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A Novel Second-order Recursive Reaching Law based Discrete-time Sliding Mode Control

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Abstract: In this paper, a novel technique of second order recursive reaching law based sliding mode control for a discrete-time system is presented. The proposed second order recurrence dictates the dynamics of sliding function using its two past terms. The synthesized control input utilises two past control inputs and results in better continuity in discrete-time, known as (p,q)-continuity.

Keywords: Sliding-mode control, discrete-time systems, Second-order reaching law, (p, q)-continuity, Lyapunov stability

1. INTRODUCTION

Sliding mode control is a robust technique which has many interesting properties like invariance to parametric uncertainty and external disturbance, order reduction etc Singh and Janardhanan (2015). The vast research of last three decades in this field has made SMC a mature technique (Utkin et al. (2009); Bandyopadhyay and Janardhanan (Oct. 2005)). The advancement of micro-controllers has encouraged its implementation on discrete-time systems. In the research of discrete-time sliding mode (DSM) control, generally there are two principal frameworks. In the first framework, new control strategies are presented for pure discrete-time systems. On the other hand, the second one is to discretise several existed continuous time control techniques (Fridman et al. (2014); Janardhanan and Bandyopadhyay (2006)).

Following the first framework, Miloslavjevic (1985) proposed a DSMC by adding a signum term to force system trajectory to remain in quasi-sliding mode (QSM). Gao et al. (1995) presented reaching law based first order DSM and suggested that for QSM, system trajectory must cross switching plane at every successive sample. Bartoszewicz (1998) modified the general condition for DSM given by Gao et al. (1995) and restricted the motion of systems trajectory in a certain band around the sliding surface. Drakunov and Utkin (1989) explained that discrete-time states are generally not able to stay on the sliding surface but stay in the neighbourhood of sliding surface, known as real sliding. The drawback of continuous-time sliding mode, chattering, can be eliminated using second order sliding mode proposed by Levant (1993), but it is impossible to achieve continuous control in discrete-time systems even using second order sliding mode (Fridman et al. (2014); Bartolini et al. (2001); Salgado et al. (2011)). Recently Sharma et al. (2015) defined relaxed version of continuity, (p,q)-continuity, to measure the continuity in discrete-time sliding function.

1.1 Motivation

The available reaching law based discrete-time sliding mode control techniques are generally based on first-order recurrence (Chakrabarty and Bandyopadhyay (2015)) and few efforts are made to design higher-order recursive reaching law. The fact that increasing order of a difference equation results in smoother dynamics, motivated us to propose a second order recursive reaching law for smoother dynamics of sliding function.

1.2 Contribution

In this paper, a novel second order recursive reaching law is proposed to design a discrete-time sliding mode. The Lyapunov stability analysis is used to prove that the proposed reaching law stabilize the states of a perturb discrete-time unstable system in a band. The control input is synthesized using proposed reaching law for relative degree-one and relative degree-two cases. To measure the smoothness of sliding function a recently defined continuity, (p,q)-continuity, is used. The conventional DSM by Gao et al. (1995) and the proposed technique are compared graphically as well as numerically.

It should be noted that unlike discretisation of existing second order continuous SMC (Bartolini et al. (2001); Fridman et al. (2014)), here DSMC designed for discrete-time system using a second-order difference equation.

After introduction in section-1, problem is defined in Section-2. Section-3 contains design of second order recursive sliding mode algorithm. Section-4 presents (p,q)-continuity of designed sliding mode. Simulation results and discussion are given in Section-5. In the end, Section-6 contains the conclusion.

2. PROBLEM STATEMENT

Consider a general controllable discrete-time system

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$$x(k+1) = Ax(k) + Bu(k) + \tilde{d}(k), \quad (1)$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $k \in \mathbb{Z}$, with the matrix A and B being of appropriate dimensions. The disturbance vector $\tilde{d}(k)$ represents the combined effect of unmodeled dynamics and external disturbances affecting the system. It is assumed that $\tilde{d}(k)$ is unknown but bounded and satisfies the matching condition i.e. $\tilde{d}(k) \in \text{Range}(B)$. Let a discrete-time sliding function defined as

$$s(k) = cx(k), \quad (2)$$

where $s \in \mathbb{R}^m$ and $c \in \mathbb{R}^{m\times n}$. Parameters of c are chosen such that sliding function has stable dynamics.

The objective is to achieve discrete-time sliding mode for system (1) by driving the system state in the vicinity of sliding surface. For this purpose, a reaching law is required to dictate the dynamics of $s(k)$. Reaching law in the discrete-time is a difference equation which has a recursive relation to its past terms. A general r-order reaching law can be represented as

$$s(k+r) = f[s(k), s(k+1), ..., s(k+r-1)], \quad (3)$$

where f is the function of the variables mentioned and r is the order of difference equation. The resulting sliding motion in the vicinity of sliding surface is known as r-order sliding mode.

Now, the aim is to find the recursive relation such that (3) is asymptotically stable at origin. Then, a control input can be synthesized using the recursive relation and (1). Asymptotic stability of (3) guarantees sliding mode or in worst case quasi-sliding mode. A discrete-time sliding mode control is designed for second order recursive sliding mode in the following section.

3. SECOND ORDER RECURSIVE SLIDING MODE

The proposed discrete-time second-order recursive reaching law is

$$s(k+1) = -\alpha(k)Sig^{\gamma}[s(k-1)] - \beta(k)Sig^{\psi}[s(k)] + d(k), \quad (4)$$

where $(\alpha(k), \beta(k) \in \mathbb{R} : 0 < \alpha(k), \beta(k))$, $(\gamma, \psi \in \mathbb{R} : 0 < \gamma < \psi < 1)$, and $Sig^{\psi}(s(k)) = |s(k)|^{\psi}Sign(s(k))$.

The disturbance $d(k) = c\tilde{d}(k)$ is bounded i.e.

$$||d(k)|| \leq \delta, \quad (5)$$

where $\delta \in \mathbb{R}$ and $||.||$ is an Euclidean norm. To assure the stability of the proposed reaching law the following lemma is stated.

Lemma 1. The nominal part of recursive reaching law (4)(i.e. reaching law without disturbance) is asymptotically stable at origin if the design parameters $\alpha(k)$, $\beta(k)$ are selected appropriately.

Proof. To prove the above theorem, let $s_1(k) = s(k-1)$ and $s_2(k) = s(k)$. The nominal part of (4) can be composed as

$$\text{S}(k+1) = G(k)\text{S}(k), \quad (6)$$

where $\text{S} = [s_1(k)\ s_2(k)]^T$ and

$$G(k) = \begin{bmatrix} 0 & 1 \\ -\alpha(k)|s_1(k)|^{\gamma-1} & -\beta(k)|s_2(k)|^{\psi-1} \end{bmatrix} \quad (7)$$

Consider a Lyapunov candidate

$$V[\text{S}(k)] = \text{S}(k)^T P(k)\text{S}(k)$$

where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$. First order difference of Lyapunov candidate

$$\Delta V[\text{S}(k)] = \text{S}(k+1)^T P(k+1)\text{S}(k+1) - \text{S}(k)^T P(k)\text{S}(k)$$

For a system (6) to be asymptotically stable at the origin, given a positive-definite Hermitian matrix Q, there exists positive-definite Hermitian matrix P such that

$$\Delta V[\text{S}(k)] \leq -\text{S}(k)^T Q\text{S}(k)$$
$$G(k)^T P(k+1)G(k) - P(k) \leq -Q \quad (8)$$

For a asymptotic stable system, as the final time k_f tends to ∞, matrix $P(k)$ attains steady state value $\bar{P}$. The values of $\alpha(k)$ and $\beta(k)$ which make P a positive-definite Hermitian matrix are the appropriate values for a asymptotically stable system. On solving (8) for $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, elements of matrix $\bar{P}$ are obtained as given in (9)-(11), where for notation simplicity $\zeta_1 = (\gamma - 1)$ and $\zeta_2 = (\psi - 1)$. $\bar{P}$ is positive-definite Hermitian if (12) is greater than zero and it is always satisfied if gain values $\alpha(k)$ and $\beta(k)$ are selected such that it satisfy (13) and (14) respectively. Thus, on choosing values of parameters $\alpha(k)$ and $\beta(k)$ appropriately, system (6) is always Lyapunov asymptotically stable. This the end of proof. □

Definition 2. Uniformly Ultimately Bounded (UUB) (Jain and Bhasin (2015)): A discrete-time system is UUB in a set, if for every initial condition of x, there exists finite sample k^* such that for any $k \geq k^*$, $x(k)$ always remain in the set.

Theorem 3. The recursive reaching law (4) reaches the vicinity of origin asymptotically and then becomes uniformly ultimately bounded in a band.

Proof. In the presence of disturbance, let the system (6) in the form of

$$\text{S}(k+1) = G(k)\text{S}(k) + d(k) \quad (15)$$

and first order difference of Lyapunov (8) is

$$G(k)^T P(k+1)G(k) + d(k)^T P(k+1)d(k) - P(k) \leq -Q \quad (16)$$

As disturbance $d(k)$ is bounded, at steady state

$$G(k)^T \bar{P}G(k) - \bar{P} \leq -(Q + \delta_d) \quad (17)$$

where $\delta_d = \bar{P}||\delta||^2$ is also bounded. Lemma-1 proves that nominal part of (4) is asymptotically stable at origin. As the disturbance $d(k)$ is non-vanishing but bounded, thus reaching law asymptotically reaches the vicinity of origin and becomes uniformly ultimately bounded in a band around origin. □