Flow Measurement and Instrumentation 35 (2014) 109–115

Contents lists available at ScienceDirect

Flow Measurement and Instrumentation

journal homepage: www.elsevier.com/locate/flowmeasinst

Discharge coefficient of rectangular sharp-crested side weirs, Part I: Traditional weir equation

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ARTICLE INFO

Available online 22 November 2013

Keywords:
Artificial neural network
Froude number
Discharge coefficient
Sensitivity analysis
Side weirs

ABSTRACT

A comprehensive study was performed to examine the flow characteristics over rectangular sharp-crested side weirs based on the traditional weir equation. To obtain a generally convenient discharge coefficient relationship, series of experiments were conducted according to manipulation of different prevailing parameters. The flow regime was consistently subcritical for upstream Froude numbers ranging from 0.08 to 0.91. Furthermore, experimental data sets of the former investigators were also applied. In order to identify the most important parameters affecting the discharge coefficient of rectangular sharp-crested side weirs, a sensitivity analysis was carried out based upon an artificial neural network modeling. Results of the sensitivity analysis indicated the Froude number to be the most influential parameter on discharge coefficient. Accordingly, a power equation is derived for estimating the discharge coefficient, which is applicable for both sub- and supercritical flow conditions simultaneously. Moreover, considering all the influential parameters, a nonlinear correlation was obtained with the highest precision to determine the discharge coefficient of sharp-crested rectangular side weirs.

1. Introduction

Weirs are normally installed either for controlling water levels in an upstream channel or for measuring flow discharge accurately. Rectangular sharp-crested weirs, which are of the most common type available, are set transversely across the width or laterally along the side walls of a channel. Flow discharge through sharp-crested weirs is a function of different dominant physical and geometrical quantities, as follows

$$Q = f_1(b, P, B, v, h, g, \mu, \sigma, \rho, S_0) \tag{1}$$

where Q is the weir flow discharge, b is the weir length, P is the weir height, B is the channel width, v is the mean velocity, h is the water depth relative to the weir crest, g is the gravitational acceleration, μ is the water viscosity, σ is the surface tension, ρ is the water density and S_0 is the channel slope. By applying the Buckingham Π theorem, the following relationship between the dimensionless parameters is achieved

$$\frac{Q}{(2/3)\sqrt{2g}bh^{1.5}} = f_2\left(F, R, W, \frac{h}{P}, \frac{h}{b}, \frac{b}{B}, S_0\right) \tag{2}$$

where F, R, and W are Froude, Reynolds, and Weber numbers. In the model experimentation for $h > 30$ mm, effects of surface tension on discharge were found to be small; hence, W was excluded from the analysis [1]. In addition, for a turbulent flow, the viscosity effect was observed to be small compared with the gravity effect [2]; therefore, R was also excluded from Eq. (2).

The dimensionless variable on the left-hand side of Eq. (2) is recognized as the discharge coefficient (C_d). Consequently, the traditional weir discharge formula in terms of the upstream total head measured relative to the weir crest elevation ($H_1 = h_1 + v_1^2/2g$) is defined as follows [3]

$$Q = \frac{2}{3} C_d b \sqrt{2g} H_1^{1.5} \tag{3}$$

Sharp-crested weirs are regularly used as frontal or lateral weirs without preferences (Fig. 1). Frontal weirs are installed transversely across the width of a channel, so that the flow approaches at right angles to the weir crest; consequently, the upstream water level increases and the upstream velocity head ($v_1^2/2g$) becomes trivial (unlike the case of side weirs). Accordingly, the spilling velocity increases and, as a consequence, a nearly two-dimensional flow arises over the crest of the frontal weir. This type of weir can readily be used as an accurate discharge-metering device in open channel flows. So far, ample investigations have been conducted to examine the hydraulic characteristics of flow over rectangular sharp-crested frontal weirs, resulting in the formulation of numerous relationships for estimating the discharge coefficient of this kind of weirs *e.g.* Rehbock [4], Rouse [5], Kindsvater and Carter [6], Swamee [7], Afzalimehr and Bagheri [8], and Aydin et al. [9]. Model

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http://dx.doi.org/10.1016/j.flowmeasinst.2013.11.005

Nomenclature

B	channel width
b	weir width
C_d	discharge coefficient
C_e	elementary discharge coefficient
C_M	De Marchi discharge coefficient
e	average percentage error
E	specific energy
F	Froude number
g	gravitational acceleration
h	measured head over the crest of weir
H	total head measured above the crest of weir
i	number of test
MSE	mean square error
N	total number of tests
P	weir height
Q	weir discharge
R^2	coefficient of determination
R	Reynolds number
S_0	channel slope
v	mean velocity in the main channel
W	Weber number
y	flow depth
ϕ	varied flow function
μ	fluid viscosity
ρ	fluid density
σ	surface tension
χ	variable

Subscripts

1	upstream section
2	downstream section

experimentation studies have indicated that the dimensionless parameter h/P in Eq. (2) is the most significant independent variable of the discharge coefficient. However, the parameter b/B, which can be used to deduce width contraction, affects the discharge coefficient of a contracted frontal weir significantly.

Side weirs are set laterally along the side walls of a channel, to divert a portion of flow from a main channel into a side channel when the water level in the main channel exceeds a specified limit. Side weirs are commonly used in irrigation and drainage canals, urban drainage systems, sewerage and wastewater networks, and storm relief systems, as well as for separation of sediment and reduction of bed load in rivers and channels. Flow over side weirs is a typical case of spatially varied flow, with the discharge decreasing along the main channel. Under a subcritical flow condition, flow depths along a side weir increase, as illustrated in Fig. 2. Characteristics of flow over side weirs have been the subject of different investigations, with a focus on the determination of the side weir discharge coefficient in most of them. According to Tults [10], the first study to conduct laboratory measurements on side weirs under a subcritical flow condition has been presented by Engels [11]. The first analogous paper to publish the effects of supercritical flow on the weirs has been authored by Coleman and Smith [12].

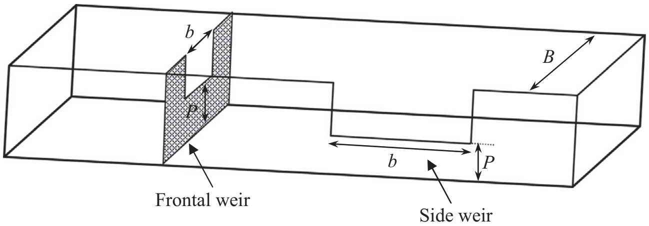

Fig. 1. Definition sketch of frontal and side weirs.

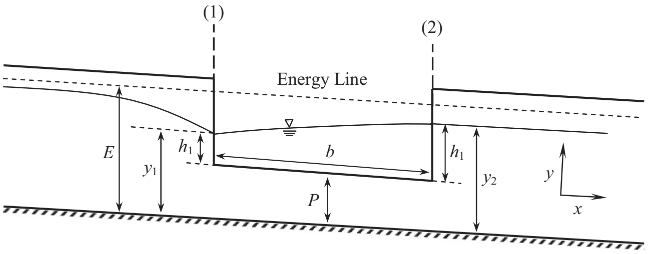

Fig. 2. Schema of the subcritical flow over a side weir.

De Marchi [13] proposed an analytical solution for the side weirs, which received considerable attention from different investigators. He formulated the following equation for the estimation of discharge coefficient of side weirs located in a rectangular channel, based on the assumption of constant specific energy (E)

$$C_M = \frac{3}{2}\frac{B}{b}(\phi_2 - \phi_1) \tag{4}$$

where C_M is De Marchi's discharge coefficient and ϕ is a function which is defined as

$$\phi = \frac{2E-3P}{E-P}\sqrt{\frac{E-y}{y-P}} - 3\sin^{-1}\sqrt{\frac{E-y}{y-P}} \tag{5}$$

where y is the flow depth inside the main channel and subscripts 1 or 2 refer to the up- and downstream sections with respect to the side weir, respectively. Subramanya and Awasthy [14] assumed De Marchi's equation to be applicable to determine the rate of the evacuating flow through the side weirs. Accordingly, they obtained the following expression for calculating the discharge coefficient of rectangular sharp-crested side weirs under a subcritical flow condition, with respect to the upstream Froude number (F_1)

$$C_M = 0.611\sqrt{1 - \frac{3F_1^2}{F_1^2+2}} \tag{6}$$

They stated that De Marchi's discharge coefficient (C_M) for a side weir with a finite height is the same as that of a side weir with zero height. Nadesamoorthy and Thamson [15] discussed the paper presented by Subramanya and Awasthy [14] and proposed a new expression for determining C_M under both sub- and supercritical flow conditions as follows

$$C_M = 0.611\sqrt{\frac{1}{1+3F_1^2/(2+F_1^2)}} \tag{7}$$

Ranga Raju et al. [16] declared that the effective width of a side weir should be 5 cm less than the exterior width of the weir, because of the creation of a separation zone at the upstream corner. They proposed the following equation for estimating the discharge coefficient of a sharp-crested side weir in a subcritical