease Δ in such a way as to drive the state to 0. More precisely, our control goal is to provide inequalities

$$\limsup_{k\to\infty} |y_k| < \epsilon,\ \limsup_{k\to\infty} |u_k| < \epsilon \quad (5)$$

with arbitrary predefined $\epsilon > 0$. This goal should be achieved for every plant (1) with coefficients from the set T by feedback controller with input signal $\hat{y}_k$ and output signal u_k. The complete solution of the problem should also include a description of an algorithm updating values of quantizer sensitivity Δ_k.

As it was already mentioned the key point of our approach is usage of the *method of recursive goal inequalities (RGI)*. It will be outlined in the next two sections.

3. RECURSIVE GOAL INEQUALITIES APPROACH

Let us assume for a while that the system under control (1) is known, i.e. the set T of possible plant parameters consists of a single point τ. In this case we find ourselves under conditions of the Theorem 1 of the paper Bondarko (2014). In this theorem it is proposed to seek polynomials $\gamma(\lambda)$ and $\delta(\lambda)$ as solutions of Diophantine equation

$$\alpha(\lambda)[1 + \lambda\gamma(\lambda)] - \beta(\lambda)\lambda\delta(\lambda) \equiv 1. \quad (6)$$

This implies

$$\begin{aligned} y_k &= \alpha(q)[1 + q\gamma(q)]y_k - q\beta(q)\delta(q)y_k = \\ &= [1 + q\gamma(q)]q\beta(q)u_k - q\beta(q)\delta(q)y_k = \\ &= q\beta(q)\{[1 + q\gamma(q)]u_k - \delta(q)y_k\}, \\ u_k &= \alpha(q)[1 + q\gamma(q)]u_k - q\beta(q)\delta(q)u_k = \\ &= \alpha(q)[1 + q\gamma(q)]u_k - \delta(q)\alpha(q)y_k = \\ &= \alpha(q)\{[1 + q\gamma(q)]u_k - \delta(q)y_k\}. \end{aligned}$$

Hence feedback control law

$$\tilde{u}_k + \gamma(q)\tilde{u}_{k-1} = \delta(q)y_k, \quad (7)$$

would be deadbeat. Unfortunately such a feedback cannot be implemented due to communication constraints. However, it is possible to find a realizible feedback

$$u_k = \delta(q)\tilde{y}_k - \gamma(q)\tilde{u}_{k-1} \quad (8)$$

and a suitable choice of Δ_k guaranteeing asymptotic stability of the closed loop system.

Thus in the case of unknown plant parameters, the only problem is to find unknown vector (2). To solve this problem we are going to use *method of recursive goal inequalities*. In order to explain the approach introduce the following notations:

$$\varphi_k = [\tilde{u}_k, \tilde{u}_{k-1}, \dots, \tilde{u}_{k-n+1}, y_k, y_{k-1}, \dots, y_{k-n+1}]',$$
$$\tilde{\varphi}_k = [\tilde{u}_k, \tilde{u}_{k-1}, \dots, \tilde{u}_{k-n+1}, \tilde{y}_k, \tilde{y}_{k-1}, \dots, \tilde{y}_{k-n+1}]',$$
$$\tilde{\phi}_k = [\tilde{u}_{k-1}, \tilde{u}_{k-2}, \dots, \tilde{u}_{k-n+1}, \tilde{y}_k, \tilde{y}_{k-1}, \dots, \tilde{y}_{k-n+1}]',$$

Introduce vector function $\theta(\cdot)$ in such a way that equation (8) is equivalent to

$$u_k = \theta'(\tau)\tilde{\phi}_k. \quad (9)$$

This means that polynomials $\alpha(\lambda)$, $\beta(\lambda)$ are constructed on a base of τ components, then $\gamma_\tau(\lambda)$, $\delta_\tau(\lambda)$ are computed as a solution of Diophantine equation (6), and then vector $\theta(\tau)$ is formed from $\gamma_\tau(\lambda)$, $\delta_\tau(\lambda)$ coefficients.

The plant equation (1) may be rewritten as follows

$$y_k - \tau'\varphi_k = 0.$$

We would use these equations to estimate τ, but φ_k is not available. Nevertheless we can try to replace φ_k by $\tilde{\varphi}_k$. This leads us to the inequalities

$$|y_k - \tau'\tilde{\varphi}_k| \leqslant \sup_{\tau\in T} |\tau|\,|\varphi_k - \tilde{\varphi}_k|. \quad (10)$$

Finitely converging algorithms of RGI method allow one to solve this infinite system of inequalities in the closed loop with control input (9). As a result, the control goal can be achieved.

4. FINITELY CONVERGING ALGORITHM "STRIPE"

In this paper we use one of the most popular among the finitely converging algorithms called "Stripe". A brief description of the "Stripe" algorithm is given below.

Consider a countable system of inequalities like (10):

$$|y_k - \tau'\tilde{\varphi}_k| \leqslant C_k. \quad (11)$$