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Parameter estimation of a real hydrological system using an adjoint method ⋆

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Abstract: This paper deals with the problem of parameter estimation in a real hydrological catchment of Niger, including Manning roughness and Horton infiltration coefficients. Firstly, the dynamics of water flow on this catchment are modeled by using the continuity equation of Saint-Venant and the Horton infiltration model whose the parameters are supposed to be unknown. The estimation approach is then based on the calculus of variations which is applied on a Lagrangian objective functional, in order to get the gradients of this functional with respect to parameters needing to be estimated. These gradients are used as inputs of a quasi-Newton optimization algorithm to solve the estimation problem. The presented approach is implemented with real data of the Niger catchment called Tondi Kiboro to get the desired values of parameters.

Keywords: Horton model; Infinite dimensional systems; Parameter estimation; Saint-Venant model; Inverse problem, Adjoint method, Tondi Kiboro catchment;

1. INTRODUCTION

Located 70 km east of the Niamey region, Niger, as presented in Fig. 1, the Banizoumbou Tondi Kiboro catchment is a typical Sahelian one which attracts attention of scientists. Some studies like Descroix et al. [2011, 2012], have been carried out on this area to investigate the hydrological consequences of land use change due to the evolution of surface runoff coefficient and the possibility to define this small region as a deep infiltration one. The present work uses the observation data provided by measurement campaigns from 2004 to 2012 on the same catchment as two previous studies but with a different purpose. The main aim of this paper is indeed to apply the optimal control theory to identify the friction coefficient and some empirical parameters of the Horton infiltration. This can be considered as an inverse problem as well. This work comes in continuation of some former studies of ours, dedicated to the estimation of Manning's roughness coefficient only and with a different infiltration model (see Nguyen and Fenton [2005]), in spirit of Nguyen et al. [2016]. In Nguyen et al. [2015], the infiltration process, as a component of the water flow source, is assumed to be perfectly known by on-site measurements and using a lookup table (presented in Descroix et al. [2011, 2012]). In the present paper, the purpose is to go further by examining the estimation of the infiltration parameters as well. The empirical Horton model is adopted instead of the physical Green-Ampt infiltration model to overcome the complexity (discontinuities due to multi-ponding time)

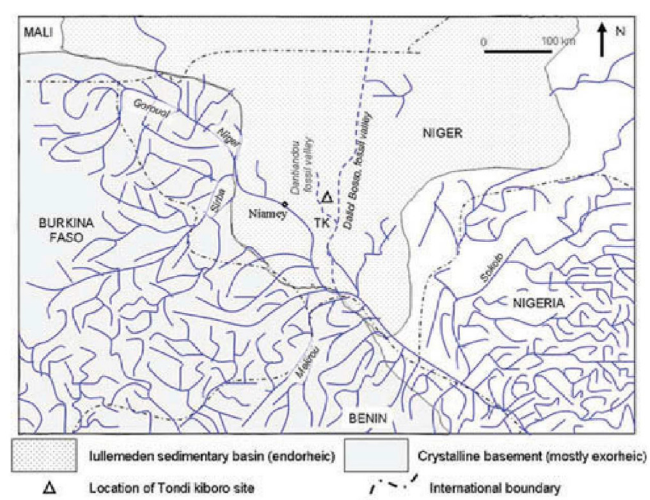

Fig. 1. Location of Tondi Kiboro catchment, Descroix et al. [2011]

of this model under variable rainfall condition. It can be noticed that some works in hydrological domain investigated the estimation of friction parameter in Saint-Venant equations: Hameed and Ali [2013] used a calibration technique with the HEC-RAS flow model of Hilla river; Atanov et al. [1999] proposed a variational approach in trapezoidal open channels; Ramesh et al. [2000] studied a sequential quadratic programming algorithm applied on open channel flow; Nguyen and Fenton [2005] employed an objective function minimization and synthetic data in compound channels.

In addition the adjoint-based method is a mature concept which has been used for the optimal control of open-

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channel hydraulic systems, see Chen and Georges [2000]; Ding and Wang [2006]; Georges [2009] for instance.

However there are very few works combining the estimation of roughness coefficient and infiltration model. In that respect, the present study can be related with the adjoint sensitivity analysis method developed by Castaings et al. [2007] for a distributed rainfall-runoff model of the different Thoré catchment, using Saint-Venant equation and Green-Ampt infiltration model. However the partial differential equation discretization and the adjoint system simulation are done by using TAPENADE automatic differentiation engine instead of using Lax-Wendroff finite difference as in our work.

This paper provides a clear demonstration of the effectiveness of an adjoint method to an hydrological model on the basis of real measurement data. Along with the previous works using Green-Ampt model, the present one can allow a comparison of applicability and suitability between Green-Ampt and Horton models for infiltration. Moreover, it facilitates also further researches dealing with hydrological changes including soil erosion, components of water cycle, fallow period ones of the major environmental challenges on this region.

The remaining of paper is organized as follows: The dynamics of overland flow at Tondi Kiboro are first formulated. Then the estimation method based on variational calculation is presented. In section 4, the estimation results based on real data are given. Finally, section 5 gives some conclusions of the paper.

2. OVERLAND FLOW MODEL OF TONDI KIBORO CATCHMENT

2.1 Overview of Tondi Kiboro catchment

The considered catchment is made of 3 small basins near the Tondi Kiboro village. Among the basins, there are two nested ones as depicted in Fig. 2 with surfaces of 46800 m^2 and 63720 m^2 for the upper and lower basins respectively. We consider only the lower part with a length of 600 m approximately (between the upper-stream end and downstream end and measurements stations). It is a sandy hill slope with average bed-slope of 3.5% . Its top soil is constructed by 10% silt and clay, and 90% sands which gives this catchment a large infiltration capacity (see Descroix et al. [2011] for further information).

2.2 Overland flow equations

From an analysis of the geometric and hydraulic characteristics of Tondi Kiboro catchment, we can use some approximations (see Nguyen et al. [2014] for more details) to neglect the momentum equation of the Saint-Venant PDEs. As a result, the dynamics of one-dimensional overland flow at Tondi Kiboro catchment can be described by the first continuity equation as follows:

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial f(h,x)}{\partial x} = r(t) - i(t) \\ h(x,0) = h_0^i(x) \\ h(0,t) = h_0^b(t) \end{cases} \tag{1}$$

where x = position, (m); t = time, (s); h = water flow depth, (m); $f(h,x) = h^{5/3}\dfrac{S_0^{1/2}}{n}$ = flow per width unit (m^2/s) with the Manning roughness coefficient n; S_0 = bed slope, (m/m); $h_0^i(x)$ and $h_0^b(t)$ are the predefined initial and boundary conditions; $r(t)$ = variable rainfall rate, (m/s); $i(t)$ = infiltration rate, (m/s). Due to the small size of Tondi Kiboro catchment, the rainfall intensity and infiltration rate can be assumed to be identical all over the spatial domain, and only time variable. Moreover, the Manning's roughness n is a constant parameter because the soil characteristic and vegetation factor do not vary a lot in a small area like this one.

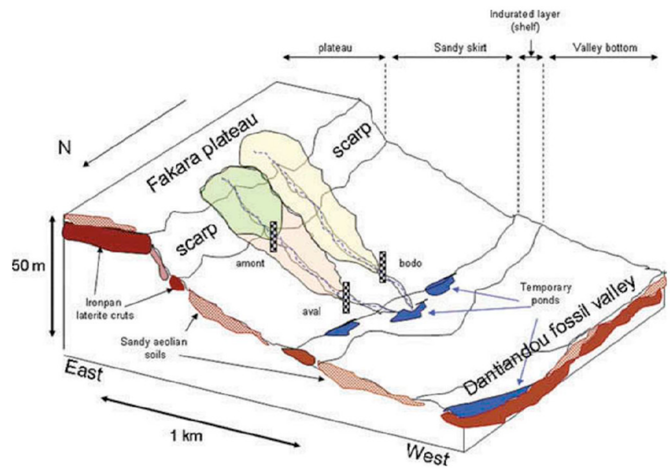

Fig. 2. Overview of Tondi Kiboro basins Descroix et al. [2011]

The water discharge $Q(x,t)$ can be described by a simple relationship between flow per width unit and the surface flow width W:

$$Q(x,t) = W h^{5/3}\frac{S_0^{1/2}}{n} \tag{2}$$

Instead of Green-Ampt model used in previous study of Nguyen et al. [2015], Horton equation, a purely empirical model, is employed to describe the dynamics of infiltration process after the rainfall intensity exceeds the infiltration capacity of soil (see Mays [2010]). It does not depend on rainfall or soil properties. According to this model, the infiltration capacity tends to decrease in an exponential manner from the initial infiltration rate, as:

$$i(t) = i_c + (i_0 - i_c)e^{-kt} \tag{3}$$

where $i(t)$ = infiltration rate at time t, (m/s); k = a constant representing the exponentiation rate, $(1/s)$; i_c = equilibrium infiltration rate, (m/s); i_0 = initial infiltration capacity, (m/s). The infiltration capacity $i(t)$ is described by a function of time rather than the soil characteristics or the rainfall rate. As a consequence, this model only makes sense if $i(t)$ is lower than $r(t)$. This model can be compared to the Green-Ampt model after the ponding time. Because of the soil characteristic of Tondi Kiboro catchment (a very large infiltration capacity), the hypotheses is that the ponding time is very small, allows the use of the Horton model.

The model of overland flow at Tondi Kiboro is adjusted by tunning 4 parameters, including Manning coefficient n, equilibrium infiltration rate i_c, initial infiltration rate i_0 and exponential rate k.