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IFAC-PapersOnLine 49-1 (2016) 041–046

Incremental Input to State Stability of Underwater Vehicle

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Abstract: This paper discusses the incremental stability of underwater vehicle based on the newly introduced contraction based input to state stability analysis. Stability analysis considered in vehicle dynamics has ability to constructing the controller and contraction metrics. The controller design is restricted to parametric-strict-feedback form to develop a back stepping design method. The proposed approach in this paper provides a recursive way of constructing a controller and it enforce incremental input to state stability of a vehicle and not just global asymptotic stability.

Keywords: incremental stability, regulatory control, exponential stability, underwater vehicle.

1. INTRODUCTION

In the past several years, there has been considerable interest in recursive designs for nonlinear control schemes. Most of these approaches are traditionally based on the construction of appropriate Lyapunov function. On the contrary, contraction theory is a recent tool for analyzing the convergence behavior of nonlinear systems (Lohmiller and Slotine (1998),Lohmiller and Slotine (2000),Majeed and Kar (2013)). It is treated as incremental form of stability since contraction analysis provides framework enabling to study the stability of nonlinear system trajectories with respect to each other(Majeed and Kar (2012)). For nonlinear systems, incremental stability is a stronger property than global exponential convergence to a single trajectory.

This paper addresses the problem of the use of an incremental approach, i.e contraction theory to the integrator back-stepping design of stabilization to the underwater vehicleMajeed and Kar (2015). The present work is the application of the newly introduced contraction based input to state stability analysis (Zamani and Tabuada (2011)). In the literature, the property of Input to state stability (ISS) has proven a valid instrument in order to study questions of robust stability for finite-dimensional nonlinear systems (Angeli (2002)).

Consider the six degrees of freedom (DOF) nonlinear underwater vehicle equations of motion in abbreviated form (Fossen (1994)).

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = B(\nu)u \tag{1}$$

$$\dot{\eta} = J(\eta)\nu \tag{2}$$

$$T\dot{u} + u = u_c \tag{3}$$

where (1) is the velocity dynamics, (2) is the kinematics and (3) is the actuator dynamics of an underwater vehicle. $\nu = (u, v, w, p, q, r)^T$ is a vector of body fixed linear and angular velocity components and $\eta = (x, y, z, \phi, \theta, \psi)^T$ is a vector of positions (x, y, z) and Euler angles (ϕ, θ, ψ). The components of ν and η corresponds to the 6 DOF motion variables in *surge, sway, heave, roll, pitch,* and *yaw*. $\mathrm{u} \in \mathbb{R}^p$ $(p \geq 6)$ is a vector of actual control inputs, and $\mathrm{u_c} \in \mathbb{R}^p$ is a vector of commanded actuator inputs. Furthermore, $g(\eta)$ is an unknown vector of restoring forces and moments while $B(\nu)$ is a known $6 \times p$ control matrix. $J(\eta)$ is a 6×6 known block diagonal transformation matrix relating to the body reference frame to the inertial reference frame. M is inertial matrix (including hydrodynamic inertia), $C(\nu)$ is the centripetal forces, and $D(\nu)$ is hydrodynamic damping matrix. $T = \mathrm{diag}([t_i])$ is a $p \times p$ diagonal matrix of positive unknown actuator time constants $(t_i > 0)$.

From the above mentioned equation of motion of underwater vehicles, Healey and Macro proposed to describe the vehicle speed equation as follows Healey and Marco (1992)

$$(m_1 - X_{\dot{u}})\dot{u}_1 = X_{u|u|}u_1\,|u_1| + X_{n|n|}n\,|n| \tag{4}$$

where u_1 is the surge velocity and n is the propeller revolution. This system can be rewritten according to

$$m\dot{\nu} + d(\nu)\nu = u, d(\nu) = d_o\,|\nu| \tag{5}$$

$$T\dot{u} + u = u_c \tag{6}$$

where $m = (m_1 - X_{\dot{u}})/X_{n|n|}$,$d_o = -X_{u|u|}/X_{n|n|}$,$u$=$n\,|n|$ and $\nu = u_1$. The last equation is included to describe the actuator dynamics. For simplicity of algebraic manipulation, here it is considered the speed stabilization of the underwater vehicle to describe the incremental input-to-state behavior of the vehicle. Interested authors can define the highly unstable situation of the vehicle at which the same proposed approach is able to stabilize the vehicle.

The paper is outlined as follows: Section 2 of the paper discusses control systems and stability notions. Section 3 describes the design of incremental speed stabilization

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.03.026

for the underwater vehicle. The numerical simulation is presented in section 4 to confirm the incremental stability of the vehicle. Finally the conclusions are given in section 5.

2. CONTROL SYSTEMS AND STABILITY NOTIONS

2.1 Notation

The symbol $\mathbb{R}, \mathbb{R}^+$ and $\mathbb{R}_0^+$ denote the set of real, positive, and nonnegative real numbers, respectively. The symbols I_m, and 0_m denote the identity and zero matrices on $\mathbb{R}^m$. Given a vector $x \in \mathbb{R}^n$, we denote by x_i and the i^{th} element of x,and by $\| x \|$ the Euclidean norm of x;we recall that $\| x \| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$. given a measurable function $f : \mathbb{R}_0^+ \to \mathbb{R}^n$, the (essential) supermom of f is denoted by $\|f\|_\infty$. we recall that $\|f\|_\infty := \text{(ess)sup}\left\{\|f\|, t \geq 0\right\}$; f is essentially bounded if $\|f\|_\infty < \infty$.for given time $\tau \in \mathbb{R}^+$,define f_τ so that $f_\tau(t) = f(t)$ for any $t \in [0, \tau)$,and $f(t) = 0$ elsewhere: f is said to be locally essentially bounded if for any $\tau \in \mathbb{R}^+, f_\tau$ is essentially bounded. A continuous function $\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said belong to class κ if it is strictly increasing and $\gamma(0) = 0; \gamma$ is said to belong to class κ_∞ if $\gamma \in \kappa$ and $\gamma(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is said belong to class ? if, for each fixed s, the map $\beta(r, s)$ belongs to class κ_∞ with respect to r and, for each fixed r, the map $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as s $\to \infty$.

2.2 Control Systems

The class of control systems that we consider in this paper is formalized in the following definition.
Definition 2.1: A control system is a quadruple:

$$\Sigma = \left(\mathbb{R}^n, U, \upsilon, f\right),$$

where

- $\mathbb{R}^n$ is the state space;
- $U \subseteq \mathbb{R}^m$ is the input space;
- υ is a subset of the set of all locally essentially bounded functions of time from intervals of the form $]a, b[\subseteq \mathbb{R}$ to U with $a < 0, b > 0$;
- $f : \mathbb{R}^n \times U \to \mathbb{R}^n$ is a continuous map satisfying the following Lipschitz assumption: for every compact set $Q \subset \mathbb{R}^n$, there exists a constant $Z \in \mathbb{R}^+$ such that $\|f(x, u) - f(y, u)\| \leq Z\|x - y\|$, $\forall x, y \in Q$ and all $u \in U$

A curve $\xi :]a, b[\to \mathbb{R}^n$ is said to be a trajectory of Σ if there exists $u \in \upsilon$ satisfying:

$$\dot{\xi}(t) = f(\xi(t), u(t)) \tag{7}$$

for almost all $t \in]a, b[$. We also write $\xi_{xu}(t)$ to denote the point reached at time t under the input u from initial condition $x = \xi_{xu}(0)$; this point is uniquely determined, since the assumptions on f ensure existence and uniqueness of trajectories Sontag (1998). We also denote an autonomous system Σ with no control inputs by $\Sigma = (\mathbb{R}^n, f)$. A control system Σ is said to be forward complete if every trajectory is defined on an interval of the form $]a, \infty[$. Sufficient and necessary conditions for a system to be forward complete can be found in Angeli and Sontag (1999). A control system Σ is said to be smooth if f is an infinitely differentiable function of its arguments.

2.3 Stability Notations

Here, we recall the notions of incremental global asymptotic stability ($\delta-$GAS) and incremental input-to-state stability ($\delta-$ISS).

Definition-1 (Angeli (2002)): A control system Σ is incrementally globally asymptotically stable ($\delta-$GAS) if it is forward complete and there exists a function β such that for any $t \in \mathbb{R}_0^+$,any $x, x' \in \mathbb{R}^n$ and any $u \in \upsilon$ the following condition is satisfied.

$$\|\xi_{xu}(t) - \xi_{x'u'}(t)\| \leq \beta\left(\|x - x'\|, t\right) \tag{8}$$

Whenever the origin is an equilibrium point for $\Sigma, \delta-$GAS implies global asymptotic stability (GAS).
Definition-2 (Angeli (2002)): A control system Σ is incrementally input-to-state stable ($\delta-$ISS) if it is forward complete and there exist a function β and a κ_∞ function γ such that for any $t \in \mathbb{R}_0^+$, any] $x, x' \in \mathbb{R}^n$, and any $u, u' \in \upsilon$, following condition is satisfied.

$$\|\xi_{xu}(t) - \xi_{x'u'}(t)\| \leq \beta\left(\|x - x'\|, t\right) + \gamma\left(\|u - u'\| \infty\right) \tag{9}$$

By observing (8) and (9), it is readily seen that δ-ISS implies δ-GAS while the converse is not true in general. Moreover, if the origin is an equilibrium point for Σ, δ-ISS implies input-to-state stability (ISS).

2.4 Descriptions of Incremental Stability

One of the methods for checking $\delta-$GAS and $\delta-$ISS properties consists in using Lyapunov functions. The Lyapunov characterizations of $\delta-$GAS and $\delta-$ISS properties were developed in Angeli (2002). In this paper we follow an alternative approach based on contraction metrics. The notion of contraction metric was popularized in control theory by the work of Slotine Lohmiller and Slotine (1998). Before going through the next definition, we need to introduce variational systems and the notion of a Riemannian metric.

The variational system associated with a smooth autonomous system $\Sigma = (\mathbb{R}^n, f)$ is given by the differential equation

$$\frac{d}{dt}(\delta\xi) = \left.\frac{\partial f}{\partial x}\right|_{x=\xi} \delta\xi, \tag{10}$$

where $\delta\xi$ is the variation [1] of a trajectory of Σ. Similarly, the variational system associated with a smooth control system $\Sigma = (\mathbb{R}^n, U, \upsilon, f)$, is given by the differential equation

$$\frac{d}{dt}(\delta\xi) = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=\xi \\ u=u}} \delta\xi + \left.\frac{\partial f}{\partial u}\right|_{\substack{x=\xi \\ u=u}} \delta u \tag{11}$$

where $\delta\xi$ and δu are variations of a state and an input trajectory of Σ, respectively.

A Riemannian metric $G : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is a smooth map on $\mathbb{R}^n$ such that, for any $x \in \mathbb{R}^n, G(x)$ is a symmetric positive definite matrix Lee (2003). For any $x \in \mathbb{R}^n$ and smooth functions $I, J : \mathbb{R}^n \to \mathbb{R}^n$, one can define the scalar function $\langle I, J\rangle_G$as $I^T(x)G(x)J(x)$. We will still use

[1] The variation $\delta\xi$ can be formally defined by considering a family of trajectories $\xi_{xu}(t, \varepsilon)$ parameterized by $\varepsilon \in \mathbb{R}$. The variation of the state is then $\delta\xi = \frac{\partial \xi_{xu}}{\partial \varepsilon}$