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Synchronization of Coupled Oscillator Dynamics

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Abstract: Synchronization is most significant phenomena to study the collective behaviour of coupled oscillators. Synchronization is said to occur if phase locking and consensus among corresponding states of coupled dynamical systems is achieved. In general, how to achieve exact conditions for synchronization is unclear. Therefore, it becomes essential to derive conditions on these coupled dynamical systems which lead to synchronization. Here, we aim to derive sufficient conditions for synchronization of selected benchmark oscillators which are linearly coupled. We have used Lyapunov approach to obtain sufficiency condition for synchronization of coupled oscillators. We study synchronization of Van der Pol oscillators and Fitzhugh Nagumo oscillators with all-to-all connectivity, for both uniformly and non-uniformly linearly coupled configurations. These results have been numerically simulated for both the above types of oscillators.

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Keywords: Van der Pol, Fitzhugh Nagumo(FHN), coupled oscillators, neuronal membrane potential v_i and recovery variable w_i.

1. INTRODUCTION

Synchronization of coupled oscillator dynamics is a fundamental problem which finds wide application in nature, science and engineering, Dorfler et.al (2014). It is therefore understanding the mutual interactions of coupled oscillators and to obtain consensus and phase locking among corresponding states of these oscillators which lead to their synchronization is a key challenge, Strogatz (2000). Thus, understanding how individual rhythms of oscillators adjust with each other such that the coupled oscillators oscillate with common frequency, is yet a problem which needs the attention of researchers. Further the synchronization of coupled oscillators become even more difficult task when it has to occur on multiple states. The synchronization of coupled oscillators therefore requires the necessary mathematical explanation using modern analytical approach.

The coupled dynamics of oscillators is described by ordinary differential equation which comprises of oscillator's dynamic state with an additional weak coupling term. In Kuramoto (1975), single state dynamical model has been considered and all to all coupling between oscillators is via single state. Where as in Bharath et.al (2013), multi state dynamical model of oscillators has been considered but the coupling is achieved via single state despite that the oscillators possessed the multiple states . The former approach helps to understand basic synchronization process in a simple dynamics, while the later one gives practical approach for dealing with problems of coupling in multistate oscillators.
As an example, to understand synchronization in coupled oscillators we consider the celebrated Kuramoto model, having n number of coupled oscillators.

$$\dot{\theta}_i = \omega_i + \frac{K}{n}\sum_{j=1}^{n} \sin(\theta_j - \theta_i) \qquad (1)$$

$$i \in 1, .., n$$

Where, θ_i is phase, ω_i is natural frequency and K is coupling gain of the oscillator.
Chopra et.al (2009) have demonstrated that Kuramoto model of oscillators, with all to all connectivity but different natural frequencies, locally exponentially synchronize. In Dorfler et.al (2011), the manner in which power network model is related to first order model of coupled oscillators has been explained. The authors have shown equivalence between swing equation of power network models and nonuniform Kuramoto model by using singular perturbation approach. In this work, pure algebraic conditions have been derived that relates Kuramoto model and transient stability of power networks under synchronization conditions. It is therefore, understanding synchronization phenomenon in second order coupled oscillators by analysing both the states is unclear and deserves to be investigated. Here, we aim to derive the sufficient condition on coupling gain K at which the multi state benchmark oscillators get synchronized. Specifically, using Lyapunov approach it is established that when the derivative of quadratic Lyapunov function of difference of states becomes negative definite, it marks synchronization of coupled oscillator dynamics. Finally, we have computed the critical coupling gain that is sufficient for synchronization of linearly cou-

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pled Van der Pol and Fitzhugh Nagumo oscillators. It is important to point out here that, the sufficiency conditions obtained using Lyapunov approach for the two models are analogous. For the case of coupled Fitzhugh Nagumo oscillators the sufficiency condition also matches with that obtained using partial contraction analysis by Wang et.al (2004), in a similar model of oscillator. These results should help to develop a framework for synchronization of second order coupled oscillators.

In section 2, we describe selected benchmark oscillators. In section 3, we give the formal definition of synchronization of coupled oscillators. In section 4 we calculate sufficient coupling gain K for synchronization of linearly coupled oscillators using Lyapunov approach. In section 5, the results are validated through simulations and summarized in section 6.

2. BENCHMARK OSCILLATOR MODELS

To illustrate the synchronization of coupled oscillators, we reproduce in this section two benchmark systems.

2.1 Van der Pol oscillator

Van der Pol oscillator is a second order model of oscillator which find large applications in modelling oscillation in electronics and in real world problems, Strogatz (1994). The dynamical equation of Van der Pol oscillator can be given as,

$$\dot{x_i} = \mu(x_i - y_i - \frac{1}{3}x_i^3) \tag{2}$$

$$\dot{y_i} = \frac{1}{\mu}x_i \tag{3}$$

Here, x_i and y_i are state variables and μ is a parameter. The oscillatory response of Van der Pol oscillator for $\mu = 1$ with initial conditions $(-1, 2)$ is shown in Fig.1.

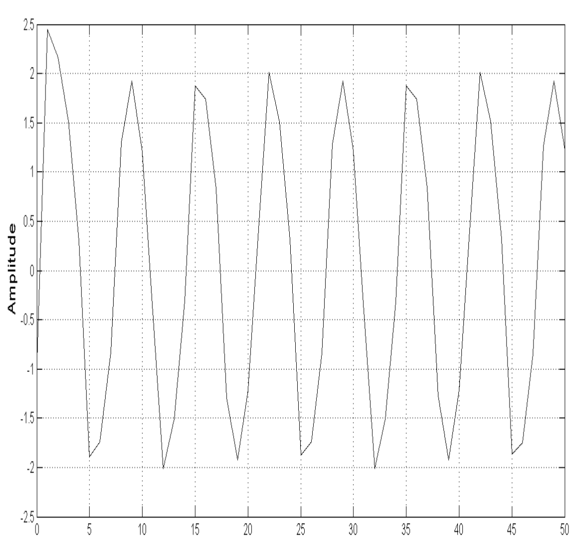

Fig. 1. Oscillations in Van der Pol oscillator.

2.2 Fitzhugh Nagumo neural oscillator

Fitzhugh Nagumo neural oscillator is a well known computational model of a biological neuron, Izhikevich (2004). It is also a second order model. Its dynamic equation is given by,

$$\dot{v_i} = c[v_i - w_i - \frac{1}{3}v_i^3] + I \tag{4}$$

$$\dot{w_i} = d[a + bv_i - w_i] \tag{5}$$

Here, v_i represents the membrane potential, w_i is the called as recovery variable which represents the channel dynamics, $I = 2$ is the excitation current and $a = 2, b = 1, c = 1$ and $d = 0.1$ are the constants. The oscillatory response of Fitzhugh Nagumo oscillator with initial conditions (-1,2) is shown in Fig.2.

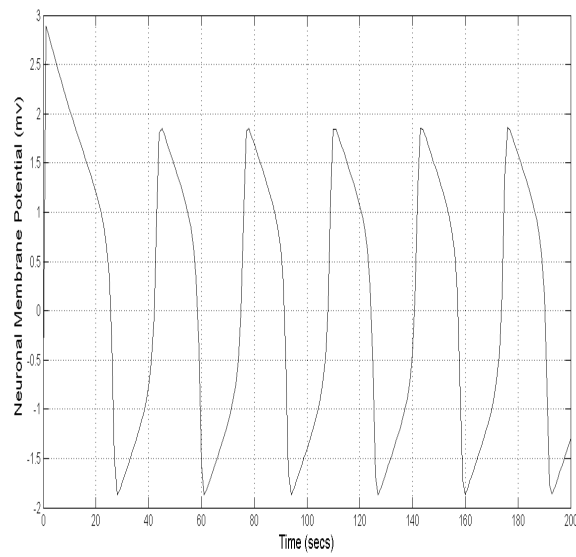

Fig. 2. Oscillations in Fitzhugh Nagumo oscillator

2.3 Linear coupling

The dynamical equation for N coupled Van der Pol oscillator be given as,

$$\dot{x_i} = \mu(x_i - y_i - \frac{1}{3}x_i^3) + \mathrm{K}\sum_{p=1}^{N}(x_p - x_i) \tag{6}$$

$$\dot{y_i} = \frac{1}{\mu}x_i, \qquad \forall \text{ i, j} = (1, \ldots, \text{N}) \tag{7}$$

The dynamical equation for N coupled Fitzhugh Nagumo oscillator is given by,

$$\dot{v_i} = c[v_i - w_i - \frac{1}{3}v_i^3] + \mathrm{I} + \mathrm{K}\sum_{s=1}^{N}(v_s - v_i) \tag{8}$$

$$\dot{w_i} = d[a + bv_i - w_i], \qquad \forall \text{ i, j} = (1, \ldots, \text{N}) \tag{9}$$

3. DEFINITION OF SYNCHRONIZATION

3.1 Definition 1

A set of coupled oscillators are said to be synchronized if the difference between corresponding states become constant asymptotically.

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