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Revisiting Approximation Techniques to Reduce Order of Interval System

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Abstract: Large interconnected modules result in complex system of higher order and often of interval structure, making the overall study and analysis, time consuming and complicated. Accepting the challenge to state an approximate model of such system, both system analysts and control engineers, headed towards the model order reduction. Continuing the same, this paper revisits few noteworthy estimation techniques for simplification of discrete-time interval system. In particular, denominator is derived through reciprocal algorithm and numerator by two varied algorithms. The proposed algorithms are validated with examples from literatures and real-time test systems via assessment of error computation. Limitation encountered during the course is also taken into count.

Keywords: Discrete-time Interval System, Bilinear Transformation, Routh Approximation, Direct Truncation, Pade Approximation, Mixed Approach, Order Reduction.

1. INTRODUCTION

Emergence of order reduction methodologies is an attractive field of research till date as seen in the survey papers [Bultheel and Barel, 1986; Genesio and Milanese, 1976; Gugercin and Antoulas, 2004; Hwang and Lee, 1997]. With the time span, system's complexity increased for accounting unmodelled dynamics, parameter variation, disturbances, actuators, etc. leading to ambiguity. Systems as flight vehicles, electric motors, and robots are formulated under continuous-time domain of vague structure. These fears are handled by considering interval system, instead of fixed coefficient mathematical representation. Algorithms notably Routh-Pade [Bandyopadhyay, Ismail and Gorez, 1994], Pade approximation [Bandyopadhyay and Ismail, 1995], Routh approximants [Sastry, Raja Rao and Rao, 2000], $\gamma-\delta$ Routh approximation [Bandyopadhyay, Upadhye and Ismail, 1997], are available for the reduction of continuous-time interval systems.

The outburst of discrete-time signals and systems, grabbed the interest of order reduction of discrete-time interval system for being considerably simple and computationally easy. Techniques for such systems are few but are proficient, specifically Pade approximation allowing dominant poles retention [Ismail, Bandyopadhyay and Gorez, 1997], accessing higher-order integrators [Hsu and Wang, 2000], μ-dependent approach [Zhang, Boukas and Shi, 2009], and pole retention with direct series expansion [Singh and Chandra, 2011]. In [Dolgin and Zeheb, 2004], finite impulse response is used for order reduction of discrete-time interval system. In recent past, algorithms by [Choudhary and Nagar, 2013 (a, b)] are applied to discrete-time interval system, showing their acceptable extension to interval structure from fixed coefficient system. Freshly, in [Choudhary and Nagar, 2015] a glimpse of algorithm similar to the methodology briefed in this paper is discovered with a significant difference between them.

Add on to the existing algorithms of discrete-time interval system is proposed in this paper. The techniques discussed here exist for fixed coefficient and continuous-time interval system, yet, state to be novel for order reduction of discrete-time interval system. The attempt in the paper is to propose new mixed methods. Precisely, denominator is computed by a new algorithm of reciprocity and numerator by two different prevailing techniques namely direct truncation and Pade approximation as explained in the next section. The algorithms are illustrated through examples from the literatures and are compared for their validation based on the error sum between the original and reduced representations in section 3. Two real-time test systems are also considered to strengthen the algorithm. Section 4 discusses the competence of the proposed algorithms taking the affordable limitation into count. Finally, paper concludes with an emergence of two varied simple and efficient mixed algorithms for obtaining reduced model of interval form.

2. METHODOLOGY

Conceive the higher order interval transfer function and its approximate lower order transfer function with $k < n$ be

$$G_n(z) = \frac{\left[M_{n-1}^-, M_{n-1}^+\right] z^{n-1} + \left[M_{n-2}^-, M_{n-2}^+\right] z^{n-2} + \ldots + \left[M_0^-, M_0^+\right]}{\left[N_n^-, N_n^+\right] z^n + \left[N_{n-1}^-, N_{n-1}^+\right] z^{n-1} + \ldots + \left[N_0^-, N_0^+\right]} = \frac{M_n(z)}{N_n(z)} \tag{1}$$

$$R_k(z) = \frac{\left[m_{k-1}^-, m_{k-1}^+\right] z^{k-1} + \left[m_{k-2}^-, m_{k-2}^+\right] z^{k-2} + \ldots + \left[m_0^-, m_0^+\right]}{\left[n_k^-, n_k^+\right] z^k + \left[n_{k-1}^-, n_{k-1}^+\right] z^{k-1} + \ldots + \left[n_0^-, n_0^+\right]} = \frac{M_k(z)}{N_k(z)} \tag{2}$$

Proceeding towards the derivation of a simple representation of higher order system, Routh Approximation is considered for obtaining denominator polynomial. Prime concern for applying this approximation is its computational simplicity and possibility to attain stable reduced model.

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Conduct of continuous-time domain algorithm over $G_n(z)$, insist its transformation to an appropriate domain and is ended by substituting $z=\frac{1+w}{1-w}$, known as bilinear or Tustin transformation. This transforms the *z*-domain system to its *w*-domain equivalent; much closer to continuous-time domain system. The transformation result in

$$G_n(w)=\frac{\left[A_n^-,A_n^+\right]w^n+\left[A_{n-1}^-,A_{n-1}^+\right]w^{n-1}++....+\left[A_0^-,A_0^+\right]}{\left[B_n^-,B_n^+\right]w^n+\left[B_{n-1}^-,B_{n-1}^+\right]w^{n-1}+.....+\left[B_0^-,B_0^+\right]}=\frac{M_n(w)}{N_n(w)} \quad (3)$$

Consider the reciprocal form of the above denominator polynomial $N_n(w)$ to obtain the reduced denominator polynomial represented as $\hat{N}_n(w)$;

$$\hat{N}_n(w)=\frac{1}{w}N_n\left(\frac{1}{w}\right)=\left[B_0^-,B_0^+\right]w^n+\left[B_1^-,B_1^+\right]w^{n-1}+.....+\left[B_n^-,B_n^+\right] \quad (4)$$

Use $\hat{N}_n(w)$ to draft the first two rows of the Routh array as shown in Table I.

Table I: Routh Approximation for Denominator

$\left[B_0^-,B_0^+\right]=\left[B_{1,1}^-,B_{1,1}^+\right]$	$\left[B_2^-,B_2^+\right]=\left[B_{1,2}^-,B_{1,2}^+\right]$	$\left[B_4^-,B_4^+\right]=\left[B_{1,3}^-,B_{1,3}^+\right]$	..
$\left[B_1^-,B_1^+\right]=\left[B_{2,1}^-,B_{2,1}^+\right]$	$\left[B_3^-,B_3^+\right]=\left[B_{2,2}^-,B_{2,2}^+\right]$	$\left[B_5^-,B_5^+\right]=\left[B_{2,3}^-,B_{2,3}^+\right]$	..
$\left[B_{3,1}^-,\ B_{3,1}^+\right]$	$\left[B_{3,2}^-,\ B_{3,2}^+\right]$		
….			
$\left[B_{n,1}^-,\ B_{n,1}^+\right]$			

Entries down the third row in the table is computed by

$$\left[B_{i,j}^-,B_{i,j}^+\right]=\left[B_{i-2,j+1}^-,B_{i-2,j+1}^+\right]-\left[\alpha_{i-2}^-,\alpha_{i-2}^+\right]\left[B_{i-1,j+1}^-,B_{i-1,j+1}^+\right] \quad (5)$$

where *i=3,4,……..,n* and *j=1,2,……*

$$\text{with}\quad \left[\alpha_i^-,\alpha_i^+\right]=\frac{\left[B_{i,1}^-,B_{i,1}^+\right]}{\left[B_{i+1,1}^-,B_{i+1,1}^+\right]}\qquad i=1,2,...k,.....n \quad (6)$$

provided $\left[B_{i+1,1}^-,B_{i+1,1}^+\right]\notin[0]$

The reduced denominator, $\hat{N}_k(w)$ is obtained according to equation (7) as stated for definite system [Hutton and Friedland, 1975]

$$\hat{N}_k(w)=\left[\alpha_k^-,\alpha_k^+\right]w\hat{N}_{k-1}(w)+\hat{N}_{k-2}(w) \quad (7)$$

with $\hat{N}_{-1}(w)=1$, $\hat{N}_0(w)=1$

For instance, if *k=1, 2* then denominator polynomial is

$$\hat{N}_1(w)=\left[\alpha_1^-,\alpha_1^+\right]w+[1,1] \quad (8a)$$

and $\hat{N}_2(w)=\left[\alpha_1^-,\alpha_1^+\right]\left[\alpha_2^-,\alpha_2^+\right]w^2+\left[\alpha_2^-,\alpha_2^+\right]w+[1,1]$ (8b)

The resulting $\hat{N}_k(w)$ is reciprocated back to $N_k(w)$ which on inverse bilinear transformation give the required $N_k(z)$.

The numerator $M_k(z)$ is computed by implicating two algorithms discussed below;

2.1 Algorithm 1

Direct Truncation [Choudhary and Nagar, 2013 b] is hired for obtaining the reduced numerator polynomial declared as

$$M_k(z)=\left[m_{k-1}^-,m_{k-1}^+\right]z^{k-1}+\left[m_{k-2}^-,m_{k-2}^+\right]z^{k-2}+....+\left[m_0^-,m_0^+\right] \quad (9)$$

2.2 Algorithm 2

Another prevailing technique; Pade approximation [Bultheel and Barel, 1986] used for obtaining the numerator polynomial is illustrated here. Once the denominator $N_k(w)$ exist, numerator $M_k(w)$ is obtained by matching first *t* time moments and *l* Markov parameters, such that $t+l=k$.

Assume the reduced model of order *k* be

$$\frac{\left[C_0^-,C_0^+\right]+\left[C_1^-,C_1^+\right]w+...+\left[C_{k-1}^-,C_{k-1}^+\right]w^{k-1}}{\left[D_0^-,D_0^+\right]+\left[D_1^-,D_1^+\right]w+...+\left[D_k^-,D_k^+\right]w^k}=\frac{M_k(w)}{N_k(w)} \quad (10)$$

Equate (10) and (3), cross multiply and compare left & right hand side for similar coefficients.

$$\frac{M_k(w)}{N_k(w)}=\frac{M_n(w)}{N_n(w)} \quad (11a)$$

$$\left[\left[C_0^-,C_0^+\right]+..+\left[C_{k-1}^-,C_{k-1}^+\right]w^{k-1}\right]N_n(w)=M_n(w)\left[\left[D_0^-,D_0^+\right]+..+\left[D_k^-,D_k^+\right]w^k\right] \quad (11b)$$

Place the obtained coefficient in equation (10) and apply inverse bilinear transformation, $w=\frac{z-1}{z+1}$ to obtain $R_k(z)$.

Reduced models are validated for their acceptable existence through the error sum computation known as performance index and expressed as

$$J=\sum_{k=0}^{\infty}\left[y(k)-y_k(k)\right]^2 \quad (12)$$

where, $y(k)$ and $y_k(k)$ are the unit step responses of original system $G_n(z)$ and its reduced model $R_k(z)$.