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Extremum Seeking-based Iterative Learning Model Predictive Control (ESILC-MPC)

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Abstract: In this paper, we study a tracking control problem for linear time-invariant systems with model parametric uncertainties under input and states constraints. We apply the idea of modular design introduced in Benosman [2014], to solve this problem in the model predictive control (MPC) framework. We propose to design an MPC with input-to-state stability (ISS) guarantee, and complement it with an extremum seeking (ES) algorithm to iteratively learn the model uncertainties. The obtained MPC algorithms can be classified as iterative learning control (ILC)-MPC.

1. INTRODUCTION

Model predictive control (MPC), e.g., Mayne et al. [2000], is a model-based framework for optimal control of constrained multi-variable systems. MPC is based on the repeated, receding horizon solution of a finite-time optimal control problem formulated from the system dynamics, constraints on system states, inputs, outputs, and a cost function describing the control objective. However, the performance of MPC based controllers inevitably depends on the quality of the prediction model used in the optimal control computation. In contrast, extremum seeking (ES) control is a well known approach where the extremum of a cost function associated with a given process performance (under some conditions) is found without the need for detailed modelling information, see, e.g., Ariyur and Krstic [2003, 2002], Nesic [2009]. Several ES algorithms (and associated stability analysis) have been proposed, Krstic [2000], Tan et al. [2006], Nesic [2009], Ariyur and Krstic [2003], Guay et al. [2013], and many applications of ES have been reported Hudon et al. [2008], Zhang and Ordóñez [2012], Benosman and Atinc [2013].

The idea that we want to theoretically analyze in this paper, is that the performance of a model-based MPC controller can be combined with the robustness of a model-free ES learning algorithm for simultaneous identification and control of linear time-invariant systems with structural uncertainties. We refer the reader to Benosman [2014], Benosman and Atinc [2013], Atinc and Benosman [2013] where this idea of learning-based modular adaptive control has been introduced in a more general setting of nonlinear dynamics. We aim at proposing an alternative approach to realize an iterative learning-based adaptive MPC. We introduce an approach for an ES-based iterative learning MPC that merges a model-based linear MPC algorithm with a model-free ES algorithm to realize an iterative learning MPC that adapts to structured model uncertainties. Due to the iterative nature of the learning model improvement, we first review some existing Iterative learning control (ILC) MPC methods. Indeed, ILC method introduced in Arimoto [1990] is a control technique which focuses on improving tracking performance of processes that repeatedly execute the same operation over time. It is of particular importance in robotics and in chemical process control of batch processes. We refer the reader to e.g., Wang et al. [2009], and Ahn et al. [2007] for more details on ILC and its applications. At the intersection of learning based control and constrained control is the ILC-MPC concept. For instance, ILC-MPC for chemical batch processes are studied in Wang et al. [2008], Cueli and Bordons [2008], and Shi et al. [2007]. As noted in Cueli and Bordons [2008] one of the shortcomings of the current literature is a rigorous justification of feasibility, and Lyapunov-based stability analysis for ILC-MPC . For example, in Wang et al. [2008] the goal is to reduce the error between the reference and the output over multiple trials while satisfying only input constraints. However, the reference signals is arbitrary and the MPC scheme for tracking such signals is not rigorously justified. Furthermore, the MPC problem does not have any stabilizing conditions (terminal cost or terminal constraint set). In Cueli and Bordons [2008], an ILC-MPC scheme for a general class of nonlinear systems with disturbances is proposed. The proof is presented only for MPC without constraints. In Shi et al. [2007], the ILC update law is designed using MPC. State constraints are not considered in Shi et al. [2007]. In Lee et al. [1999] a batch MPC (BMPC) is proposed, which integrates conventional MPC scheme with an iterative learning scheme. A simplified static input-output map is considered in the paper as opposed to a dynamical system. Finally, the work of Aswani et al. [2013, 2012a,b], studies similar control objectives as the one targeted in this paper using a learning-based MPC approach. The main differences are in the control/learning design methodology and the proof techniques. In summary, we think that there is a need for more rigorous theoretical justification attempted in this paper. Furthermore, to the best of our knowledge, the literature on ILC-MPC schemes do not

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consider state constraints, do not treat robust feasibility issues in the MPC tracking problem, rigorous justification of reference tracking proofs for the MPC is not present in the literature and stability proofs for the combination of the ILC and MPC schemes are not established in a systematic manner.

The main contribution of this work is to present a rigorous proof of an ILC-MPC scheme using existing Lyapunov function based stability analysis established in Limon et al. [2010] and extremum seeking algorithms in Khong et al. [2013b], to justify the ILC-MPC method in Benosman et al. [2014], where an ES-based modular approach to design ILC-MPC schemes for a class of constrained linear systems is proposed.

The rest of the paper is organized as follows. Section 2 contains some useful notations and definitions. The MPC control problem formulation is presented in Section 3. Section 4 is dedicated to a rigorous analysis of the proposed ES-based ILC-MPC. Finally, simulation results and concluding comments are presented in Section 5 and Section 6, respectively.

2. NOTATION AND BASIC DEFINITIONS

Throughout this paper, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{Z}$ denotes the set of integers. State constraints and input constraints are represented by $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U} \subset \mathbb{R}^m$, respectively. $\mathbb{B}$ refers to a closed unit ball in $\mathbb{R}^n$. The optimization horizon for MPC is denoted by $N \in \mathbb{Z}_{\geq 1}$. The feasible region for the MPC optimization problem is denoted by $\mathcal{X}_N$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $\alpha(0) = 0$ belongs to class $\mathcal{K}$ if it is increasing and bounded. A function β belongs to class $\mathcal{K}_\infty$ if it belongs to class $\mathcal{K}$ and is unbounded. A function $\beta(s,t) \in \mathcal{KL}$ if $\beta(\cdot,t) \in \mathcal{K}$ and $\lim_{t\to\infty} \beta(s,t) = 0$. Given two sets A and B, such that $A \subset \mathbb{R}^n$, $B \subset \mathbb{R}^n$, the Minkowski sum is defined as $A \oplus B := \{a + b | a \in A, b \in B\}$. The Pontryagin set difference is defined as $A \ominus B := \{x | x \oplus B \in A\}$. Given a matrix $M \in \mathbb{R}^{m\times n}$, the set $MA \subset \mathbb{R}^m$, is defined as $MA \triangleq \{Ma : a \in A\}$. A positive definite matrix is denoted by $P > 0$. The standard Euclidean norm is represented as $|x|$ for $x \in \mathbb{R}^n$, $|x|_P := \sqrt{x^T P x}$ for a positive definite matrix P, $|x|_{\mathcal{A}} := \inf_{y\in\mathcal{A}} |x - y|$ for a closed set $\mathcal{A} \subset \mathbb{R}^n$ and $\|A\|$ represents an appropriate matrix norm where A is a matrix. $\mathbb{B}$ represents the closed unit ball in the Euclidean space. Also, a matrix $M \in \mathbb{R}^{n\times n}$ is said to be Schur iff all its eigenvalues are inside the unitary disk.

3. PROBLEM FORMULATION

We consider linear systems of the form

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k), \quad (1)$$

$$y(k) = Cx(k) + Du(k), \quad (2)$$

where ΔA and ΔB represent the uncertainty in the system model. We will assume that the uncertainties are bounded as follows:

Assumption 1. The uncertainties $\|\Delta A\| \leq \ell_A$ and $\|\Delta B\| \leq \ell_B$ for some $\ell_A, \ell_B > 0$.

Next, we impose some assumptions on the reference signal r.

Assumption 2. The reference signal $r : [0,T] \to \mathbb{R}$ is a piecewise constant trajectory for some $T > 0$.

Due to the iterative control design methodology, the initial condition x_0 for the system is fixed over multiple trials and at the end of each trial the state is reset to the initial condition. The goal is design the sequence of control inputs $\{u(k)\}_{k=0}^{T-1}$ using MPC to track the reference trajectory r while satisfying the state and input constraints, and the update laws for parameter estimation of the uncertainties $\Delta A, \Delta B$ after each trial or iteration. We also implicitly assume that the reference signal r is slowly varying and the time T is sufficiently large to allow learning from previous trials. Next, we will explain in detail the optimization problem associated with the MPC based controller. The results stated here are from Limon et al. [2010]. We exploit the analysis results in Limon et al. [2010] to establish that the closed-loop system has an ISS property with respect to the parameter estimation error. We first observe that since the reference trajectory r is a piecewise constant trajectory, the problem of tracking the signal r is simplified to the problem of tracking multiple constant and feasible set points during successive time intervals in $[0,T]$ in the presence of uncertainties.

Since the value of ΔA and ΔB are not known a priori, the MPC uses a model of the plant based on the current estimate $\hat{\Delta}A$ and $\hat{\Delta}B$.

We will now formulate the MPC problem with a given estimate of the uncertainty for a particular iteration of the learning process. We will rewrite the system dynamics as

$$x(k+1) = f(x,u) + g(x,u,\Delta) = F(x,u,\Delta), \quad (3)$$

where $f(x,u) = Ax + Bu$ and $g(x,u,\Delta) = \Delta Ax + \Delta Bu$.

Assumption 3. The state constraint set $\mathcal{X} \subset \mathbb{R}^n$ and control constraint set $\mathcal{U} \subset \mathbb{R}^m$ are compact, convex polyhedral sets.

The MPC model is generated using an estimate $\hat{\Delta}A$, $\hat{\Delta}B$ and is expressed as

$$x(k+1) = f(x,u) + g(x,u,\hat{\Delta}) = F(x,u,\hat{\Delta}). \quad (4)$$

We can now rewrite the actual model as

$$x(k+1) = f(x,u) + g(x,u,\hat{\Delta}) + (\Delta A - \hat{\Delta}A)x + (\Delta B - \hat{\Delta}B)u. (5)$$

This system can now be compared to the model in Limon et al. [2010]. So we have

$$x(k+1) = F(x(k),u(k),\hat{\Delta}) + w(k), \quad (6)$$

where

$$w(k) = (\Delta A - \hat{\Delta}A)x(k) + (\Delta B - \hat{\Delta}B)u(k), \quad (7)$$

and $x(k) \in \mathcal{X}, u(k) \in \mathcal{U}$. The following assumption will be justified in the next section.

Assumption 4. The estimates of the uncertain parameters are bounded with $\|\hat{\Delta}A\| \leq \ell_A$ and $\|\hat{\Delta}B\| \leq \ell_B$ for all iterations of the extremum seeking algorithm.

We now impose certain conditions on the disturbance $w(k)$ and system matrices in accordance with [Limon et al., 2010, Assumption 1].

Assumption 5. The pair $(A+\hat{\Delta}A, B+\hat{\Delta}B)$ is controllable for every realization of $\hat{\Delta}A$ and $\hat{\Delta}B$.

We will denote the actual model using (x,u) and the MPC model through $(\bar{x},\bar{u})$. Hence we have