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IFAC-PapersOnLine 49-1 (2016) 379–384

Posture Stabilization of Unicycle Mobile Robot using Finite Time Control Techniques

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Abstract: In this paper, a finite time control strategy is proposed for the posture stabilization of a unicycle type mobile robot. The underlying bilinear structure of the chained system of the unicycle is exploited to develop the control strategy which involves switching between two continuous finite time controllers. The chained system is analysed as two subsystems. A homogeneity based finite time controller is used to stabilize the first subsystem in finite time and thereafter the Super Twisting Algorithm is employed for the finite time stabilization of the second subsystem. Simulations are performed for the proposed scheme and the mathematical results are validated.

Keywords: Unicycle, Homogeneity, Super Twisting Algorithm (STA).

1. INTRODUCTION

Over the past few years wheeled mobile robots (WMRs) have become increasingly important in a wide variety of applications such as transportation, security, inspection, planetary exploration, etc. WMRs are increasingly present in industrial and service robotics, particularly when specific motion capabilities are required. The control of WMRs has been, and still is, the subject of numerous research studies. In particular, the nonholonomic constraints associated with these systems have motivated the development of nonlinear control techniques. Beyond the relevance in applications, the problem of autonomous motion planning and control of WMRs has attracted the interest of researchers in view of its theoretical challenges. In the absence of workspace obstacles, the basic motion tasks assigned to a WMR could be reduced to regulation to a point, trajectory tracking and path following. In the point or posture (orientation and position) regulation problem, the objective is to bring the robot to a desired goal posture starting from an arbitrary initial posture.

From a control viewpoint, the peculiar nature of nonholonomic kinematics makes the problem of posture stabilization challenging; in particular, feedback stabilization at a given posture cannot be achieved via differentiable or even continuous pure state feedback control [Brockett et al., 1983]. A number of approaches have been proposed for the problem, which can be classified as (i) discontinuous time-invariant stabilization [Astolfi, 1996], (ii) time-varying stabilization [Samson, 1993], [Pomet, 1992] and (iii) hybrid stabilization [Sørdalen and Egeland, 1995].

A common approach for controller design of nonholonomic systems is to convert, with appropriate state and input transformations, the original system into a canonical form for which controller can be designed [Ge et al., 2003]. One such canonical form is the *chained form*. Using the special algebraic structure of the chained form, various feedback strategies have been proposed to stabilize nonholonomic systems. A discontinuous time invariant control law has been used to yield exponential stabilization of nonholonomic chained systems [Astolfi, 1996]. Sliding mode control (SMC) has been employed as a discontinuous control strategy for stabilizing the origin for a class of nonlinear driftless systems of the form $\dot{x} = B(x)u$ [Bloch and Drakunov, 1996]. Taking into account the robustness properties of the SMC, it has been used to stabilize the nonholonomic system in the event of perturbations [Floquet et al., 2000]. These control laws guarantee asymptotic convergence.

In practice, finite time stabilization is an important requirement. A finite time sliding mode controller based on homogeneity has been designed to drive the system to zero dynamics in known finite time [Defoort and Djemaï, 2010]. Robust stabilization has been achieved through two different sliding mode control strategies in [Floquet et al., 2003] with one of them providing a finite time convergence. Finite time stabilization under input disturbances has been presented in [Guerra et al., 2014]. Certain outputs of the system have been robustly stabilized in the uniform output stability (uOS) sense.

In [Samson, 1995], the underlying linear structure of the nonlinear chained system has been established. The two input nonlinear system in the chained form has been converted into a single input linear time varying system by setting one of the inputs as a time-varying function. A methodology has been suggested for the finite time stabilization of the nonholonomic chained system through the suitable selection of control inputs. This methodology has been exploited to design a finite time controller for a class of WMRs [Zhu et al., 2006]. The potential linear

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.03.083

structure of the chained form system together with the finite time convergence property of the terminal sliding mode control method has been adopted for a tractor-trailer system [Binazadeh and Shafiei, 2014]. This paper presents a homogeneous finite time controller and the Super Twisting Algorithm (STA) for the finite time stabilization of a unicycle like WMR, a representative of the family of nonholonomic systems.

In this paper, a control law is proposed to stabilize the posture vector $\zeta = (x(t), y(t), \theta(t))$ of a unicycle at the origin. The main contributions are summarized as follows:

- The underlying linear structure of the chained form is exploited to design a finite time control law which involves switching between two continuous controllers.
- The controllers are derived using a homogeneous finite time control technique and the STA respectively.
- In the proposed strategy, an analytical expression for the explicit time of switching that corresponds to the settling time estimate of the homogeneity based finite time controller is obtained.

This paper is organised as follows. Section 2 describes the problem addressed in the work and Section 3 analyses the chained form of a nonholonomic system. In Section 4, a stabilizing control law is developed for the finite time stabilization of the chained form of a unicycle system. Section 5 validates the results obtained from the mathematical formulations and Section 6 concludes the paper.

2. PROBLEM STATEMENT

The paper discusses the problem of stabilizing a fixed posture of a mobile robot at the origin which refers to the problem of stabilization of equilibria of controllable driftless systems with lesser number of control inputs than state variables. The mobile robot is modelled as a unicycle with the kinematics given as:

$$\dot{x}(t) = v(t)\cos\theta(t) \quad (1)$$
$$\dot{y}(t) = v(t)\sin\theta(t) \quad (2)$$
$$\dot{\theta}(t) = \omega(t) \quad (3)$$

where $(x(t), y(t))$ are the instantaneous position coordinates of the mobile robot, $\theta(t)$ is the heading angle of the robot, $v(t)$ and $\omega(t)$ are the linear and angular velocities of the robot respectively. Fig. 1 shows the instantaneous posture of the robot with respect to the final posture. The final posture is assumed to be the origin without loss of generality. $R(x, y, \theta)$ is the instantaneous robot posture while $O(0, 0, 0)$ is the final posture.

3. ANALYSIS OF CHAINED SYSTEM

Consider the single chained system

$$\left.\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= u_1 x_3 \\ &\vdots \\ \dot{x}_{n-1} &= u_1 x_n \\ \dot{x}_n &= u_2 \end{aligned}\right\} \quad (4)$$

where $x = (x_1, x_2, \cdots, x_n)^T \in R^n$ denotes the state vector with $n \geq 3$ and $u = (u_1, u_2)^T \in R^2$ denotes the input vector. The kinematic model of the unicycle vehicle ((1)-(3)) can be transformed into the chained form

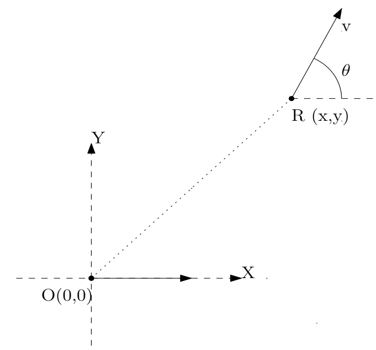

Fig. 1. Instantaneous posture of the robot with respect to the final posture (origin)

$$\left.\begin{aligned} \dot{x_1} &= u_1 \\ \dot{x_2} &= u_1 x_3 \\ \dot{x_3} &= u_2 \end{aligned}\right\} \quad (5)$$

by applying the coordinate and feedback transformation as in (6) and (7) respectively.

$$x_1 = x, \quad x_2 = y, \quad x_3 = tan\theta \quad (6)$$

$$v_1 = u_1 sec\theta, \quad v_2 = u_2 cos^2\theta \quad (7)$$

Due to the accessibility rank condition [Ravi N. Banavar, 2006], the chained system is a controllable system. The underlying bilinear structure of the chained system is obtained as follows. Denoting $y_1 = x_1$ and $y_2 = (x_2, x_3, \cdots, x_n)^T$, we get

$$\dot{y}_1 = u_1 \quad (8)$$
$$\dot{y}_2 = A(u_1)y_2 + Bu_2 \quad (9)$$

where

$$A(u_1) = \begin{bmatrix} 0 & u_1(t) & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ \cdots & \cdots & \cdots & u_1(t) \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

For brevity, (8) and (9) are treated as two subsystems denoted by Σ_1 and Σ_2 respectively. The Σ_1 subsystem is a first order linear system controlled by u_1 only whereas the Σ_2 subsystem is of $(n-1)^{th}$ order and is controlled by u_1 and u_2 simultaneously. Treating u_1 as a time-varying function rather than as a control variable, the subsystem Σ_2 can be viewed as a linear time-varying system with its controllability determined by function u_1.

The control variable u_1 causes coupling between the two subsystems Σ_1 and Σ_2. To remove this coupling, the following control scheme has been suggested in [Zhu et al., 2006]:

$$u_2(t) = \begin{cases} u_{21}(t) & ,t < T \\ 0 & ,t \geq T \end{cases} \quad (10)$$

where T is a finite positive constant. This is under the assumption that the control law $u_{21}(t)$ stabilizes the Σ_2 subsystem to its equilibrium $y_2 = 0$ in time T. When $t \geq T$, $\dot{y_2} \equiv 0$ from (9) and hence the state of the subsystem Σ_2 will remain at its equilibrium $y_2 = 0$ despite the influence of $u_1(t)$. Thus decoupling is achieved.

The essence of this control scheme lies in the finite time stabilization of the subsystem Σ_2. Initially $u_1(t)$ is chosen