Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-1 (2016) 154–159

Robust stabilization of uncertain neural networks with additive time-varying delays ⋆

K. Subramanian *, **P. Muthukumar** **, **S. Lakshmanan** ***.

* *Department of Mathematics, The Gandhigram Rural Institute-Deemed University, Gandhigram- 624302, Tamil Nadu, India. (e-mail:subramaniangri@gmail.com).*

** *Department of Mathematics, The Gandhigram Rural Institute-Deemed University, Gandhigram- 624302, Tamil Nadu, India. (e-mail: pmuthukumargri@gmail.com).*

*** *Center for Intelligent Systems Research, Geelong Waurn Ponds Campus, Deakin University, Australia, (e-mail: lakshm85@gmail.com).*

Abstract: This study is mainly concerned with the problem of robust stabilization of neural networks with two additive time-varying delays via Wirtinger-based double integral inequality. The main purpose of this paper is to design a memoryless state feedback controller which ensuring that the global asymptotic stability of closed loop system. By constructing a suitable Lyapunov-Krosovskii functionals, utilizing Wirtinger-based double integral inequality, the sufficient criteria are derived in terms of linear matrix inequalities technique which ensuring the global asymptotic stability of the proposed neural networks with norm bounded uncertainties. The desired controllers can be calculated by solving the linear matrix inequalities with the help of some standard numerical packages. Finally, the numerical examples are given to demonstrate the effectiveness of the theoretical results.

Keywords: Global asymptotic stability, Linear matrix inequalities, Neural networks, Stabilization, Time-varying delays.

1. INTRODUCTION

In the past few decades, neural networks have been widely studied due to their successful applications in various fields such as power systems (Beaufays et al. (1994)), pattern recognition (Galicki et al. (1997)), smart antenna arrays (Rawat et al. (2012)) and so on. On the other hand, time delay is a common phenomenon that often occur in neural networks and the existence of time delay can cause system's instability and sometimes degrade system's performance. Since stability is an important property of the dynamical systems. During the transmission process, signal must go through various nodes which may cause time delay with some different characteristics in practical applications. As a result, time delay is no longer in a single or simple form. Based on this, a new model for neural networks with two additive time-varying delays has been proposed in Wang et al. (2014).

As is well known that, due to environmental noise, uncertain or slowly varying parameters, it is very difficult to obtain an exact mathematical model. Therefore, it is an important to ensure that the model is stable with respect to the uncertainties and the robust stability analysis of neural networks has gained much research attention. In this regard, many researchers have focused on stability and stabilization issues of delayed neural networks with existence of uncertain parameters (*see*, *e.g.*,Lakshmanan et al. (2013), *Yang et al.* (2009), *Zhang et al.* (2013, *a*)).

To solve the stability issue, some effective controllers have been designed to stabilize the states of the dynamical systems. In particular, study of stabilization problem for different kinds of delayed neural networks have become an active research topics. As an example, *Huang et al.* (2013), the authors discussed the exponential stabilization of delayed recurrent neural networks based on state estimation approach. *Rakkiyappan et al.* (2015), proposed the stochastic sample data control for the stabilization problem of delayed neural networks.

Motivated by the above discussion, in this paper the problem of robust stabilization of neural networks with two additive time-varying delays is considered under the memoryless state feedback controller design. From the existing literature, many authors have derived the stability criteria through the Jensen's inequality, free weighting matrices method and some integral inequalities (for example, *Sun et al.* (2010), *Song et al.* (2015), *Gu* (2000)). Jensen's integral inequality approach is important one, which was first introduced by *Gu* (2000) for stability analysis of time-delay systems. Based on this paper, the authors *Ramakrishnan et al.* (2011), *Sun et al.* (2010), are further extended the Jensen's integral inequality to some new forms and the authors also studied the stability problems of discrete time systems with time-varying de-

⋆ This work was supported by SERB project DST, New Delhi, India under the F. No: YSS/2014/000447 dated 20.11.2015.

2405-8963
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.03.045

lays, linear systems with time-varying delays respectively. Moreover, Song et al. (2015) deals with a new methodology of robust state feedback fuzzy controller based on free weighting matrix method for a class of uncertain Markovian jump nonlinear systems. However, to the best of our knowledge, up to now there is no results on the robust stabilization problem of uncertain neural networks with two additive time-varying delays using the Wirtinger-based double integral inequality.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean vector space and the set of all $n\times n$ real matrices, respectively. The superscript T denotes the transposition and the notation $X \geq Y$ (similarly, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (similarly, positive definite). The notation $*$ always denotes the symmetric term in a matrix. $x_t := \{x(t+s) : s \in [-h, 0]\}$. The notation diag$\{\ldots\}$ stands for a block-diagonal matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider the uncertain neural networks with additive time-varying delays as follows:

$$\begin{cases} \dot{x}(t) = -(A + \varDelta A(t))x(t) + (B + \varDelta B(t))f(x(t)) \\ \qquad +(C + \varDelta C(t))f(x(t - h_1(t) - h_2(t))) + Du(t), \\ x(t) = \phi(t),\ t \in [-h, 0], \end{cases} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vectors, $A = \text{diag}\{a_i\}$ with $a_i > 0$, $(i = 1, 2, \cdots n)$. B, $C \in \mathbb{R}^{n\times n}$ are the connection weight matrix and delayed connection weight matrix respectively. $D \in \mathbb{R}^{n\times n}$ is the control input matrix. $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))]^T \in \mathbb{R}^n$ denotes the neuron activation functions. The initial condition $\phi(t)$ is a continuous and differentiable vector-valued function, where $t \in [-h, 0]$. In addition, we address the uncertainty, suppose that matrices A, B, C have parameter perturbations $\varDelta A(t)$, $\varDelta B(t)$, $\varDelta C(t)$, which are of the form

$$[\,\varDelta A(t) \quad \varDelta B(t) \quad \varDelta C(t)\,] = LM(t)\,[\,N_a \quad N_b \quad N_c\,].$$

Here L, N_a, N_b, N_c are known real constant matrices of appropriate dimensions with an unknown time-varying matrix $M(t) \in \mathbb{R}^{i\times j}$ with Lebesque measurable elements and satisfying $M^T(t)M(t) \leq I$, where I denotes the identity matrix of appropriate dimension. The following assumption are adopted throughout this paper.

Hypothesis 1. The delays $h_1(t)$, $h_2(t)$ represents the time varying continuous function and satisfy: $0 \leq h_1(t) \leq h_1$, $\dot{h}_1(t) \leq \mu_1$, $0 \leq h_2(t) \leq h_2$, $\dot{h}_2(t) \leq \mu_2$,
where h_1, h_2 and μ_1, μ_2 are constants. We denote $h(t) = h_1(t) + h_2(t)$, $h = h_1 + h_2$.

Hypothesis 2. Each neuron activation function $f_i(\cdot)$ in (1) satisfies the following condition:

$$k_i^- \leq \frac{f_i(\alpha_1) - f_i(\beta_2)}{\alpha - \beta} \leq k_i^+, \quad i = 1, 2, \ldots, n, \forall \alpha, \beta \in \mathbb{R},$$

$\alpha \neq \beta$, where k_i^-, $k_i^+ \geq 0$, $f_i(0) = 0$.

Define the following control law to tackle the robust stabilization problem:

$$u(t) = Kx(t),$$

where K is the controller gain matrix. The system (1) can be rewritten in this form:

$$\begin{aligned} \dot{x}(t) &= -(A + \Delta A(t))x(t) + (B + \Delta B(t))f(x(t)) \\ &\quad +(C + \Delta C(t))f(x(t - h_1(t) - h_2(t)) + DKx(t). \end{aligned} \tag{2}$$

Thus, to prove the system (2) is robust globally asymptotically stable.

Lemma 3. [Seuret et al. (2013)]**(Wirtinger based integral inequality):** For any constant matrix $M > 0$, the following inequality holds for all continuously differentiable function ϕ in $[a,\ b] \to \mathbb{R}^n$;

$$(b-a)\int_a^b \phi(s)M\phi(s)ds \geq \left(\int_a^b \phi(s)ds\right)^T M \left(\int_a^b \phi(s)ds\right) + 3\Theta^T M\Theta$$

$$\text{where } \Theta = \int_a^b \phi(s)ds - \frac{2}{b-a}\int_a^b \int_a^s \phi(u)duds.$$

Lemma 4. [Kwon et al. (2015)]**(Wirtinger based double integral inequality):** For any constant matrix $M > 0$, the following inequality holds for all continuously differentiable function ϕ in $[a,\ b] \to \mathbb{R}^n$;

$$\frac{(b-a)^2}{2}\int_a^b \int_s^b x^T(u)Mx(u)duds \geq \left(\int_a^b \int_s^b x(u)duds\right)^T \times M\left(\int_a^b \int_s^b x^T(u)Mx(u)duds\right) + 2\Theta_d^T M\Theta_d$$

$$\Theta_d = -\int_a^b \int_s^b x(u)duds + \frac{3}{b-a}\int_a^b \int_s^b \int_u^b x(u)dvduds.$$

Lemma 5. [Yang et al. (2009)] Let M, N and F be matrices of appropriate dimensions, and $F^T F \leq I$, then for any scalar $\epsilon > 0$,

$$MFN + N^T F^T M^T \leq \epsilon MM^T + \epsilon^{-1}N^T N.$$

3. MAIN RESULTS

In this section, we investigate the globally asymptotic stability of uncertain neural networks with two additive time-varying delays described by the system (2).

Theorem 6. For given positive scalars h_1, h_2, μ_1 and $\mu_2, \epsilon_1, \epsilon_2, \epsilon_3$, the system (2) is globally asymptotically stable if there exist positive definite symmetric matrices P, X, Q_m, R_m $(m = 1, 2 \cdots, 4)$, $Y = \text{diag}(y_{11}, y_{12}, \cdots, y_{1n}) > 0$, such that the following linear matrix inequalities hold:

$$\Phi_1 = \begin{bmatrix} \Omega & \zeta \\ * & \Psi \end{bmatrix} < 0, \tag{3}$$

where $\Omega = \Omega^T = (\Omega_{i,j})_{13\times 13}$, $\zeta = (\zeta_{i,j})_{13\times 3}$ and $\Psi = (\Psi_{i,j})_{3\times 3}$ with $\Omega_{1,1} = -2PA + 2DX + \epsilon_1 N_a N_a^T + Q_1 + Q_3 + h_1^2 R_1 + h^2 R_2 - 2K^- Y K^+ + \left(\frac{h_1^2}{2}\right)^2 R_3 + \left(\frac{h_2^2}{2}\right)^2 R_4$, $\Omega_{1,4} = PB + K^- Y + K^+ Y$, $\Omega_{1,6} = PC$, $\Omega_{2,2} = -(1-\mu_1)Q_1$, $\Omega_{3,3} = -Q_3$, $\Omega_{4,4} = -2Y + Q_2 + Q_4 + \epsilon_2 N_b^T N_b$, $\Omega_{5,5} = -Q_4$, $\Omega_{6,6} = -(1-\mu_2)Q_2 + \epsilon_3 N_c^T N_c$, $\Omega_{7,7} = -4R_1$, $\Omega_{7,9} = \frac{6}{h_1}R_1$, $\Omega_{8,8} = -4R_2$, $\Omega_{8,11} = \frac{6}{h}R_2$, $\Omega_{9,9} = \frac{-12}{h_1^2}R_1 - 3R_3$, $\Omega_{9,12} = \frac{6}{h_1}R_3$, $\Omega_{10,10} = -3R_4$, $\Omega_{10,13} = \frac{6}{h_2}R_4$,