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Using Stochastic Approximation Type Algorithm for Choice of Consensus Protocol Step-Size in Changing Conditions ⋆

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Abstract: In the paper a multi-agent network system of different computing nodes is considered. A problem of load balancing in the network is addressed. The problem is formulated as consensus achievement problem and solved via local voting protocol. Agents exchange information about their states in presence of noise in communication channels. At certain moment network system topology changes and new step size of control protocol is chosen to meet new conditions. Step size adjustment is done by stochastic approximation type algorithm. Analytically obtained optimal step size values are given. Simulation example demonstrating step size adjustment is provided.

Keywords: load balancing, consensus achievement, stochastic approximation, multi-agent networks

1. INTRODUCTION

Important practical problem in network systems is problem of load balancing. It may arise in such network systems as computer, production, transport, logistics, and other service networks. In computational networks load balancing is needed to improve system efficiency. A multi-agent approach is used to address this problem in network systems. A possible goal for control in such systems is to improve the network speed of operation using communication among agents in the system. In Amelina et al. (2015a) it was shown that the problem of almost optimal task distribution among agents could be reformulated as a problem of the consensus achievement in the network.

The consensus approach was widely applied for solving various practical problems such as cooperative control of multivehicle networks Ren et al. (2007); Granichin et al. (2012), distributed control of robotic networks Bullo et al. (2009), flocking problem Yu et al. (2010a); Virágh et al. (2014), optimal control of sensor networks Kar and Moura (2010) and others. Works Ren and Beard (2007); Chebotarev and Agaev (2009); Li and Zhang (2009); Yu et al. (2010b); Huang (2012); Proskurnikov (2013); Lewis et al. (2014) consider formulating the conditions for achieving consensus in such systems.

In Amelina et al. (2015b) a choice of an optimal step-size of consensus-type protocol for task redistribution among agents in a stochastic network with randomized priorities is considered. It is shown that a trade-off is made between noise sensitivity and the rate of convergence of control protocol while choosing its step-size. The paper proposes a way of choosing step-size to maximize convergence precision.

An optimal step-size of control protocol could be chosen for the network system under certain conditions such as parameters of noise during information exchange, delays occurring in communication channels, system topology etc. But the value of optimal step-size corresponding to different conditions of system operation might be different. In Granichin and Amelina (2015) a problem of tracking under influence of disturbances via simultaneous perturbation stochastic approximation is considered. The paper addresses the problem in general formulation with little assumption about disturbances.

Simultaneous perturbation stochastic approximation (SPSA) was proposed by Spall Spall (1992) and can be used for solving optimization problems in case when it is difficult or impossible to obtain a gradient of the objective function with respect to the parameters being optimized.

In this paper we propose a way to adjust step-size parameter of control strategy in changing conditions. We use a stochastic approximation type procedure to update values of step-size during multi-agent system operation. Obtained analytically optimal step-sizes for the proposed control protocol are provided.

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The paper is organized as follows. Notation used in the paper and the problem formulation are given in Section II. The control protocol for achieving the consensus is introduced in Section III. In Section IV the main assumptions and main results are presented. Simulation results are given in Section V. Section VI contains conclusion remarks.

2. PROBLEM STATEMENT

Let's consider a dynamic network system of n agents, which exchange information among themselves during tasks processing. Tasks may come to different agents of the system in different discrete time instants $t = 1, \ldots$. Agents process incoming tasks in parallel. Tasks can be redistributed among agents based on a feedback.

Without loss of generality, agents in the system are numbered. Assume $N = \{1, \ldots, n\}$ denotes the set of agents in the network system. Let $i \in N$ be the number of an agent. The network topology may switch over time. Let the dynamic network topology be modeled by a sequence of digraphs $\{(N, \mathscr{E}_t)\}_{t \geq 0}$, where $\mathscr{E}_t \subset \mathscr{E}$ denotes the set of edges at time t of topology graph $(N, \mathscr{E}_t)$. The corresponding adjacency matrices are denoted as $A_t = [a_t^{i,j}]$, where $a_t^{i,j} > 0$ if agent j is connected with agent i and $a_t^{i,j} = 0$ otherwise. Here and below, an upper index of agent i is used as the corresponding number of an agent (not as an exponent). Denote $\mathscr{G}_{A_t}$ as the corresponding graph.

To introduce some properties of the network topology, the following definitions from the graph theory will be used. Define *the weighted in-degree* of node i as the sum of i-th row of matrix A: $indeg^i(A) = \sum_{j=1}^n a^{i,j}$; $\mathscr{D}(A) = \mathrm{diag}\{indeg^i(A)\}$ is the corresponding diagonal matrix; $indeg_{\max}(A)$ is the maximum in-degree of graph $\mathscr{G}_A$. Let $\mathscr{L}(A) = \mathscr{D}(A) - A$ denote the *Laplacian* of graph $\mathscr{G}_A$; $\cdot^{\mathrm{T}}$ is a vector or matrix transpose operation; $||A||$ is the Euclidian norm: $||A|| = \sqrt{\sum_i \sum_j (a^{i,j})^2}$; $Re(\lambda_2(A))$ is the real part of the second eigenvalue of matrix A ordered by the absolute magnitude; $\lambda_{\max}(A)$ is the maximum eigenvalue of matrix A.

It is said that digraph $\mathscr{G}_B$ is a subgraph of a digraph $\mathscr{G}_A$ if $b^{i,j} \leq a^{i,j}$ for all $i, j \in N$.

Digraph $\mathscr{G}_A$ is said to contain a *spanning tree* if there exists a directed tree $\mathscr{G}_{tr} = (N, \mathscr{E}_{tr})$ as a subgraph of $\mathscr{G}_A$ which includes all vertices of $\mathscr{G}_A$.

The behavior of agent $i \in N$ is described by characteristics of two types:

- lengths of queue of tasks at time instant t: q_t^i,
- productivity: p^i.

Let random variable η_j denote complexity (or a number of computational operation needed to execute the task) of a task which came to the system. Dynamics of the system can be written in the following way:

$$\sum_{q_{t+1}^i} \eta_j = \sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''},$$

where $\sum_{q_{t+1}^i} \eta_j$ is number of computational operations needed to execute all tasks in the queue of agent i at time instant $t+1$, p^i is productivity of agent i or the number of computational operations it can perform during one tact of the system (assume it is constant), $\sum_{z_t^i} \eta_{j''}$ is the complexity of tasks which came to the system on agent i at time instant t and $\sum_{u_t^i} \eta_{j'''}$ is the complexity of tasks which already came to other agents at previous time instants and were redistributed to agent i according to control protocol.

Assume random variable η has mathematical expectation $\bar{\eta} < \infty$. Let's take expectation of left and right parts of the equation of system dynamics.

$$E\left(\sum_{q_{t+1}^i} \eta_j\right) = E\left(\sum_{q_t^i} \eta_{j'} - p^i + \sum_{z_t^i} \eta_{j''} + \sum_{u_t^i} \eta_{j'''}\right)$$

$$\sum_{q_{t+1}^i} \bar{\eta} = \sum_{q_t^i} \bar{\eta} - p^i + \sum_{z_t^i} \bar{\eta} + \sum_{u_t^i} \bar{\eta}$$

Left and right parts are now equal to number of tasks at agent i multiplied by their average complexity.

$$\bar{\eta} \sum_{q_{t+1}^i} 1 = \bar{\eta} \sum_{q_t^i} 1 - p^i + \bar{\eta} \sum_{z_t^i} 1 + \bar{\eta} \sum_{u_t^i} 1$$

$$\bar{\eta} q_{t+1}^i = \bar{\eta} q_t^i - p^i + \bar{\eta} z_t^i + \bar{\eta} u_t^i$$

Divide both parts of the equation by constant value $\bar{\eta}$. We get discrete model which allows as to analyze system dynamics without information about complexities of each task in the system (but with assumption their average value is bounded). For all $i \in N$, $t = 0, 1, \ldots$, the dynamics of the network system in a vector form is as follows

$$q_{t+1}^i = q_t^i - \tilde{p}^i + z_t^i + u_t^i, \tag{1}$$

where $\tilde{p}^i = p^i / \bar{\eta}$, z_t^i the amount of new tasks, which came to the system and were received by agent i at time instant t; u_t^i is control action (redistributed tasks to agent i at time instant t), which is chosen based on some information about queue lengths of neighbors q_t^j, $j \in N_t^i$, where N_t^i is the set $\{j \in N : a_t^{i,j} > 0\}$.

Denote

$$x_t^i = \frac{q_t^i}{\tilde{p}^i} \tag{2}$$

the *load* of agent $i \in N$. Assume, that $\tilde{p}^i \neq 0$, $\forall i \in N$. In Amelina et al. (2015a) it was proven that from all possible options for the redistribution of all tasks the minimum operation time of the system is achieved when loads x_t^i are equalized throughout the network. Hence, it is important to consider the achievement of the following goal.

It is required to maintain balanced (equal) loads across the network under conditions of changing network topology.

At this setting we can consider the consensus problem for states x_t^i of agents, where x_t^i is a state of agent $i \in N$. We use the following definitions.

Definition *1.* n agents of a network are said to reach a *consensus* at time t if $x_t^i = x_t^j \ \forall i, j \in N, i \neq j$.

Definition *2.* n agents are said to achieve *asymptotic mean square ε-consensus* for $\varepsilon > 0$ when

$$\overline{\lim}_{t \to \infty} \mathrm{E} \|x_t^i - x_t^j\|^2 \leq \varepsilon.$$

To ensure balanced loads across the network (e.g., in order to increase the overall throughput of the system and to reduce the execution time), it is naturally to use a redistribution protocol over time. We assume that to form the control (redistribution)