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Robust Dynamic Event-triggered Control for Linear Uncertain System

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Abstract: Event-triggered control has a greater significance in networked control system (NCS) as it minimizes communication cost. The present paper introduces a novel event-based robust control framework for linear uncertain system. The robust control law is designed by solving an optimal control problem and realized thorough a dynamic event-triggering mechanism to reduce the communication traffic. A dynamic variable is used to generate the event-triggering law using Input-to-State Stability (ISS) criteria . Proposed control strategies ensure stability in the presence of bounded system uncertainties. Derivation of dynamic event-triggering rule with a non-zero positive inter-event time and corresponding stability criteria for uncertain system are the key contributions of this paper. The validation of proposed algorithm is carried out through a numerical simulation.

Keywords: Event-triggered control, robust control, dynamic event-triggered control, aperiodic control, input to state stability.

1. INTRODUCTION

Aperiodic sensing, communication and computation play a crucial role for controlling resource constrained Cyber-Physical Systems. It is shown in [Astrom et al. (2002); Heemels et al. (2012); Tabuada (2007); Marchand (2013)] that aperiodic sampling has more benefits over periodic sampling, which motivates control researchers towards event-triggered control. In event-triggered control, sensing, communication and computation happens only when any predefined event condition is violated. This control strategy finds application in different control problems like tracking [Tallapragada (2013)], estimation [Tallapragada (2012); Trimpe et al. (2012)] etc. Event-triggered system is modeled as a perturbed system in continuous and discrete time domain respectively [Tabuada (2007); Eqtami et al. (2010)]. Also the behaviour of such system is described by an impulsive dynamics in literature [Donkers et al. (2012); Sahoo et al. (2013)]. To achieve larger average inter-event time, [Girard (2015)] proposes a dynamic event-generating rule over the static approach [Tabuada (2007)]. The input to state stability (ISS) property [Sontag (2008); Nesic et al. (2004)] is exploited to prove the closed loop stability and to define triggering condition for event-triggered system. Sahoo et al. [Sahoo et al. (2013)] proposed an event based adaptive control approach for uncertain systems. A neural network is used to estimate the nonlinear function to generate the control law. In event based robust control problems, the uncertainty is mainly considered in the communication channel in the form of time-delay or data-packet loss [Garcia et al. (2013)]. The main shortcoming of the classical event-triggered system lies in the fact that one must know the exact model of the plant apriori. A plant with an uncertain (system) model is a more realistic scenario and has far greater significance. However, there are open problems of designing a control law and triggering conditions to deal with system uncertainties. These uncertainties mainly arise due to system parameter variations, unmodeled dynamics, disturbances etc. and necessitates the design of robust controller. To deal with uncertainties, an optimal control approach to robust controller design for the uncertain system has been reported in [Lin et al. (2000, 1998); Adhyaru et al. (2009)] and find applications in tracking problem of robot manipulator [Lin et al. (1998); Tripathy et al. (2014)], set-point regulation in CSTR system etc. To achieve an optimal solution to the robust control problem there is a need to minimize a cost functional. In this direction, a non-quadratic cost functional is utilized to solve robust control problem with input constraint [Adhyaru et al. (2009)]. In the above mentioned approach, event-trigger based implementation of robust control law is not considered which is essential in the context of NCS.

This paper considers a robust control strategy of linear uncertain system with limited state and input information. The limited state information is considered to address the channel unreliability or bandwidth constraint which is a very common phenomena in NCS. To capture the channel uncertainty, event-triggered control strategy is adopted in [Xia et al. (2013)] without considering system uncertainty explicitly. The primary motivation for this work is that with limited information, existing robust control results in [Lin et al. (2000, 1998); Kar (2002)] can not be simply extended to the event-triggered system. This paper proposes a novel event based robust control strategy for matched uncertain systems where it is assumed that the

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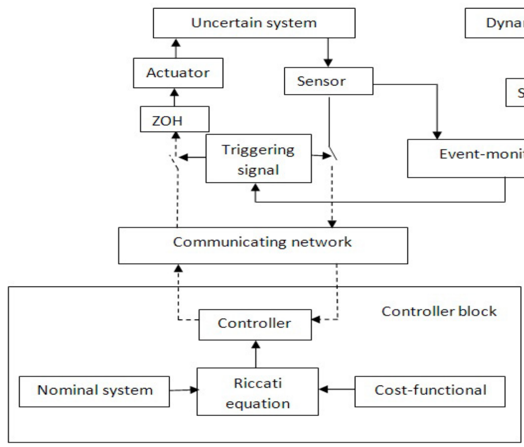

Fig. 1. Conceptual Block diagram of proposed event-trigger based robust control. Dotted line represents the aperiodic information transmission through the communication channel.

unknown uncertainty is in the range space of control input matrix. A conceptual block diagram of the proposed event-trigger based robust control framework is illustrated in Figure 1. Here the system, sensor and actuator are co-located but the controller is connected thorough a communication network. A dedicated computing unit monitors the event condition at the sensor end. The control input is computed and updated only when an event is generated. A zero-order-hold (ZOH) at the actuation end holds the last transmitted control input until the transmission of next input. The aperiodic state transmission to controller and control input update instant $\{t_k\}_{k\in I}$ over the network is decided by the same event-triggering law. For simplification, it is assumed that there is no communication, computation and actuation delay in the system. To design robust control law, an equivalent optimal control problem is formulated with an appropriate cost functional which takes care of the upper bound of system uncertainty. The nominal system dynamics is used to compute the optimal controller gain which minimizes the cost-functional. The analysis of this system is done in continuous time domain. The proposed method is also extended to design dynamic event-triggering rule in order to increase inter-event time. The corresponding triggering rule and their stability criteria for matched uncertain system have been derived. The advantage of the proposed control strategy is that it significantly reduces the number of control input transmission and computation in spite of system uncertainties.

Summary of contribution: The main contributions of this paper are summarized as follows.

- Defining an optimal control problem to design a robust control law for matched uncertain system.
- Deriving a dynamic event-triggering rule for uncertain system using the upper bound of system uncertainty.
- Ensuring stability of closed loop system using ISS Lyapunov function.
- Deriving a positive non-zero lower bound of inter-execution time. Results are verified through simulation studies.

Organization of paper The paper is organized as follows. In Section 2, we briefly review an optimal approach to robust control design for uncertain system. Section 3 discuss the optimal control approach to solve the robust stabilization problem for event-triggered system with system uncertainty. A dynamic event triggering conditions is stated in the form of theorems and their corresponding proofs are reported. Also the expressions of the minimum positive inter-event time is defined in Section 3. An academic example with simulation results is discussed in Section 4 to validate the proposed control algorithm. Section 5 concludes the paper.

Notation The notation $\|x\|$ is used to denote the Euclidean norm of a vector $x \in \mathbb{R}^n$. Here $\mathbb{R}^n$ denotes the n dimensional Euclidean real space and $\mathbb{R}^{n\times m}$ is a set of all $(n \times m)$ real matrices. $\mathbb{R}_0^+$ and $\mathbb{I}$ denote the all possible set of positive real numbers and non-negative integers. $X \leq 0$, X^T and X^{-1} represent the negative definiteness, transpose and inverse of matrix X, respectively. Symbol I represents an identity matrix with appropriate dimensions and time t_∞ implies $+\infty$. Symbols $\lambda_{min}(P)$ and $\lambda_{max}(P)$ denote the minimum and maximum eigenvalue of symmetric matrix $P \in \mathbb{R}^{n\times n}$ respectively. A function $f\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is K_∞ if it is continuous and strictly increasing and it satisfies $f(0) = 0$ and $f(s) \to \infty$ as $s \to \infty$.

2. PRELIMINARIES & PROBLEM STATEMENT

2.1 Preliminaries

Input to state stability In state space form, a linear system with external disturbance $d(t) \in \mathbb{R}^n$ is expressed as

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t) \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$ are system's state and control input respectively. For simplification from now onwards, $x(t)$ and $u(t)$ are denoted by x and u respectively. Disturbance $d(t)$ is assumed to be bounded by a known function $d_m(t)$ i.e. $\|d(t)\| \leqslant d_m(t)$. The above system (1) is said to be ISS with respect to $d(t)$ if there exist an ISS Lyapunov function. To analyze the ISS of system (1) with respect to $d(t)$, following definition is introduced [Sontag (2008); Nesic et al. (2004)].

Definition 1. A continuous function $V(x) : \mathbb{R}^n \to \mathbb{R}$ is an ISS Lyapunov function for system (1) if there exist class k_∞ functions $\alpha_1, \alpha_2, \alpha_3$ and γ for all $x, d \in \mathbb{R}^n$ and it satisfy

$$\alpha_1(\|x(t)\|) \leq V(x(t)) \leq \alpha_2(\|x(t)\|) \tag{2}$$

$$\nabla V(x)\dot{x} \leq -\alpha_3(\|x(t)\|) + \gamma(\|d(t)\|) \tag{3}$$

System with matched uncertainty: A linear system having system-uncertainty is described as

$$\dot{x} = A(p)x + Bu \tag{4}$$

where $p \in P$ is an uncertain parameter vector. In general system uncertainty is classified in two categories namely matched and mismatched uncertainty [Lin et al. (2000); Kar (2002)]. The system (4) has matched uncertainty if there exists a bounded uncertain matrix $\phi(p) \in \mathbb{R}^{m\times n}$ such that

$$A(p) - A(p_0) = B\phi(p) \tag{5}$$