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Robust finite frequency design of iterative learning control schemes ⋆

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Abstract: This paper considers the problem of designing an iterative learning control law for discrete linear systems using repetitive process stability theory. The resulting design produces a stabilizing output feedback controller in the time domain and a feedforward controller that guarantees monotonic convergence in the trial-to-trial domain. An extension to design with limited frequency range specifications for uncertain plants is developed. All design computations required for the new results in this paper can be computed using linear matrix inequalities (LMIs). A simulation example is given to illustrate the theoretical developments.

Keywords: Learning control, Linear control systems, Convex optimisation, Robustness.

1. INTRODUCTION

Iterative learning control (ILC) is a method of iteratively updating the control signal to a given system that repeats the same finite duration task. Each execution is known as a trial, or pass, and the sequence of operations is that a trial is completed, the system resets to the starting position and then the next trial begins, either immediately after the resetting is complete or a further period of time has elapsed. The novel feature of this control law design method is the use information from the previous trial to update the control input applied on the next trial and thereby improve performance from trial-to-trial. In particular, the control objective is to find a control input such that the corresponding output tracks a reference signal that is specified over a finite time interval.

Since the original work of Arimoto et al. (1984), ILC has remained as a significant area of control systems research with many algorithms experimentally verified in the research laboratory and applied in industrial applications. An overview of developments and possible application areas up to their publications dates can be found in, e.g., the survey papers Ahn et al. (2007); Bristow et al. (2006).

One common approach to ILC design for discrete dynamics, see, e.g., Ahn et al. (2007); Bristow et al. (2006) as starting points for the literature, is to first apply a feedback control law to stabilize and/or produce acceptable along the trial dynamics and then apply ILC to force trial-to-trial error convergence of the resulting system. This method is also known as lifting. An alternative approach to ILC design is to use the two-dimensional/repetitive systems setting, i.e., systems that propagate information in two independent directions where for ILC these directions are from trial-to-trial and along each trial respectively.

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Repetitive processes Rogers et al. (2007) are a distinct class of two-dimensional systems where information in the temporal domain is limited to a finite duration and hence a more natural match to ILC. The result of repetitive process setting is a one step design for trial-to-trial error convergence and transient response along the trials and hence simultaneous treatment of the trial-to-trial error and transient response along the trials is possible. Moreover, unlike the lifting approach, the repetitive process extends to differential dynamics, i.e., design by emulation.

This paper deals with simultaneous synthesis of both feedback and learning controllers in an ILC scheme for error convergence and performance, starting with a new result for the trial-to-trial error convergence. The result is derived by converting the problem to one of stability along the trial for a discrete linear repetitive process, leading to design based on LMI computations with an extension to robust design. In addition, the new control law is based on output feedback and hence state measurements or an observer are not required for implementation.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I, respectively, and the notation $X \succ Y$ means that the matrix $X - Y$ is positive definite. Also sym{M} is used to denote the symmetric matrix $M + M^\top$ and $\rho(\cdot)$ denotes the spectral radius of its matrix argument, i.e., if $\lambda_i, 1 \leq i \leq q$, denote the eigenvalues of a $q \times q$ matrix, say H, $\rho(H) = \max_{1\leq i\leq q} |\lambda_i|$. The superscript $*$ denotes the complex conjugate transpose of a matrix and $\otimes$ the matrix Kronecker product.

Use will also be made of the following results, where the first is the generalized KYP lemma and the second the Projection Lemma.

Lemma 1. Iwasaki et al. (2005) Consider matrices $\mathbb{A}$, $\mathbb{B}_0$, Θ and

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$$\Phi = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \Psi = \begin{bmatrix} 0 & \mathrm{e}^{\mathrm{j}\omega_c} \\ \mathrm{e}^{-\mathrm{j}\omega_c} & -2\cos(\omega_d) \end{bmatrix}, \quad (1)$$

with $\omega_c = (\omega_l + \omega_u)/2$, $\omega_d = (\omega_u - \omega_l)/2$ and ω_l, ω_u satisfying $-\pi \leq \omega_l \leq \omega_u \leq \pi$. Suppose also that $\det(\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A}) \neq 0$ for all $\omega \in [\omega_l, \omega_u]$. Then the following statements are equivalent.

i) $\forall \omega \in [\omega_l, \omega_u]$

$$\begin{bmatrix} (\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A})^{-1}\mathbb{B}_0 \\ I \end{bmatrix}^* \Theta \begin{bmatrix} (\mathrm{e}^{\mathrm{j}\omega} I - \mathbb{A})^{-1}\mathbb{B}_0 \\ I \end{bmatrix} \prec 0. \quad (2)$$

ii) There exist $\mathcal{Q} \succ 0$ and a symmetric $\mathcal{P}$ such that

$$\begin{bmatrix} \mathbb{A} & \mathbb{B}_0 \\ I & 0 \end{bmatrix}^\top (\Phi \otimes \mathcal{P} + \Psi \otimes \mathcal{Q}) \begin{bmatrix} \mathbb{A} & \mathbb{B}_0 \\ I & 0 \end{bmatrix} + \Theta \prec 0. \quad (3)$$

Lemma 2. Gahinet et al. (1994) Given a symmetric matrix $\Gamma \in \mathbb{R}^{p \times p}$ and two matrices Λ, Σ of column dimension p, there exists a matrix $\mathcal{W}$ such that the following inequality holds

$$\Gamma + \mathrm{sym}\{\Lambda^\top \mathcal{W} \Sigma\} \prec 0, \quad (4)$$

if, and only if the following two projection inequalities with respect to $\mathcal{W}$ are satisfied

$$\Lambda^{\perp\top} \Gamma \Lambda^{\perp} \prec 0, \; \Sigma^{\perp\top} \Gamma \Sigma^{\perp} \prec 0, \quad (5)$$

where $\Lambda^\perp$ and $\Sigma^\perp$ are arbitrary matrices whose columns form a basis of null spaces of Λ and Σ, respectively.

2. BACKGROUND AND PROBLEM FORMULATION

The plant dynamics are assumed to be discrete linear time-invariant and described in the ILC setting as

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p), \\ y_k(p) &= Cx_k(p), \end{aligned} \quad (6)$$

where $x_k(p) \in \mathbb{R}^n$ is the state vector, $y_k(p) \in \mathbb{R}^m$ is the output vector and $u_k(p) \in \mathbb{R}^l$ is the control input vector. Also define z as the standard forward shift operator along discrete-time axis, i.e.,

$$zx_k(p) = x_k(p+1),$$

see, e.g., Bristow et al. (2006) for the details of how the z-transform can be applied over the finite trial length without errors arising from the basic definition of this transform over an infinite interval.

The form of ILC considered in this paper is shown in the block diagram of Figure 1 and consists of a unity negative feedback control loop with controller C applied on the current trial k and the ILC law as shown within the shaded part of this figure. In this paper no loss of generality arising from assuming that the initial state vector on each trial is zero and the memory block in Figure 1 represents the use of previous trial information in the computation of the current trial control input and y_d denotes the supplied reference. In the literature L is often termed the learning filter and Q the robustness filter. All computations within the shaded part of Figure 1 are completed in the time elapsed between the end of one trial and the beginning of the next, i.e., off-line.

The first task is to find guidelines for the choice of the C, L and Q filters, where to improve learning and robustness a basic rule is to choose Q as a low-pass filter whose magnitude is unity for the frequency range where reference

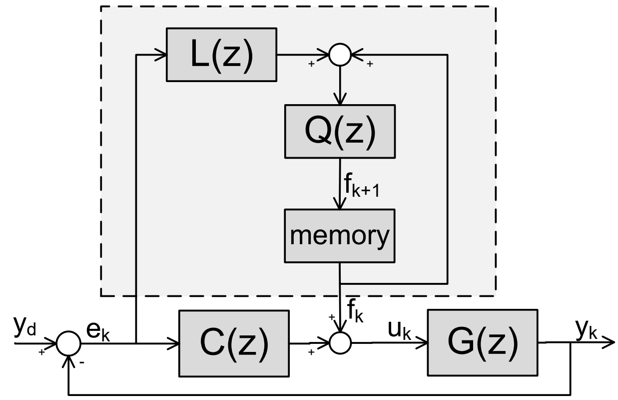

Fig. 1. ILC block diagram representation.

tracking is required and zero at all other frequencies. From the block diagram of Figure 1, the ILC law is

$$F_{k+1}(z) = Q(z)\left(F_k(z) + L(z)E_k(z)\right),$$

and the previous trial error feedforward contribution (assuming $Y_d(z) = 0$) to the current trial error is

$$E_k(z) = -\left[(I + G(z)C(z))^{-1}G(z)\right]F(z) = -S_P(z)F_k(z),$$

where $S_P(z) = (I + G(z)C(z))^{-1}G(z)$ denotes the sensitivity function and the propagation of the error from trial-to-trial is

$$E_{k+1}(z) = M(z)E_k(z), \quad (7)$$

where

$$M(z) = Q(z)\left(I - S_P(z)L(z)\right). \quad (8)$$

Given (7) it follows immediately that the tracking error converges as $k \to \infty$, i.e., the error reduces from trial-to-trial successive if and only if

$$\rho\left(M(\mathrm{e}^{\mathrm{j}\omega})\right) < 1, \; \forall \omega \in [-\pi, \pi]. \quad (9)$$

Practical experience has revealed that some ILC laws have poor transients during the convergence phase even if the above condition is satisfied, e.g., the tracking error may grow over a number of trials before beginning to decrease with each successive trial. To avoid these problems, a stronger convergence criteria is required for engineering practice. To ensure that the Euclidean norm of the tracking error decreases monotonically for each trial requires that $M(\mathrm{e}^{\mathrm{j}\omega})$ satisfies the sufficient stability condition

$$\overline{\sigma}(M(\mathrm{e}^{\mathrm{j}\omega})) < 1, \; \forall \omega \in [-\pi, \pi], \quad (10)$$

where $\overline{\sigma}(\cdot)$ denotes the maximum singular value of its matrix argument. This condition is often used in design for applications and also in this paper.

To satisfy (10) for a given case requires the selection of Q, L and C which is a non-trivial task. Some simplification in the selection procedure can be introduced by selecting Q to be frequency-independent and equal to qI where q is a real constant, see Pinte et al. (2010) for more details on this type of filtering with particular emphasis on mechanical systems. For this case, the convergence condition (10) can be rewritten as

$$\overline{\sigma}(q\left(I - S_P(z)L(z)\right)) < 1, \; \forall \omega \in [-\pi, \pi]. \quad (11)$$

2.1 ILC as a repetitive process

This section formulates the ILC design problem considered in the repetitive process setting, starting with the dynamics of these processes. The state-space model of a discrete