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Influence of wall roughness on discharge coefficient of sonic nozzles

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ABSTRACT

To research the influence of roughness on discharge coefficient of axisymmetric sonic nozzles systematically, a turbulence model was established, and standard k–ε model was used in the turbulent core region while Wall Functions was carried out in the boundary layer region. A series of numerical simulations were conducted to research discharge coefficients of 6 critical flow Venturi nozzles with throat diameter ranging from 0.5 to 100 mm when Reynolds numbers ranges from 10^4 to 10^9 and relative roughness from 10^{-2} to 10^{-6}. The validity of the simulation model was confirmed by both the experimental data of Stewart and ISO 9300 empirical equation. According to the simulation results and theoretical analysis, the relations between discharge coefficient and relative roughness were obtained. It is recommended that the dimensionless parameter relative roughness should be used in ISO 9300 rather than absolute roughness. Additionally, when the machining of nozzle cannot satisfy the ISO 9300 requirement or the Reynolds numbers exceed the upper limits of the ISO 9300 equation, the effect of roughness should be considered, and the relative roughness of sonic nozzle should be provided clearly in the further experiment of discharge coefficient.

1. Introduction

Generally there exist three methods to research discharge coefficient of sonic nozzles namely theoretical analysis, experimental verification and numerical simulation. Theoretical analysis began in 1960s. Stratford [1] divided the flow field of sonic nozzles into two regions, namely the turbulent core region owing to non-one-dimensional flow and the boundary layer region due to viscous effects. The boundary layer was divided into laminar and turbulent layer further. Finally the calculation equations were obtained and its structure became the prototype of the discharge coefficient equations hereafter. Experiments using four different gases including air were conducted by Arnberg [2]. The fitted curve had a good match with the equation deduced by Stratford. The turbulent core influenced by non-one-dimensional flow was analyzed and the calculation equation of the inviscid discharge coefficient was obtained by Hall [3]. The relation between the displacement thickness δ^* and the characteristic length L for the laminar boundary layer region was derived from fundamental theories of gas dynamics by Geropp [4] after a series of complicated deductions and simplifications. The research results of Hall and Geropp were integrated by Ishibashi [5] and the equation of the discharge coefficient caused by the core flow distribution and the laminar boundary layer was obtained. Furthermore, the equation was verified through experiments with super accurately machined nozzles by pVTt gas flow standard device of National Research Laboratory of Metrology (NRLM) in Japan [5,6].

According to the request of surface roughness to critical-flow toroidal-throat Venturi nozzles by ISO 9300 [7], the discharge coefficient equations can be divided into 'normally machined' and 'accurately machined'. The demand for the 'normally machined' nozzle is the relative surface roughness Ra/d below 1.5×10^{-5} while for the 'accurately machined' is the absolute roughness Ra less than 0.04 μm. The 'normally machined' equation was summarized by Ishibashi [8] using experimental data from Physikalisch-Technische Bundesanstalt (PTB) [9], NRLM [5], National Institute of Standards and Technology (NIST) [10] and other national institutes of metrology during the year from 1962 to 2000 as well as the theoretical analysis results of Stratford [1], and most data are within $\pm0.3\%$ of the fitted curve. Meanwhile the 'accurately machined' equation was the fitted equation by Ishibashi and Takamoto [11] using data through experiments of 23 super-accurate nozzles whose absolute roughness is less than 0.03 μm.

For the past few years, an increasing number of researchers take notice of the effect of roughness on the discharge coefficient characteristics of nozzles. Ishibashi [5] minimized the roughness effect by using super-accurately machined nozzles. Through experiments, Wendt [9] found that, when absolute roughness ranges from 0.1 to 1 μm and Reynolds number is below 1.5×10^5, there is no systematic or significant influence on discharge coefficient. Meanwhile, Wendt put forward that the requests of roughness are hard to meet for both 'normally machined' and 'accurately machined' nozzles. By comparing

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discharge coefficients of normally machined' and 'accurately machined', C. H. Li found the effect of roughness on discharge coefficient is relatively small at normal temperature and atmospheric pressure conditions [12]. Stewart [13] found that discharge coefficient is not only related to Reynolds number but also influenced by the existence of roughness; however, no theoretical analysis was presented. Up to now, the influence of roughness on discharge coefficient mainly focuses on orifice plate and Venturi tube [14,15]. The structures and operation conditions of orifice or Venturi tube with ISO 9300 sonic nozzles are distinctly different. However, the research of the roughness on discharge coefficient of sonic nozzle is still in the stage of experimental exploration. The effect of roughness on the boundary layer transition of nozzle was studied by Anthony [16]. The inlet Mach and Nusselt numbers were influenced by surface roughness according to Alper [17]. To sonic nozzle, ISO 9300 simply makes a distinction based on roughness and does not refer the specific relation between discharge coefficient of sonic nozzle and roughness parameters. Currently, major research organizations start to notice that roughness has a certain influence on discharge coefficient. However, in order to further improve the accuracy of discharge coefficient equations of ISO 9300 sonic nozzles, the relation of roughness on discharge coefficient should be studied. In this paper, to analyze the influence of roughness on discharge coefficients of ISO 9300 sonic nozzle, numerical calculations to nozzles with diameter from 0.5 to 100 mm, when Reynolds numbers ranges from 10^4 to 10^9 and relative roughness from 10^{-2} to 10^{-6} were conducted. Meantime, the flow physics revealed in the paper were theoretically analyzed.

2. Theory of discharge coefficient

2.1. Influence factors of discharge coefficient

Fig. 1 is the sketch map of toroidal-throat Venturi nozzle according to the regulation of ISO 9300. Given that the gas is ideal gas and the flow is one-dimensional and isentropic, under critical flow conditions, the ideal flow-rate q_{mi} is calculated by the following equation [7,18].

$$q_{mi} = \frac{A_* C_* p_0}{\sqrt{R_m T_0}} \tag{1}$$

where, the cross-sectional area of nozzle throat A_* is $\pi d^2/4$; d is the throat diameter of nozzle; gas constant R_m is R/M, R is molar gas constant, and M is the gas molar mass; p_0, T_0 are the absolute stagnation pressure and temperature; critical flow function C_* is given by Eq. (2). [25]

$$C_* = \sqrt{\gamma \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{2}$$

where, γ is isentropic exponent. However, due to non-one-dimensional flow and viscous effects, the real flow-rate q_m is less

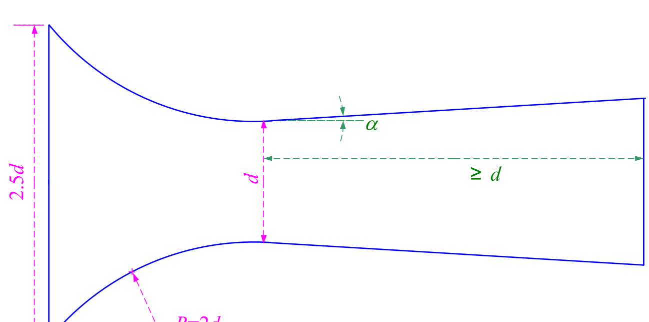

Fig. 1. ISO 9300 toroidal-throat Venturi nozzle.

than the ideal flow-rate. Discharge coefficient C_d is a dimensionless ratio of the actual flow-rate to the ideal flow-rate of non-viscous gas that would be obtained with one-dimensional isentropic flow for the same upstream stagnation conditions [7].

$$C_d = \frac{q_m}{q_{mi}} \tag{3}$$

The research of C_d is divided into three parts [19]: viscous region, namely viscous discharge coefficient C_{d1} affected by viscous effects of boundary layer; core flow region, namely inviscid discharge coefficient C_{d2} influenced by non-one-dimensional flow; virial discharge coefficient C_{d3} influenced by physical properties. The influence of C_{d3} is negligible when gas is ideal approximately [19]. Thus, discharge coefficient can be expressed by Eq. (4) [20,21]

$$C_d = 1-(1-C_{d1})-(1-C_{d2}) \tag{4}$$

To analyze the effect of roughness on discharge coefficient at different operating conditions, the important dimensionless parameters are determined by normalizing the governing gas dynamic equations [22,24]. First, solution vector [p, v_x, v_r, T] is normalized by [p^*, c^*, c^*, T^*]. The dimensionless variables can be described as follow.

$$geom^0 = \frac{geom}{d}, \quad v_x^0 = \frac{v_x}{c^*}, \quad v_r^0 = \frac{v_r}{c^*}, \quad p^0 = \frac{p}{p^*}$$
$$T^0 = \frac{T}{T^*}, \quad \Lambda^0 = \frac{\Lambda}{\Lambda^*}, \quad c_p^0 = \frac{c_p}{c_{pi}}, \quad \mu^0 = \frac{\mu}{\mu_0} \tag{5}$$

where, superscripts 0 and $*$ represent the dimensionless variables and critical condition variables, respectively. Parameters including axial coordinates x, radial coordinate r, the nozzle curvature R_c are integrated by parameter '$geom$'; v_x and v_r are axial and radial velocity; The critical sound speed is $c^*=(\gamma R_m T^*)^{0.5}$; Λ is the thermal conductivity. The specific heat of ideal gas is $c_{pi}=\gamma R_m/(\gamma-1)$. μ_0 is gas dynamic viscosity at the stagnation condition. The dimensionless density is $\rho^0=p^0/(ZT^0)$. Z is the compressibility factor. According to the governing equations in dimensionless form, the normalized solution vector $[p^0, v_x^0, v_r^0, T^0]$ can be described as follows.

$$[p^0, v_X^0, v_r^0, T^0] = f(gemo^0, Re_i, \gamma, \Lambda^0, Pr, Z, c_p^0, \beta T, T_w/T^*) \tag{6}$$

where, Pr is Prandtl number; Re_i is the Reynolds number along the nozzle; T_w/T^* is standard radial temperature distribution normalized by the temperature at the nozzle throat; βT is normalized coefficient of volumetric expansion. And, the function of discharge coefficient is given by the following dimensionless parameters

$$C_d = f(geom^0, Re_{nt}, \gamma, \Lambda^0, Pr, Z, c_p^0, \beta T, T_w/T^*) \tag{7}$$

where, Re_{nt} is the dimensionless parameter of throat Reynolds number.

The compressibility factor Z should be taken into consideration in the real critical mass flow-rate equation. However, under atmospheric or low pressure condition, the influence of compressibility factor Z is negligible [7,25]. On the other hand, in this paper, the research focuses on the influence of roughness on discharge coefficient of sonic nozzle under atmospheric conditions. In order to reduce the model number, the operating pressure is changed to obtain wide range of Reynolds number. According to the similarity principle, to ensure the similarity of the flow field and get the influence of roughness on discharge coefficient, the compressibility effect is not considered. In ISO 9300 and related literature, either theoretical or experimental equation, equations of discharge coefficient usually adopt the form used by Stratford, namely,

$$C_d = a - bRe_{nt}^{-n} \tag{8}$$

Major research achievements are shown in Table 1. Fig. 2 shows the corresponding discharge coefficient of the above equations