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Adaptive Algorithm for Linear Systems with Input Delay[1]

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Abstract: The paper deals with the adaptive algorithm for single input single output (SISO) linear plants with any relative degree and input delay. The control system design is based on modified high-order tuners. The resulting algorithm ensures required accuracy of difference between the plant output and the reference signal. The modeling results illustrate the effectiveness of the proposed algorithm.

Keywords: Adaptive control, input delay, Lyapunov-Krasovskii functional, high-order tuner.

1. INTRODUCTION

The control of plants under uncertainties is one of a fundamental problem of automatic control. This problem becomes much more complicated if the plant has delay in the input signal. Moreover, the closed loop system can become unstable if delay is ignored (Kolmanovski and Nosov, 2009, Fridman, 2014).

First, the prediction problem was solved in (Smith, 1959). The solution of (Smith, 1959) is based on introduction of a special loop (Smith predictor) parallel to the plant. Another result is a Resvik controller (Kolmanovski and Nosov, 2009). However, if the relative degree of the plant model more than one, than Resvik controller can be not implementable. For discrete systems Tsypkin predictor is proposed in (Tsypkin, 1986). Note, that the solutions (Smith, 1959, Tsypkin, 1986) are obtained for stable plants with known parameters.

Other widely spread result is the method of (Manitius and Olbort, 1979). This algorithm allows to control of unstable plants with input delay. In (Kristic, 2009) application of the control algorithm (Manitius and Olbort, 1979) is considered with using backstepping method. The algorithms (Manitius and Olbort, 1979, Manitius and Olbort, 1979) are designed for plant with known parameters.

The paper (Lozano et. al, 2004) describes a resetting Smith predictor for control of linear plants under parametric uncertainties. In (Tsykunov, 2000) the adaptive modification of Smith predictor with one adjustable parameter is proposed for plants under measurable state vectors. In (Niculescu and Annaswamy, 2003) the results of (Smith, 1959, Manitius and Olbort, 1979) are generalized for control of single input single output (SISO) plants under parametric uncertainties with relative degree is not exceed two. The results of (Niculescu and Annaswamy, 2003) are generalized for control of stable plants with any relative degree in (Furtat, 2014). The papers (Bazylev, Pyrkin, 2013, Bazylev, Zimenko, Margun et al., 2014, Bazylev, Margun, Zimenko, 2014, Pyrkin, Bobtsov, Kolyubin et al., 2013) application of adaptive control under delays and constraints.

Unlike the described results, we will consider the problem of control of linear SISO plants with any relative degree and with input delay under parametric uncertainties. The control system design is based on modified high-order tuners, first proposed in (Tsykunov, 2006) and generalized for stable plants with input delay in (Furtat, 2014). Unlike higher order tuners proposed in (Morse, 1992, Nikiforov, 1999), the modified high order tuners have one adjustable parameter and allow reducing the dynamical order of the closed loop system.

The paper is organized as follows. The problem statement is presented in Section 2. In Section 3 the parameterization of the plant equation is considered. In Section 4 we design the control algorithm for SISO plants with any relative degree and input delay under parametric uncertainties. The proposed algorithm ensures small difference between plant output and reference model output. In Section 5 we consider simulation results. Concluding remarks are given in Section 6. Appendix A gives the proof of the control system.

2. PROBLEM STATEMENT

Consider a plant model in the form

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$$Q(p)y(t) = kR(p)u(t-h),$$
$$p^i y(0) = y_{i0}, \quad i = 0,...,n-1, \quad u(s) = 0, \quad s \in [-h, 0), \tag{1}$$

where $y(t) \in R$ is an output, $u(t) \in R$ is an input, $Q(p)$ and $R(p)$ are linear differential operators with unknown coefficients, $\deg Q(p) = n$, $\deg R(p) = m$, $k > 0$, y_{i0}, $i = 0, ..., n-1$ are unknown initial conditions, $p = d/dt$.

Assume that the coefficients of operators $Q(p)$, $R(p)$ and coefficient k belong to a known compact set Ξ. The polynomial $R(\lambda)$ is Hurwitz, where λ is a complex variable.

Consider the reference model

$$Q_m(p)y_m(t) = k_m R_m(p)r(t), \tag{2}$$

where $y_m(t) \in R$, $r(t)$ is a piecewise continious function, $k_m > 0$ is a known, $Q_m(p)$, $R_m(p)$ are known, $\deg Q_m(p) = n$, $\deg R_m(p) = m$, $Q_m(\lambda)$ and $R_m(\lambda)$ are Hurwitz polynomials.

The problem is to design the control system such that the following condition holds

$$|y(t) - y_m(t-h)| < \delta \text{ for } t > T, \tag{3}$$

where $\delta > 0$ is prespecified sufficiently required accuracy, $T > 0$ is a transient time.

3. PARAMETRIZATION OF A PLANT MODEL

Rewrite the operators $Q(p)$ and $R(p)$ as follows

$$Q(p) = Q_m(p) + \Delta Q(p), \quad R(p) = R_m(p) + \Delta R(p), \tag{4}$$

where $\Delta Q(p)$ and $\Delta R(p)$ are operators with unknown coefficients and the orders of these operators does not exceed $n-1$ and $m-1$ respectively.

Substituting (4) into (1) and taking into account (2), rewrite the equation for the tracking error $e(t) = y(t) - y_m(t-h)$:

$$e(t) = \frac{kR_m(p)}{Q_m(p)}\left[u(t-h) + \frac{\Delta R(p)}{R_m(p)}u(t-h) - \right.$$
$$\left. - \frac{\Delta Q(p)}{kR_m(p)}y(t) - \frac{k_m}{k}r(t-h)\right]. \tag{5}$$

Introduce the control law

$$u(t) = \frac{Q_m(p)}{R_m(p)}\bar{v}(t), \tag{6}$$

where $\bar{v}(t)$ is an estimate of the auxiliary control law $v(t)$. Taking into account control law (6), rewrite the equation (5):

$$e(t) = k\left[v(t-h) + \frac{\Delta R(p)}{R_m(p)}v(t-h) - \frac{\Delta Q(p)}{kQ_m(p)}y(t)\right.$$
$$\left. - \frac{k_m R_m(p)}{kQ_m(p)}r(t-h) + \varepsilon(t-h)\right], \tag{7}$$

where $R_m(p)\varepsilon(t) = R(p)(\bar{v}(t) - v(t))$.

Introduce the filters

$$\dot{\theta}_1(t) = F_1\theta_1(t) + bv(t-h), \quad \theta_1(0) = 0,$$
$$\dot{\theta}_2(t) = F_2\theta_2(t) + by(t), \quad \theta_2(0) = 0, \tag{8}$$
$$\dot{\theta}_3(t) = F_2\theta_3(t) + br(t-h), \quad \theta_3(0) = 0.$$

Here $\theta_1(t) \in R^m$, $\theta_2(t) \in R^n$, $\theta_3(t) \in R^n$, F_1 and F_2 are matrices in Frobenius form with characteristic polynomials $R_m(\lambda)$ и $Q_m(\lambda)$ respectively, $b = [0, \ldots, 0, 1]^{\mathrm{T}}$.

Taking into account (8), rewrite equation (7) as follows

$$e(t) = k\left[v(t-h) + c_{01}^{\mathrm{T}}\theta_1(t) - c_{02}^{\mathrm{T}}\theta_2(t)\right.$$
$$\left. - c_{03}^{\mathrm{T}}\theta_3(t) + \varepsilon(t-h)\right], \tag{9}$$

where c_{01}, c_{02}, c_{03} are vectors of unknown constant parameters depending on the coefficients of $\Delta R(p)$, $\Delta Q(p)/k$ and $k_m R_m(p)/k$ accordingly.

Taking into account the first expression of (4), rewrite the plant (1) in the form

$$y(t) = \frac{kR(p)}{R_m(p)}v(t-h) - \frac{\Delta Q(p)}{Q_m(p)}y(t) + k\varepsilon(t-h)$$
$$= \bar{c}_{01}^{\mathrm{T}}\theta_1(t) - kc_{02}^{\mathrm{T}}\theta_2(t) + k\varepsilon(t-h), \tag{10}$$

where $\bar{c}_{01}$ is a vector composed of the coefficients of $kR(p)$.

Substituting (10) into (8), we get

$$\dot{\theta}_2(t) = \left(F_2 - kbc_{02}^{\mathrm{T}}\right)\theta_2(t) + b\bar{c}_{01}^{\mathrm{T}}\theta_1(t) + kb\varepsilon(t-h). \tag{11}$$

Introduce the notation

$$c_0 = -\left[c_{01}^{\mathrm{T}} \ -c_{02}^{\mathrm{T}} \ -c_{03}^{\mathrm{T}}\right]^{\mathrm{T}}, \ w(t) = \left[\theta_1^{\mathrm{T}}(t) \ \theta_2^{\mathrm{T}}(t) \ \theta_3^{\mathrm{T}}(t)\right]^{\mathrm{T}}. \tag{12}$$

Taking into account (11) and (12), rewrite equations (8) and (9) in the form

$$\dot{w}(t) = Aw(t) + Bf(t-h) + D\varepsilon(t-h), \tag{13}$$

$$e(t) = k\left[v(t-h) - c_0^{\mathrm{T}}w(t) + \varepsilon(t-h)\right], \tag{14}$$

where

$$A = \begin{bmatrix} F_1 & 0 & 0 \\ b\bar{c}_{01}^{\mathrm{T}} & F_2 - kbc_{02}^{\mathrm{T}} & 0 \\ 0 & 0 & F_2 \end{bmatrix}, \ B = \begin{bmatrix} b & 0 \\ 0 & 0 \\ b & 0 \end{bmatrix}, \ D = \begin{bmatrix} 0 \\ kb \\ 0 \end{bmatrix},$$

$$f(t-h) = \left[v(t-h) \quad r(t-h)\right]^{\mathrm{T}}.$$

Let us find the solution of (13) in the form

$$w(t+h) = e^{Ah}w(t) + \int_{-h}^{0} e^{-Ag}Bf(t+g)dg$$
$$+ \int_{-h}^{0} e^{-Ag}D\varepsilon(t+g)dg. \tag{15}$$

Substituting (15) into (14), we get

$$e(t) = k\left[v(t-h) - \int_{-h}^{0}\beta_0^{\mathrm{T}}(g)f(t+g-h)dg\right.$$
$$\left. - \alpha_0^{\mathrm{T}}w(t-h)\right] + \varphi(t). \tag{16}$$

Here $\varphi(t) = \int_{-h}^{0}\gamma_0^{\mathrm{T}}(g)\varepsilon(t+g-h)dg + k\varepsilon(t-h)$, $\alpha_0^{\mathrm{T}}(g) = c_0^{\mathrm{T}}e^{Ah}$, $\beta_0^{\mathrm{T}}(g) = c_0^{\mathrm{T}}e^{-Ag}B$, $\gamma_0^{\mathrm{T}}(g) = kc_0^{\mathrm{T}}e^{-Ag}D$ are new vectors and matrix of unknown constant parameters.

4. THE MODIFIED HIGH ORDER TUNERS FOR PLANT WITH INPUT DELAY

Introduce the auxiliary control law in the form