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A Multiresolution Wavelet based Subspace Identification

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Abstract:
This paper presents multiresolution subspace identification as an extension of classical subspace modeling, thus inheriting features of robust subspace identification with added advantages of wavelet based modeling enabling multiresolution state-space model development. Identification of a noisy process in presence of mild nonlinearity can be approximated by estimating multiple multiresolution time invariant models. Parameter estimation in projection space at appropriate scales is achieved using least squares method. The efficacy of the proposed approach has been demonstrated by modeling nuclear reactor in prediction as well as simulation environment. It is shown that root mean squared error reduces significantly as compared to their single scale counterparts providing better modeling performances.

Keywords: Wavelets, System Identification, Subspace Modeling, Nuclear Reactor.

1. INTRODUCTION

Process modeling is one of the core activities in the study of control systems. To develop control mechanism for the process a simple model description is a necessary prerequisite. A process may be complex, nonlinear, time varying, or multiscale in nature. For any kind of processes the model governed by physical laws may be of high order and it could be demanding to have a simplified model for control purposes. Sometimes process behavior is not known a priori and measurement data is available only as a source of information. This measured data can then be utilized for building mathematical models of dynamical systems. The task of constructing model from measured data is known as system identification.

System identification using generalized basis functions, in particular wavelet basis functions have specific advantages over classical methods as the estimated model with wavelet basis functions usually turns out to be of lower order. In general, wavelet bases are preferred compared to other bases due to its excellent approximation ability in multiresolution. Wavelet transform can extract local information present in the data simultaneously in time and frequency. In most of the reported literature, the problem of system modeling using wavelet basis function has been addressed mainly through two approaches. The first approach tries to estimate a parametric impulse response function by expansion on wavelet bases, Doroslovacki and Fan (1996), Zhao and Bentsman (2001), and Dorfan et al. (2004). Another approach identifies models in multiresolution projection space, Carrier and Stephanopoulos (1998), Sureshbabu and Farrell (1999), Wei and Billings (2002), and Li et al. (2011). Both of these approaches usually solve the output error minimization in least squares sense for system parameter estimation.

The work by Basseville et al. (1992) laid the foundation of stochastic modeling in a multiresolution framework. Their work is based on dividing the complex modeling problem into simple stochastic models on homogeneous trees. Chou et al. (1994) constructed a class of time-invariant multiscale dynamic models in state space on dyadic trees. The works by Tsatsanis and Giannakis (1993) and Doroslovacki and Fan (1996) discuss linear time-varying (LTV) system identification by expanding the time-varying coefficients of the parametric model. Zhao and Bentsman (2001) formalizes the theory of Doroslovacki and Fan (1996) in the function space background and demonstrates the convergence of approximation. Estimation of reduced order models are described in Carrier and Stephanopoulos (1998), and a scale selection measure based on open-loop cross over frequency has been suggested. Reis (2009) proposes a multiscale multivariable scheme, in prediction framework, for real time implementation and advocates exploratory data analysis (EDA) techniques for modeling. Mukhopadhyay and Tiwari (2010) employs consistency in output estimate in projection space and suggests alternate projection for reconstruction.

The present work describes subspace identification in a multiresolution framework using fast wavelet transform. The methodology is suitable for modeling a class of multiscale systems with multiple input and multiple output.

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The technique estimates models at appropriate scales giving better modeling performances for multiple time-scale processes. The suitability of the proposed approach is illustrated for modeling a nuclear reactor which is an ideal example of complex multiscale process. A comparative case study shows that multiscale subspace identification possesses better prediction capability over the classical approach of single scale (measurement space) subspace modeling, and the multiscale autoregressive with an exogenous (ARX) input model. Further the development of state-space models in projection space is very useful for design and implementation of multiscale control laws. Moreover, the multiscale subspace identification technique generally gives a reduced order model of a multiscale process.

The rest of the paper is organized as follows. Section 2 presents fundamentals. Section 3 describes the mathematical modeling of a system using wavelet basis function. Section 4 formalizes the subspace identification in wavelet domain. Section 5 discusses simulation results of the proposed methodology. Finally conclusions are drawn in section 6 indicating main contribution and future work.

2. PRELIMINARIES

2.1 Fundamentals

The function space $L^2(R)$ is a Hilbert space of square integrable functions i.e. all real functions which have finite energy and whose L^2-norms are finite. The L^2-norm of a function A is given by

$$\|A\| = \left(\int_{-\infty}^{+\infty} |A(t)|^2 dt \right)^{1/2} = \sqrt{\langle A, A \rangle} \tag{1}$$

Assuming that the input-output signals belong to L^2 space and V_u, V_y be the corresponding subspaces of L^2 containing approximation of input and output signals respectively. These subspaces can be called as projection spaces and are spanned by the shift invariant basis function ζ and ξ respectively. Cardinal series expansions of input and output on to generalized basis functions are given by

$$u(x) = \sum_{k \in Z} C_u[k]\, \zeta(x - kT) \tag{2}$$

$$y(x) = \sum_{k \in Z} C_y[k]\, \xi(x - kT) \tag{3}$$

where $C_u[k]$ and $C_y[k]$ are coefficients of basis function belonging to l^2 (l^2 is vector space of square summable discrete sequences). An orthogonal projection of input/output on subspace V_U /V_Y with minimum error is given by

$$\tilde{u}(x) = \sum_{k \in Z} \langle u(x), \zeta(x - kT) \rangle\, \zeta(x - kT) \tag{4}$$

$$\tilde{y}(x) = \sum_{k \in Z} \langle y(x), \xi(x - kT) \rangle\, \xi(x - kT) \tag{5}$$

$C_u[k] = \langle u(x), \zeta(x - kT) \rangle$ and $C_y(k) = \langle y(x), \xi(x - kT) \rangle$ give the signal contribution along the direction of specified basis function. The inner product can be implemented in terms of filtering and sampling

$$(u * \zeta)(x)|_{x=kT} = \sum_{k \in Z} u(x)\zeta(kT - x) = \langle u(x), \bar{\zeta}(x - kT) \rangle \tag{6}$$

$$(y * \xi)(x)|_{x=kT} = \sum_{k \in Z} y(x)\xi(kT - x) = \langle y(x), \bar{\xi}(x - kT) \rangle \tag{7}$$

where $\bar{\zeta}(x) = \zeta(-x)$ and $\bar{\xi}(x) = \xi(-x)$. The inner product of measurement function with the integer shift of time reversed impulse response is equivalent to first low pass filtering and sampling thereafter. In Shannon′s sampling theory orthonormal *sinc* basis function are chosen for function expansion. In general, basis functions are not necessarily need to be orthogonal. They can be biorthogonal or spline in nature as discussed in Unser (2000).

2.2 Discrete Wavelet Transform

A brief introduction to wavelet transform is given here. For detailed studies readers are advised to refer Daubechies (1992), Meyer (1993), Vetterli and Kovačević (1995), Strang and Nguyen (1997), and Mallat (2008). Wavelet being a time-frequency tool has been widely used for the analysis of signals and systems for past two decades and still enjoys wide applications. A signal is projected onto different subspaces with the help of a scaling (mother wavelet) and a wavelet (father wavelet) function. The approximation of signal is obtained by moving from a finer subspace to the coarser subspace which is spanned by the scaling function ϕ. The information left out during this transition has been resided in the detail subspace which is spanned by wavelet function ψ. The projection of signals can be processed further for better representation of information. To reconstruct the original signal at finer resolution the approximation can be added to the detail which will give a processed version of original signal. In terms of signal processing this task can be achieved efficiently by quadrature mirror filter bank in orthogonal system and by perfect reconstruction filter bank in biorthogonal system.

The continuous wavelet transform (CWT) of a signal $A(t)$ is given as an inner product of $A(t)$ with dilates of wavelet $\frac{1}{\sqrt{m}}\psi\left(\frac{t-n}{m}\right)$.

$$\left\langle A(t), \frac{1}{\sqrt{m}}\psi\left(\frac{t-n}{m}\right) \right\rangle = \frac{1}{\sqrt{m}} \int_{-\infty}^{\infty} A(t)\psi\left(\frac{t-n}{m}\right)^* dt \tag{8}$$

where $n \in R$ being translation parameter and $m \in R^+$ being dilation parameter. The CWT results into discrete wavelet transform (DWT) when computed at dyadic grids. In DWT $m = 2^j$ and $n = 2^j * k$, where j and k are scale and position indexes respectively with $j, k \in Z$.

2.3 Subspace Identification

Subspace identification is a class of linear state space model identification technique and falls in the purview of time-domain identification. Subspace identification algorithms employ robust decomposition procedure like singular value decomposition (SVD) and QR factorization making them computationally amenable. The classical way of system identification first identifies a model and then states are derived from the estimated model. Subspace