Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-13 (2016) 181–186

Dealing with Transients due to Multiple Experiments in Nonlinear System Identification ⋆

Rishi Relan * **Koen Tiels** * **Johan Schoukens** *

* *Department ELEC, Vrije Universiteit Brussel 1050 Brussels, Belgium (e-mail: firstname.surname@vub.ac.be)*

Abstract: Although in many static estimation problems the data are collected from repeated experiments, the default underlying (assumption) setting in most of the system identification tasks is that the data are generated from a single experiment. The rationale for this is that more data can be obtained by increasing the measurement time instead of doing more experiments. As the measurement time tends to infinity, under suitable assumptions it is possible to estimate consistently the model parameters. In most of the real life cases, however, increasing the measurement time is either not possible or it does not cover the whole operating range of the system to be identified, hence data from multiple experiments need to be combined. Furthermore, most of the real life systems are nonlinear and a large variety of nonlinear systems can be described by a nonlinear state space model structure. In this paper, a methodology to deal with the transients arising due to concatenating data from multiple experiments during the identification of Polynomial nonlinear state space (PNLSS) models is described. The methodology is validated on data generated from a laboratory experimental set-up.

Keywords: System identification; Multiple experiments; Nonlinear models; Transients; Nonlinear systems.

1. INTRODUCTION

There is an evident need of good system modelling techniques in many branches of engineering. Mathematical (linear or nonlinear) models are needed in various applications, for example, to understand and analyse the system under test, to simulate or predict the behavior of the system during the design phase or to design and implement a controller. System identification provides us with a variety of methods to derive accurate mathematical descriptions of the underlying system, based on a set of input/output measurements.

1.1 *Nonlinear System Identification*

The recent years have witnessed the shift from linear system identification (Ljung (1998); Pintelon and Schoukens (2012); Van Overschee and De Moor, 1996) to nonlinear system identification methods, driven by the need to capture the inherent nonlinear effects of real-life systems. Nonlinear system identification constantly faces the challenge of deciding between the flexibility of the fitted model and its parsimony. Flexibility refers to the ability of the model to capture complex nonlinearities, while parsimony is its ability to possess a low number of parameters. A general framework for nonlinear system identification does not exist (Giannakis and Serpedin (2001)), however, modeling nonlinear systems is covered in different fields like statistical learning and machine learning (Hastie et al. (2009); Rasmussen and Williams (2006); Suykens et al. (2002, 2012)), but most of these methods are typically not specifically developed to deal with dynamics and often have limited means for dealing with noise. Within the system identification community two major approaches to nonlinear system identification can be distinguished: black-box nonlinear system identification (Sjöberg et al. (1995), Billings (2013)) and block-oriented system identification (Giri and Bai (2010), Mzyk (2013)). In this paper, we focus mainly on the black-box nonlinear system identification methods, especially on the identification of nonlinear state space model structures (Paduart et al. (2010)) .

1.2 *Multiple Experiments*

There are many situations that can lead to a series of subrecords of equal (Markovsky and Pintelon (2015)) or unequal lengths (Schoukens et al. (2012)). A first illustration is a long experiment where some parts in the data have extremely poor quality due to a sensor failure or due to very large disturbances coming from other processes. Eliminating these bad parts results in a series of broken subrecords of the data. In other experiments, it might be impossible to measure for a very long time without interruption; only a series of shorter tests can be performed. Finally, we can consider systems that vary slowly due to changing operational conditions, e.g., a

⋆ The authors* are with the ELEC Department of the Vrije Universiteit Brussel (VUB),Brussels B-1050, Belgium. The corresponding author can be contacted at the (Rishi.Relan@vub.ac.be). This work was supported in part by the IWT-SBO BATTLE 639, Fund for Scientific Research (FWO-Vlaanderen), by the Flemish Government (Methusalem), the Belgian Government through the Inter university Poles of Attraction (IAP VII) Program, and by the ERC advanced grant SNLSID, under contract 320378.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.948

varying temperature, pressure etc. In that case, a series of subrecords that are collected under similar conditions can be grouped. If, for one of these reasons, a set of shorter subrecords is available, it turns out that it is advantageous to process them all at once, considering the data to come from one single experiment. Therefore in this paper we concentrate on developing a methodology to handle data from multiple experiments (as well as transients due to concatenation of data records) within the nonlinear system identification framework. The structure of the paper is the following: In Section 2, first along with the definition of a generic nonlinear state space model structure, the structure of the PNLSS model is presented. Next, in Section 3, an identification procedure of PNLSS models is explained in detail. Furthermore, it is explained how this identification method can be used to handle transients arising due to concatenating the data from multiple experiments. Section 4 presents the results and finally the conclusions are stated in Section 5.

2. NONLINEAR STATE SPACE

Physical interpretation of the system under test is not always required, for instance in control or prediction problems. In that case, the user prefers a flexible and an easy-to-initialize black-box model. Moreover, the model should preferably be able to describe Multiple-Input Multiple-Output (MIMO) systems in a compact way. A good base for such a model is a state space representation of the system under consideration. A general n_a^{th} order discrete-time state space model is described by the following equations:

$$x(t+1) = f(x(t), u(t)) \tag{1}$$

$$y(t) = g(x(t), u(t)) \tag{2}$$

with $u(t) \in \mathbb{R}^{n_u}$ the vector containing the n_u inputs at time t, and $y(t) \in \mathbb{R}^{n_y}$ the vector containing the n_y outputs. The state vector $x(t) \in \mathbb{R}^{n_a}$ represents the memory of the dynamical system.

2.1 *Polynomial Nonlinear State-Space Models*

A nonlinear state space model (where $f(\cdot), g(\cdot)$ are approximated by polynomial basis functions) is a natural representation for a MIMO system. Moreover, it can handle nonlinear feedback without any problem. The PNLSS model structure (Paduart et al. (2010)) is described as:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + E\zeta(t) \\ y(t) &= Cx(t) + Du(t) + F\eta(t) + e(t) \end{aligned} \tag{3}$$

The coefficients of the linear terms in $x(t) \in \mathbb{R}^{n_a}$ and $u(t) \in \mathbb{R}^{n_u}$ are given by the matrices $A \in \mathbb{R}^{n_a \times n_a}$ and $B \in \mathbb{R}^{n_a \times n_u}$ in the state equation, $C \in \mathbb{R}^{n_y \times n_a}$ and $D \in \mathbb{R}^{n_y \times n_u}$ in the output equation. The vectors $\zeta(t) \in \mathbb{R}^{n_\zeta}$ and $\eta(t) \in \mathbb{R}^{n_\eta}$ contain nonlinear monomials in $x(t)$ and $u(t)$ of degree two up to a chosen degree P and $e(t)$ is the measurement noise. The coefficients associated with these nonlinear terms are given by the matrices $E \in \mathbb{R}^{n_a \times n_\zeta}$ and $F \in \mathbb{R}^{n_y \times n_\eta}$.

3. IDENTIFICATION OF PNLSS FROM MULTIPLE EXPERIMENTS

The identification procedure for the PNLSS model in Section 2.1 consists of three major steps. The structure of the black-box state space model given in (3) lends itself to an efficient, three steps identification procedure. First, initial estimates of the A, B, C and D matrices are obtained. In order to do so, first, a nonparametric estimate of the system's frequency response function (FRF) is determined in mean square sense. Then, a parametric linear model (linear subspace A, B, C, D matrices) is estimated from this nonparameteric Best Linear Approximation (BLA). Second, the subspace estimates are optimised in maximum likelihood sense by applying a nonlinear minimisation routine. The last step consists in estimating the full nonlinear model (also including the polynomial coefficients) by using again a nonlinear search routine. Bounded input-bounded output (BIBO) stability is required for this optimization procedure. In the subsections below, these steps as well as the framework involved in these steps are described briefly. The whole procedure is carried out in the frequency domain, which opens the possibility to apply user-defined weighting functions in specific frequency bands.

3.1 *Best Linear Approximation*

Definition 1. The Best Linear Approximation (BLA) of a nonlinear system is defined as the model G belonging to the set of linear models $\mathcal{G}$, such that

$$G_{BLA} = \arg\min_{G \in \mathcal{G}} \mathbb{E}\left(|y(t) - Gu(t)|^2\right) \tag{4}$$

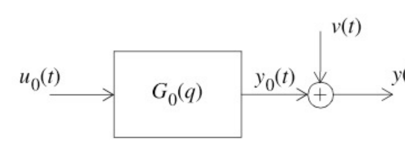

Fig. 1. Time domain representation of the problem

3.1.1. Set Up In this section, we focus for simplicity on (the estimation of) a nonparametric linear discrete-time single-input-single-output (SISO) model $G_0(q)$, which is excited with signals belonging to the Riemann equivalence class of asymptotically normally distributed excitation signals (Pintelon and Schoukens (2012)), see Fig. 1.

$$y(t) = G_0(q)u_0(t). \tag{5}$$

with q^{-1} the backward shift operator ($q^{-1}x(t) = x(t-1)$). All results apply also to continuous time systems. The exact input $u_0(t)$ is assumed to be known, while the output is disturbed with additive noise $v(t)$:

$$y(t) = y_0(t) + v(t). \tag{6}$$

The noise $v(t)$ is assumed to be filtered white noise ,

$$v(t) = H_0(q)e(t). \tag{7}$$

where $H_0(q)$ is the noise model. For an infinitely long data record $t = -\infty, ..., N-1$, the input-output relation is

$$y(t) = G_0(q)u_0(t) + H_0(q)e(t). \tag{8}$$

For a finite record length $t = 0, ..., N-1$, as it is in practical applications, this equation has to be extended with the initial conditions (transient) effects of the dynamic plant and noise system t_G , t_H:

$$y(t) = G_0(q)u_0(t) + H_0(q)e(t) + t_G(t) + t_H(t). \tag{9}$$

Using the discrete Fourier transform (DFT)

$$X(k) = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} x(t)e^{-j2\pi kt/N}, \tag{10}$$