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Enhanced Energy Harvesting from Nonlinear Oscillators via Chaos Control

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Abstract: Modern electronic devices require less energy on-board and could be powered by energy harvested from the environment. Broadband vibration energy harvesting is one of the widely explored options for powering such devices. One such harvester that exploits this technique is the nonlinear inverted elastic pendulum. This system has a double-well potential and can undergo large amplitude inter-well oscillations. Piezoelectric patches, pasted near the base of the beam, convert these mechanical oscillations into electrical energy. However, the large amplitude responses can deteriorate over time into low energy chaotic oscillations, thereby reducing the harvested power. Under such conditions, forcing the system to follow a high energy orbit would improve the energy harvested from the system. However, it is not wise to invest a large control force to stabilize the high energy orbits. This manuscript aims to exploit the chaotic nature of the system to stabilize the unstable periodic orbits. The chaotic behavior of the system allows even small perturbations to alter its response dramatically. Taking advantage of this property, a low power controller, based on the method of Ott, Grebogi, and Yorke (OGY), is used to stabilize the unstable high energy periodic orbits. The control strategy is implemented on the linearized system, about the operating point corresponding to the chosen orbit. LQR is used to determine the optimum control force. The devised strategy is numerically simulated, which favors the implementation of OGY control to improve their energy harvesting capabilities.

Keywords: Chaos control; OGY control; Unstable periodic orbits; LQR; Broadband energy harvesting; Vibrations.

1. INTRODUCTION

The realm of wireless devices is witnessing an exponential growth ever since its inception. But a common setback to the extent of services rendered by these devices is the limited lifetime of batteries powering them. Presence of a self-sustainable power source would thus help us to exploit such devices exhaustively. Due to the current technological revolution, devices continue to shrink and less energy is required on-board. This has invoked the interest of researchers to efficiently harvest even a small amount of electrical energy available in the environment. Ambient vibrations, temperature gradients and radiations are some of the sources from which electrical energy could be harvested. Among these, low frequency mechanical vibrations (1-100 Hz) are commonly available and could act as a long term source of power for miniature devices.

The classical design of a vibration energy harvester consists of a cantilever beam carrying a tip mass. This model exploits the linear resonance of the system and uses a transducer to convert mechanical vibrations to electrical energy (Inman and Erturk, 2011). Ambient vibrations are random in nature and under such excitations, the efficiency of resonant harvesters is drastically reduced (Ali et al., 2010). Hence, modern research focuses on broadband energy harvesting where the harvester gives a fairly good output over a broad range of frequencies.

Various broadband harvester designs have been proposed based on nonlinear structural systems in the past decade (Gammaitoni et al., 2009; Erturk et al., 2009; Friswell et al., 2012; Kumar et al., 2015). The key aspect of the nonlinear harvesters is the use of a double well potential, so that the device will have two stable equilibrium positions. Erturk et al. (2009); Stanton et al. (2010) and Masana and Daqaq (2011) highlighted the advantages of a double well potential for energy harvesting, particularly when inter-well dynamics are excited. The simplest equation of motion with a double well potential is the well-known Duffing oscillator. The dynamics is complex, sometimes with coexisting periodic solutions and sometimes exhibiting a chaotic response. The Duffing oscillator model has been used for many energy harvesting systems along with electromechanical coupling for the harvesting circuit.

Favorable bistable harvester behavior involves steady-state large amplitude inter-well oscillations (Friswell et al., 2012). Unfortunately, overlapping dynamic attractors can result in low energy chaotic oscillations for matching excitation criteria (Harne and Wang, 2013; Daqaq et al., 2014). Harvesting can be improved during the low energy chaotic motion by forcing the system to follow a high energy periodic orbit (Geiyer and Kauffman, 2015). However, it is also not wise to invest a large control force to stabilize the high energy orbits.

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Chaotic systems display extreme sensitivity to initial conditions, thus allowing small perturbations to alter their behavior dramatically. In a chaotic attractor, stretching and folding of the trajectories in time give rise to a theoretically infinite number of unstable periodic orbits existing within a bounded region of the state space. Ott et al. (1990) theorized that by using only small parameter perturbations it is possible to stabilize these unstable periodic orbits present in a chaotic attractor. This perturbation method of controlling chaos is since known as the OGY (Ott, Grebogi and Yorke) method.

In this manuscript, an attempt has been made to enhance the energy harvesting capabilities of inverted beam harvester proposed by Friswell et al. (2012) through OGY control. The control input for the system has been optimized based on LQR technique. Numerical simulations are performed to validate the efficacy of the proposed method.

The content of this manuscript is organized as follows. A concise mathematical background of OGY control is presented in Section 2. Section 3 provides a brief description of the inverted elastic pendulum energy harvester and its modeling. The discussions are then extended to design a LQR controller to stabilize the unstable periodic orbits in the harvester in Section 4. Section 5 deals with the numerical simulations carried out to validate the concept. Conclusions are drawn in Section 6.

2. CHAOS CONTROL: MATHEMATICAL BACKGROUND

The prime motive behind application of control to a system is to regulate its behavior as desired. The presence of chaos in physical systems has been extensively demonstrated. However, in practical applications, a system is expected to behave in an ordered fashion. Hence, chaos is strictly avoided from an application standpoint. On the other hand, flexibility is an important property inherently present in chaotic systems that makes us appreciate chaotic motion. A chaotic trajectory is extremely sensitive to the effect of perturbations. Hence, a small perturbation applied at the right time is sufficient to change the system's trajectory and guide it along the desired path. Thus, flexibility of a chaotic system rules out the need for the application of a large control force to drive it to the desired operating point in a brute force manner. Instead, it makes possible to let the system fluctuate and eventually change its dynamics as little as possible to reach the desired state (Macau et al., 2008).

The concept of chaos control came about when Ott et al. (1990) theorized that it is possible to stabilize unstable periodic orbits present in a chaotic system through small, tailored, time-specific parameter perturbations. Also, the flexibility of the chaotic system makes it possible to stabilize different orbits, depending on the situation, in real-time. In the context of energy harvesting, this is very useful in real-time stabilization of a large amplitude orbit that is accessible from a wide range of excitation frequencies, thus enhancing the broadband capabilities of the harvester.

The first step in chaos control is the visualization of the chaotic attractor and the underlying unstable periodic orbits. A Poincare section is helpful in visualizing a chaotic system. Poincare sections (for example, refer Figs. (3&4)) are created from periodic sampling of the system response, with the sampling frequency being equal to the excitation frequency. Hence, each successive point in the Poincare section represents the state that is one period later in time.

Consider a dynamical system as described by (1):

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}) \tag{1}$$

where $\boldsymbol{X} \in \Re^n$ is the state vector.

A Poincare map relates the state of a dynamical system at any instant to another instant that is one period later in time. Hence, mathematically, a Poincare section can be described as (Strogatz, 2001):

$$\boldsymbol{X_{k+1}} = \boldsymbol{F}(\boldsymbol{X_k}) \tag{2}$$

where k and $k+1$ denote time instances one period apart.

When the response of a conservative system is periodic, a point $\boldsymbol{P^*}$ in the Poincare map maps onto itself the next period. Hence, essentially the Poincare section in this case is a single point. But, during chaotic motion, the motion is non-repetitive. Hence, the existence of a dense set of points in the Poincare section indicates the presence of a chaotic behavior. These dense set of points represent the strange attractor (or) the chaotic attractor of the system. This strange attractor is the bounded structure on which the unstable periodic orbits lie. Hence, isolation of unstable periodic orbits may be done by inspection of the underlying data used to plot the Poincare section.

Let $\boldsymbol{X^*}$ ($\in \Re^n$) be the desired operating point in the Poincare map that corresponds to the chosen orbit. The effectiveness of OGY technique lies in choosing an orbit that has high energy. Let u ($\in \Re^1$) be the control force required to control the dynamical system about the desired operating point $\boldsymbol{X^*}$. The state equation and Poincare section are given by:

$$\dot{\boldsymbol{X}} = f(\boldsymbol{X}, u) \tag{3}$$

$$\boldsymbol{X_{k+1}} = \boldsymbol{F}(\boldsymbol{X_k}, u_k) \tag{4}$$

Typically in a chaotic system, in the absence of a control force, the point $\boldsymbol{X^*}$ maps onto some other point the next period. But the purpose of control force is to make the point $\boldsymbol{X^*}$ map onto itself during the next period, thus stabilizing the periodic orbit that passes through $\boldsymbol{X^*}$. Since the point $\boldsymbol{X^*}$ maps onto itself, it acts as a stable fixed point of the controlled system. Hence,

$$\boldsymbol{X^*} = \boldsymbol{F}(\boldsymbol{X^*}) \tag{5}$$

Let $\boldsymbol{X_k}$ be the state of the system at some instant. Then, the difference between the current state and the fixed point is given as,

$$\boldsymbol{e_k} = \boldsymbol{X_k} - \boldsymbol{X^*} \tag{6}$$

One period later, the difference can be given as,

$$\boldsymbol{e_{k+1}} = \boldsymbol{X_{k+1}} - \boldsymbol{X^*} \tag{7}$$

Substituting (4) and (5) in (7), we get,

$$\boldsymbol{e_{k+1}} = \boldsymbol{F}(\boldsymbol{X_k}, u_k) - \boldsymbol{F}(\boldsymbol{X^*}) \tag{8}$$

$$\boldsymbol{e_{k+1}} = \boldsymbol{F}(\boldsymbol{X^*} + \boldsymbol{X_k} - \boldsymbol{X^*}, u_k) - \boldsymbol{F}(\boldsymbol{X^*}) \tag{9}$$

On expanding up to first order approximation using Taylor Series, we get,

$$\boldsymbol{e_{k+1}} = \left.\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{X}}\right|_{\boldsymbol{X}=\boldsymbol{X^*}} (\boldsymbol{X_k} - \boldsymbol{X^*}) + \left.\frac{\partial \boldsymbol{F}}{\partial u}\right|_{\boldsymbol{X}=\boldsymbol{X^*}} (u_k - 0) \tag{10}$$