

FEBRUARY 1-5, 2016. Indianal Conference online at www.sciencedirect.com

IFAC-PapersOnLine 49-1 (2016) 041–046

IFAC-1 apersoneline 49-1 (2010) 041-040 tal Input to State S Incremental Input to State Stability of Incremental Input to State Stability of Underwater Vehicle Underwater Vehicle Underwater Vehicle Incremental Input to State Stability of

Majeed Mohamed ∗ Swaroop S ∗∗ Majeed Mohamed ∗ Swaroop S ∗∗

∗ Senior Scientist, Flight Mechanics and Control Division, ∗ Senior Scientist, Flight Mechanics and Control Division, CSIR-National Aerospace Laboratories, Bengaluru-560017, India.

(e-mail: majeed_md_123@rediffmail.com,majeed@nal.res.in).

* Besearch Scholar, Department of Electrical Engineering, UVCL e-mail: majeed_md_123@rediffmail.com,majeed@nal.res.in). ^{e-man}. majeed_mall23@reatyman.com, majeed@nalles.inf.
** Research Scholar, Department of Electrical Engineering, UVCE, Bengaluru- 560001, India. (e-mail: swaroop.s.1992@ieee.org). ∗∗ Research Scholar, Department of Electrical Engineering, UVCE, Research Beholdt, Department by Electrical Engineering, 0 VOD,
Bengaluru- 560001, India. (e-mail: swaroop.s.1992@ieee.org). Bengaluru- 560001, India. (e-mail: swaroop.s.1992@ieee.org).

Abstract: This paper discusses the incremental stability of underwater vehicle based on the **ADStract:** This paper discusses the incremental stability of underwater venicle based on the
newly introduced contraction based input to state stability analysis. Stability analysis considered in vehicle dynamics has ability to constructing the controller and contraction metrics. The in venicle dynamics has ability to constructing the controller and contraction metrics. The
controller design is restricted to parametric-strict-feedback form to develop a back stepping controller design is restricted to parametric-strict-reedback form to develop a back stepping
design method. The proposed approach in this paper provides a recursive way of constructing design method. The proposed approach in this paper provides a recursive way of constructing
a controller and it enforce incremental input to state stability of a vehicle and not just global asymptotic stability. newly introduced contraction based input to state stability analysis. Stability analysis considered
in orbital, demonstrates and illustrate contraction the controller and contraction metrics. The a controller and it enforce incremental input to state stability of a vehicle and not just global
construction of constructions controller design is restricted to parametric-strict-feedback form to develop a back stepping design method. The proposed approach in this paper provides a recursive way of constructing

© 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved. c 2016 IFAC (Interna

Keywords: incremental stability, regulatory control, exponential stability, underwater vehicle. Keywords: incremental stability, regulatory control, exponential stability, underwater vehicle. Keywords: incremental stability, regulatory control, exponential stability, underwater vehicle. Keywords: incremental stability, regulatory control, exponential stability, underwater vehicle.

1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

In the past several years, there has been considerable in the past several years, there has been considerable
interest in recursive designs for nonlinear control schemes. interest in recursive designs for nonlinear control schemes.
Most of these approaches are traditionally based on the contrary, contraction theory is a recent tool for analyzing construction of appropriate Lyapunov function. On the construction of appropriate Lyapunov function. On the Most of these approaches are traditionally based on the construction of appropriate Lyapunov function. On the
contrary, contraction theory is a recent tool for analyzing contrary, contraction theory is a recent tool for analyzing
the convergence behavior of nonlinear systems (Lohmiller the convergence behavior of hominear systems (Londminer and Slotine (1998), Lohmiller and Slotine (2000), Majeed and Sound $(1335),$ communer and Sound $(2000),$ Majed and Kar (2013)). It is treated as incremental form of stability since contraction analysis provides framework enabling to study the stability of nonlinear system trajectories with to study the stability of nominear system trajectories with
respect to each other (Majeed and Kar (2012)). For nonrespect to each other (majeed and Kar (2012)). For hon-
linear systems, incremental stability is a stronger property than global exponential convergence to a single trajectory. than global exponential convergence to a single trajectory. construction or appropriate Lyapunov function. On the and Slotine (1998),Lohmiller and Slotine (2000),Majeed and Slotine (1998),Lohmiller and Slotine (2000),Majeed and Kar (2013)). It is treated as incremental form of stability in the stability of stability of the stability of stability of the stabil linear systems, incremental stability is a stronger property than global exponential convergence to a single trajectory. linear systems, incremental stability is a stronger property
than global exponential convergence to a single trajectory.
This paper addresses the problem of the use of an incre-In the past several velocity components of *v* and γ and γ and γ and γ and Fuler mights (α and Fuler mights (α and β of α and γ of α by $(\rho \ge 0)$ is a vector of antangular velocity contraction depends o

This paper addresses the problem of the use of an increback-stepping design of stabilization to the underwater mental approach, i.e contraction theory to the integrator mental approach, i.e contraction theory to the integrator mental approach, i.e contraction theory to the integrator back-stepping design of stabilization to the underwater application of the newly introduced contraction based vehicleMajeed and Kar (2015). The present work is the vehicleMajeed and Kar (2015). The present work is the back-stepping design of stabilization to the underwater vented and Tabuary introduced contraction based application of the newly introduced contraction based (2011)). In the literature, the property of Input to state input to state stability analysis (Zamani and Tabuada input to state stability analysis (Zamani and Tabuada (2011)). In the literature, the property of Input to state (2011)). (2011) . In the herature, the property of liput to state
stability (ISS) has proven a valid instrument in order to s stability (155) has proved a valid instrument in order to study questions of robust stability for finite-dimensional nonlinear systems (Angeli (2002)). nonlinear systems (Angeli (2002)). mental approach, i.e contraction theory to the integrator back-stepping design of stabilization to the underwatervehicleMajeed and Kar (2015). The present work is the application of the newly introduced contraction based application of the newly introduced contraction based input to state stability analysis (Zamani and Tabuada) (2011)). In the literature, the property of Input to state stability (ISS) has proven a valid instrument in order to stability (ISS) has proven a valid instrument in order to study questions of robust stability for finite-dimensional nonlinear systems (Angeli (2002)).

Consider the six degrees of freedom (DOF) nonlinear un- $\frac{d}{dx}$ derwater vehicle equations of motion in abbreviated form $(v394)$. (Fossen (1994)). (Fossen (1994)). Consider the six degrees of freedom (DOF) nonlinear underwater vehicle equations of motion in abbreviated form (Fossen (1994)).

$$
M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = B(\nu)u \tag{1}
$$

$$
\dot{\eta} = J(\eta)\nu\tag{2}
$$

$$
T\dot{u} + u = u_c \tag{3}
$$

where (1) is the velocity dynamics, (2) is the kinematics where (1) is the velocity dynamics, (2) is the kinematics
and (3) is the actuator dynamics of an underwater vehicle. $\nu = (u, v, w, p, q, r)^T$ is a vector of body fixed linear and and $\overline{3}$ is the actuator dynamics of an underwater vehicle. $\nu = (u, v, w, p, q, r)^{2}$ is a vector of body fixed linear and where (1) is the velocity dynamics, (2) is the kinematics

 τ angular velocity components and $\eta = (x, y, z, \phi, \theta, \psi)^T$ is a angular velocity components and $\eta = (x, y, z, \phi, \theta, \psi)$ is a
vector of positions (x, y, z) and Euler angles (ϕ, θ, ψ) . The vector of positions (x, y, z) and Euler angles $(\varphi, \theta, \varphi)$. The
components of ν and η corresponds to the 6 DOF motion components of ν and η corresponds to the σ DOT motion
variables in *surge*, *sway*, *heave*, *roll*, *pitch*, and *yaw*. $u \in \mathbb{R}^n$ variables in *surge*, *sway*, *heave*, *rott*, *pitch*, and *yaw*. $u \in \mathbb{R}^p$
 \mathbb{R}^p ($p \ge 6$) is a vector of actual control inputs, and $u_c \in \mathbb{R}^p$ \mathbb{R}^{μ} ($p \ge 0$) is a vector of actual control inputs, and $u_c \in \mathbb{R}^{\mu}$
is a vector of commanded actuator inputs. Furthermore, is a vector of commanded actuator inputs. Furthermore,
 $g(\eta)$ is an unknown vector of restoring forces and moments $g(\eta)$ is an unknown vector of restoring forces and moments
while $B(\nu)$ is a known $6 \times p$ control matrix. $J(\eta)$ is a 6×6
known block diagonal transformation matrix relating to
the body reference frame to the inertia wille $D(\nu)$ is a known $0 \times p$ control matrix. $J(\eta)$ is a 0×0
known block diagonal transformation matrix relating to khown block diagonal transformation matrix relating to the body reference frame to the inertial reference frame. M is inertial matrix (including hydrodynamic inertia), $C(\nu)$ is the centripetal forces, and $D(\nu)$ is hydrodynamic damping
the centripetal forces, and $D(\nu)$ is hydrodynamic damping matrix. $T = \text{diag}([t_i])$ is a $p \times p$ diagonal matrix of positive matrix. $I = \text{diag}(\lvert \ell_i \rvert)$ is a $p \times p$ diagonal matrix of positive unknown actuator time constants $(t_i > 0)$. $\frac{1}{m}$ is a components of ν and η corresponds to the 6 DOF motion the body reference frame to the inertial reference frame. M is inertial matrix (including hydrodynamic inertia), $C(\nu)$ is the centripetal forces, and $D(\nu)$ is hydrodynamic damping unknown actuator time constants $(t_i > 0)$. Assume that control and a second term of the control and contribute of contribute of the contribute of the control and contribute of the control and contribute of the control and control and control and control and contro

From the above mentioned equation of motion of under-From the above mentioned equation of motion of under-
water vehicles, Healey and Macro proposed to describe the vehicle speed equation as follows Healey and Marco (1992) vehicle speed equation as follows Healey and Marco (1992) water vehicles, Healey and Macro proposed to describe the
water vehicles, Healey and Macro proposed to describe the vehicle speed equation as follows Healey and Marco (1992)

$$
(m_1 - X_u) \dot{u}_1 = X_{u|u|} u_1 |u_1| + X_{n|n|} n |n| \tag{4}
$$

where $u_1 = \lambda_u u_1 = \lambda_u u_1 u_1 + \lambda_{n|n|} u_1 u_1$ (4)
where u_1 is the surge velocity and n is the propeller revolution. This system can be rewritten according to
revolution. This system can be rewritten according to where u_1 is the surge velocity and n is the propeller revolution. This system can be rewritten according to

$$
m\dot{\nu} + d(\nu)\nu = u, d(\nu) = d_o |\nu|
$$
 (5)

$$
T\dot{u} + u = u_c \tag{6}
$$

 $I u + u = u_c$ (0 where $m = (m_1 - X_u)/X_{n|n|}, d_o = -X_{u|u|}/X_{n|n|}, u=n|n|$ where $m = (m_1 - \Lambda_u)/\Lambda_n|n|, a_0 = -\Lambda_u|u|/\Lambda_n|n|, a=n|n|$
and $\nu = u_1$. The last equation is included to describe the and $\nu = u_1$. The last equation is included to describe the actuator dynamics. For simplicity of algebraic manipulaactuator dynamics. For simplicity or algebraic mamputation, here it is considered the speed stabilization of the tion, here it is considered the speed stabilization of the
underwater vehicle to describe the incremental input-tounder water venicle to describe the incremental input-to-
state behavior of the vehicle. Interested authors can define state behavior of the venicle. Interested authors can define
the highly unstable situation of the vehicle at which the same proposed approach is able to stabilize the vehicle. same proposed approach is able to stabilize the vehicle. $T\dot{u} + u = u_c$ (6)
where $m = (m_1 - X_u)/X_{n|n|}, d_o = -X_{u|u|}/X_{n|n|}, u=n |n|$ actuator dynamics. For simplicity of algebraic manipula-
tional have it is separated the grand stabilization of the the highly unstable situation of the vehicle at which the same proposed approach is able to stabilize the vehicle.

The paper is outlined as follows: Section 2 of the paper 3 describes the design of incremental speed stabilization discusses control systems and stability notions. Section discusses control systems and stability notions. Section 3 describes the design of incremental speed stabilization 3 describes the design of incremental speed stabilization The paper is outlined as follows: Section 2 of the paper discusses control systems and stability notions. Section 3 describes the design of incremental speed stabilization

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved. Peer review under responsibility of International Federation of Automatic Control. 10.1016/j.ifacol.2016.03.026

for the underwater vehicle. The numerical simulation is presented in section 4 to confirm the incremental stability of the vehicle. Finally the conclusions are given in section 5.

2. CONTROL SYSTEMS AND STABILITY NOTIONS

2.1 Notation

The symbol \mathbb{R}, \mathbb{R}^+ and \mathbb{R}_0^+ denote the set of real, positive, and nonnegative real numbers, respectively. The symbols I_m , and 0_m denote the identity and zero matrices on \mathbb{R}^m . Given a vector $x \in \mathbb{R}^n$, we denote by x_i and the i^{th} element of x,and by $\parallel x \parallel$ the Euclidean norm of x; we recall that $|| x || = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ given a measurable function $f: \mathbb{R}^+ \to \mathbb{R}^n$, the (essential) supermom of f is denoted by $||f||_{\infty}$, we recall that $||f||_{\infty} := (\text{ess})\text{sup} \{||f||, t \ge 0\}$; f is essentially bounded if $||f||_{\infty} < \infty$ for given time $\tau \in \mathbb{R}^+$. \mathbb{R}^+ , define f_τ so that $f_\tau(t) = f(t)$ for any $t \in [0, \tau)$, and $f(t) = 0$ elsewhere: f is said to be locally essentially bounded if for any $\tau \in \mathbb{R}^+, f_\tau$ is essentially bounded. A continuous function $\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said belong to class κ if it is strictly increasing and $\gamma(0) = 0; \gamma$ is said to belong to class κ_{∞} if $\gamma \in \kappa$ and $\gamma(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said belong to class ? if, for each fixed s, the map $\beta(r, s)$ belongs to class κ_{∞} with respect to r and, for each fixed r, the map $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

2.2 Control Systems

The class of control systems that we consider in this paper is formalized in the following definition.

Definition 2.1 : A control system is a quadruple:

$$
\Sigma = (\mathbb{R}^n, U, \nu, f),
$$

where

- \mathbb{R}^n is the state space;
- $U \subseteq \mathbb{R}^m$ is the input space;
- v is a subset of the set of all locally essentially bounded functions of time from intervals of the form $[a, b] \subseteq \mathbb{R}$ to U with $a < 0, b > 0$;
- $f : \mathbb{R}^n \times U \to \mathbb{R}^n$ is a continuous map satisfying the following Lipschitz assumption: for every compact set $Q \subset \mathbb{R}^n$, there exists a constant $Z \in \mathbb{R}^+$ such that $|| f(x, u) - f(y, u) || \leq Z ||x - y||, \forall x, y \in Q$ and all $u \in U$

A curve $\xi : a, b \mapsto \mathbb{R}^n$ is said to be a trajectory of Σ if there exists $u \in v$ satisfying:

$$
\dot{\xi}(t) = f(\xi(t), u(t))\tag{7}
$$

for almost all $t \in]a, b[$. We also write $\xi_{xu}(t)$ to denote the point reached at time t under the input u from initial condition $x = \xi_{xu}(0)$; this point is uniquely determined, since the assumptions on f ensure existence and uniqueness of trajectories Sontag (1998). We also denote an autonomous system Σ with no control inputs by $\Sigma = (\mathbb{R}^n, f)$. A control system Σ is said to be forward complete if every trajectory is defined on an interval of the form $[a,\infty]$. Sufficient and necessary conditions for a system to be forward complete can be found in Angeli and Sontag (1999). A control system Σ is said to be smooth if f is an infinitely differentiable function of its arguments.

2.3 Stability Notations

Here, we recall the notions of incremental global asymptotic stability $(δ–GAS)$ and incremental input-to-state stability $(δ–ISS)$.

Definition-1 (Angeli (2002)): A control system Σ is incrementally globally asymptotically stable $(δ–GAS)$ if it is forward complete and there exists a function β such that for any $t \in \mathbb{R}_0^+$, any $x, x' \in \mathbb{R}^n$ and any $u \in v$ the following condition is satisfied.

$$
\|\xi_{xu}(t) - \xi_{x'u'}(t)\| \le \beta \left(\|x - x'\| \right), \tag{8}
$$

Whenever the origin is an equilibrium point for Σ , δ -GAS implies global asymptotic stability (GAS).

Definition-2 (Angeli (2002)): A control system Σ is incrementally input-to-state stable $(δ–ISS)$ if it is forward complete and there exist a function β and a κ_{∞} function γ such that for any $t \in \mathbb{R}_0^+$, any] $x, x' \in \mathbb{R}^n$, andanyu, $u' \in v$, following condition is satisfied.

 $\|\xi_{xu}(t) - \xi_{x'u'}(t)\| \le \beta (\|x - x'\|, t) + \gamma (\|u - u'\|\infty)$ (9) By observing (8) and (9), it is readily seen that δ -ISS implies δ -GAS while the converse is not true in general. Moreover, if the origin is an equilibrium point for Σ , δ -ISS implies input-to-state stability (ISS).

2.4 Descriptions of Incremental Stability

One of the methods for checking δ –GAS and δ –ISS properties consists in using Lyapunov functions. The Lyapunov characterizations of δ -GAS and δ -ISS properties were developed in Angeli (2002). In this paper we follow an alternative approach based on contraction metrics. The notion of contraction metric was popularized in control theory by the work of Slotine Lohmiller and Slotine (1998). Before going through the next definition, we need to introduce variational systems and the notion of a Riemannian metric.

The variational system associated with a smooth autonomous system $\Sigma = (\mathbb{R}^n, f)$ is given by the differential equation

$$
\frac{d}{dt}(\delta \xi) = \frac{\partial f}{\partial x}\bigg|_{x=\xi} \delta \xi,\tag{10}
$$

where $\delta \xi$ is the variation¹ of a trajectory of Σ . Similarly, the variational system associated with a smooth control system $\Sigma = (\mathbb{R}^n, U, v, f)$, is given by the differential equation

$$
\frac{d}{dt}\left(\delta\xi\right) = \frac{\partial f}{\partial x}\bigg|_{\substack{x=\xi\\u=u}} \delta\xi + \frac{\partial f}{\partial u}\bigg|_{\substack{x=\xi\\u=u}} \delta u \tag{11}
$$

where $\delta \xi$ and δu are variations of a state and an input trajectory of Σ, respectively.

A Riemannian metric $G: \mathbb{R}^n \to \mathbb{R}^{n \times n}$ is a smooth map on \mathbb{R}^n such that, for any $x \in \mathbb{R}^n$, $G(x)$ is a symmetric positive definite matrix Lee (2003). For any $x \in \mathbb{R}^n$ and smooth functions $I, J : \mathbb{R}^n \to \mathbb{R}^n$, one can define the scalar function $\langle I,J\rangle_{G}$ as $I^{T}(x)G(x)J(x)$. We will still use

¹ The variation $\delta \xi$ can be formally defined by considering a family of trajectories $\xi_{xu}(t,\varepsilon)$ parameterized by $\varepsilon \in \mathbb{R}$. The variation of the state is then $\delta \xi = \frac{\partial \xi_{xu}}{\partial \varepsilon}$

Download English Version:

<https://daneshyari.com/en/article/708861>

Download Persian Version:

<https://daneshyari.com/article/708861>

[Daneshyari.com](https://daneshyari.com)