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Output Robust Control with Anti-Windup Compensation for Quadcopters ⋆

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Abstract: In the paper an output control approach for quadcopters under the condition of the bounded input signal is presented. This algorithm is based on the high-gain principle "consecutive compensator", which was augmented by an auxiliary integral loop in order to implement the anti-windup scheme. The mathematical model describing quadcopters is decomposed on two parts: a static MIMO transformation and six SISO dynamical channels. The controller generates virtual input signals for these channels, which after inverse MIMO transformation are allocated between the actuators as real bounded control signals.

Keywords: Robust control, Output regulation, Saturation control, MIMO systems, Control applications.

1. INTRODUCTION

This investigation develops simple control algorithms under condition of bounded input with application to quadcopters. Output control is significantly useful, when output derivatives are unmeasurable. Parameters of real systems often can be unknown, so design of robust regulators is very important from practical point of view. Inputs of all real systems are limited due to hardware constraints. So, one of the main motivation of this paper is design saturated control for quadcopters representing MIMO systems reducing (or even avoiding) loss of performance. This becomes available by redesign of consecutive compensator approach and addition of an integral loop together with anti-windup scheme (AW), which was first introduced in Fertik and Ross (1967). These problems are investigated in this work.

In papers Bobtsov (2005); Bobtsov and Nikolaev (2005); Bobtsov et al. (2007); Pyrkin et al. (2013) models with parametrically uncertain linear parts and static nonlinear blocks are considered. The proposed approach is based on the pacification approach Fradkov (1974, 2003). Controller has a simple structure and can be implemented as a feedback providing strictly positive realness of the closed-loop system. The developed control algorithm is close to results described in Bar-Kana (1987); Barkana (2004); Kaufman et al. (2012), but with weaker requirements for the plant, which are: the known relative degree and minimum phasness of the plant. In Bobtsov et al. (2011) this approach is used to compensate sinusoidal disturbance in uncertain nonlinear systems.

Input saturation is one of the significant problems related with implementation of theoretical results in real systems. If the control signal exceeds the input limits it may lead to undesirable behavior of the system and damage its performance. Anti-windup is frequently used approach in wide range of applications to solve this problem. The analysis of the aircraft control system with input saturation conducted in Andrievsky et al. (2013) has shown existence of hidden oscillations, which may be compensated by the anti-windup scheme. In Van den Berg et al. (2006) the anti-windup approach based on the property of convergency for marginally stable linear plants with saturated input is proposed. The latter was applied to the aircraft control problem in Pogromsky et al. (2009); Leonov et al. (2012). In van den Bremer et al. (2008) a PI controller with anti-windup was used to control a manufacturing machine.

Recently quadcopters became popular in applied control theory, as they represent a convenient experimental platform for testing of various control approaches for robotic MIMO systems (Pyrkin et al. (2014a,b); Bazylev et al. (2014)). Quadcopters are controlled by varying rotor speed changing the lift force. It is the most maneuver vertically taking off air vehicle. It belongs to a class of autonomous robots which movement occurs without any contact with some supporting surface. A quadcopter as a plant is a MIMO dynamical system. Its mathematical model will be divided on two parts: static MIMO transformation and a six SISO channels that allows to design the control law in two steps. At the first step we design the saturated

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virtual controls for each SISO channel and then after the inverse MIMO transformation we can get the control law for considered system. The designed regulator generates bounded control signal with windup compensation, which becomes possible by adding the auxiliary integral loop in the structure of consecutive compensator. Also it is assumed that unknown wind disturbances act on the each channel of the quadcopter. Their values and directions change negligibly slowly. So, we can consider them as unknown constant signals which must be canceled by the new structure of the controller. The integral loop allows to remove a static error, which may occur, and cancel these wind disturbances. This result is extension of the works Pyrkin et al. (2014a,b) enhanced by the condition of bounded input and performance improvement.

2. PROBLEM FORMULATION

Consider the class of MIMO systems

$$y_i(t) = G_i(u_1, u_2, \dots, u_r, f_i), \tag{1}$$

where $i = 1..l$, $y_i(t)$ are the output variables, $u_1, \dots, u_r$ are the actuator inputs, G_i are the nonlinear differential operators in common case, and f_i are the unknown wind disturbances, which change their values and directions negligibly slowly.

Perform decomposition of the model (1) as follows

$$y_i(t) = \overline{G}_i(U_i, f_i), \quad U_i = \widetilde{G}_i(u_1, u_2 \dots, u_r), \tag{2}$$

where $\overline{G}_i, i = 1, ..l$ are the differential SISO channels and $\widetilde{G}_i, i = 1, ..l$ are the MIMO static functions.

Consider the model of the separate SISO system

$$y_i(t) = \frac{b(p)}{a(p)} u_i(t) + f_i, \tag{3}$$

where $p = d/dt$ is the differential operator, $y_i(t)$ are the measurable output variables (theirs derivatives are unmeasurable), $b(p) = b_m p^m + \dots + b_1 p + b_0$, and $a(p) = p^n + \dots + a_1 p + a_0$ are polynomials with unknown coefficients, the number $r \le n-1$, $\rho = n - m$ is the known relative degree of the transfer function $\frac{b(p)}{a(p)}$, the polynomial $b(p)$ is Hurwitz and its parameter $b_m > 0$, the unknown constants f_i represent negligibly slow-varying wind disturbances for all the channels.

Since all real systems work under the obvious condition of bounded control, in this study it is worth to consider the plant with saturated input

$$y_i(t) = \frac{b(p)}{a(p)} \hat{u}_i(t) + f_i, \tag{4}$$

where $\hat{u}(t) = \operatorname{sat} u(t) \in \mathbb{R}^1$ is the saturated (bounded) input signal satisfying

$$\hat{u}(t) = \operatorname{sat} u(t) = \begin{cases} u_{upp}, & \text{if } u(t) \ge u_{upp}; \\ u(t), & \text{if } u_{low} < u(t) < u_{upp}; \\ u_{low}, & \text{if } u(t) \le u_{low}. \end{cases} \tag{5}$$

where u_{low} and u_{upp} are limits of the input caused by the hardware capabilities.

The first goal is to provide the exponential stability of linear system (3). The second one is to extend this result for quadcopters with slowly varying wind disturbance cancellation under condition of bounded input. The purpose of control is to stabilize the quadcopter in the specified point with the specified orientation.

3. CONTROL DESIGN

In this section the notation i which corresponds to particular SISO channel of the model (2) will be omitted for conciseness.

From Bobtsov (2002) it is known that there exist the parameters $\mu > \mu_0 > 0$, $\sigma > \mu$ and $\delta > 0$ (possibly small) such that with the control algorithm

$$u(t) = -\mu \alpha(p) \hat{y}(t), \tag{6}$$

$$\dot{\xi}(t) = \sigma(\Gamma \xi(t) + d k_1 y(t)) \tag{7}$$

$$\hat{y}(t) = h^T \xi(t), \tag{8}$$

ensures asymptotic stability of the linear plant $y(t) = \frac{b(p)}{a(p)} u(t)$ with unbounded control. Here $\alpha(p)$ is the Hurwitz polynomial of the degree $(\rho - 1)$, $\hat{y}(t)$ is the estimate of the output variable, Γ, d, h are matrices and vectors of corresponding dimensions

$$\Gamma = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_{\rho-1} \end{bmatrix}, d = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, h = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \tag{9}$$

$k = \{k_1, k_2, \dots, k_{\rho-1}\}$ are chosen for the estimation model (7), (8) to be stable.

Saturation of the control signal is very important to properly apply the obtained algorithm to real systems. In the same time anti-windup schemes allow decrease overshoot of the output variable caused by bounded input. So, in order to do these both tasks first rewrite the control law (6) in the form

$$u(t) = -\mu \frac{\beta(p)}{p} \hat{y}(t), \tag{10}$$

where $\beta(p)$ is the Hurwitz polynomial of the degree ρ.

Substitute the control law (10) to the plant model (3) as follows

$$y(t) = \frac{b(p)}{a(p)} \left(-\mu \frac{\beta(p)}{p} \hat{y}(t) \right). \tag{11}$$

Expressing the error between the output and its estimate $\varepsilon(t) = y(t) - \hat{y}(t)$ rewrite the previous equation

$$y(t) = \frac{b(p)}{a(p)} \left(-\mu \frac{\beta(p)}{p} y(t) + \mu \frac{\beta(p)}{p} \varepsilon(t) \right). \tag{12}$$

After simple transformations obtain

$$y(t) = \frac{\mu b(p) \beta(p)}{a(p) p + \mu b(p) \beta(p)} \varepsilon(t), \tag{13}$$

where the transfer function of the closed-loop system $\frac{\mu b(p)\beta(p)}{a(p)p + \mu b(p)\beta(p)}$ is strictly positive real (SPR).

Transform (13) to a state-space model

$$\dot{x}(t) = A x(t) + B \varepsilon(t), \tag{14}$$

$$y(t) = C^T x(t), \tag{15}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, A, B, C are matrices of the corresponding dimensions.