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A Cooperative Target-centric Formation with Bounded Acceleration

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Abstract: In this paper, we propose a consensus based multi-agent distributive controller to form and maintain a target-centric formation. It is assumed that only a subset of agents has target information and agents are subject to limited accelerations. We show that if there is at least one agent which has complete target information and the corresponding communication graph is weakly connected, then a target-centric formation can be maintained around the target. Lyapunov analysis is done to show the stability of the formation under the proposed control law. The performance of the proposed controller is demonstrated through numerical simulations.

Keywords: Formation control, Consensus, Target-centric formation, Double integrator, Multi-agent systems.

1. INTRODUCTION

Multi-agent systems (MAS) is getting interest in recent years because of its advantageous applications over a single agent system. Weiss and Sen (1996) discussed that the MAS improvise the scalability, robustness, and parallelism of a system. The MAS is successfully applied to many defence and civilian application such as surveillance, search and rescue, fire monitoring.

Distributed control for the multi-agent system like unmanned aerial vehicles (UAVs) and the applications of the cooperative control in consensus, flocking/ rendezvous, and formation control are gaining importance now a days. Discussion on consensus were found in Olfati-Saber et al. (2007), Wen et al. (2014), Zhou et al. (2015). Their works considered first order dynamics for consensus. Consensus with double integrator (Ren and Atkins, 2007), (Cao and Ren, 2011) were also reported in literature. Abdessameud and Tayebi (2013) proposed a consensus based algorithm with bounded input for the agents with double integrator dynamics and the agents were tracking a desired reference velocity where the reference velocity and the reference acceleration were known to all agents. Ren (2008) proposed a consensus based controller with bounded input to track a desired trajectory which is known to a subset of agents. Applications in consensus were found in flocking (Zhu et al., 2013), rendezvous (Sahoo et al., 2013), attitude synchronization (Min et al., 2012).

Kawakami and Namerikawa (2009) proposed a consensus based target-centric formation controller based on dynamic network topology. Kothari et al. (2013) designed a consensus based target-centric formation controller for multiple UAVs. They have provided double integrator consensus algorithm to obtain desired formation. However, according to their control law the acceleration becomes very large when the position and/or velocity error is very large, which is not desirable for the practical purpose.

There are several strategies for making a desired formation among the multiple agents. Among them behavior-based approach (Balch and Arkin, 1998), potential function approach , virtual structure approach (Ren and Beard, 2004), and leader-follower strategy (Tanner et al., 2004), (Consolini et al., 2008) are very popular. Chen and Wang (2005) discussed several formation control strategies briefly. A particular behavior is assigned to the agents in case of behavior-based approach (Monteiro and Bicho, 2002). The controller is designed in such a way that the agents can follow the pre-assigned behavior (such as 'orientation'). In the case of potential function technique (Hengster-Movric et al., 2010), the formation pattern is represented by a mathematical potential function. The minimum potential point denotes the desired formation. Leader-follower approach (Shao et al., 2005) consists of two types of agents, leaders and followers. A subset of agents in the MAS group act as leaders and the remaining are followers. In this approach, followers are trying to follow their respective leaders. As there is no information flow from the follower to the leader, if any leader deviates from the desired trajectory, the desired formation can be hampered. In the case of virtual control approach (Lalish et al., 2006), it is assumed that all agents are connected by a virtual link. Each agent tries to track their self-generated reference trajectory to maintain the virtual geometry. As the tracking of the self generating trajectory is done in a centralized manner, the approach is not suitable in the case of distributed control design.

Our work is motivated from Kothari et al. (2013) and Ren (2008). We propose a cooperative target-centric formation

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controller with bounded acceleration effort. We assume that the communication graph topology within the agents has a directed spanning tree and at least one agent in the group has the complete target information (position, velocity and acceleration). The proposed controller provides a bounded acceleration effort. The acceleration effort of the agents are bounded around the acceleration of the target.

The rest of the paper is organized as follows. In Section 2, we give a basic preliminary of the graph theory and Section 3 consists of the problem formulation. In Section 4, we derive a consensus based target-centric formation controller. Instantaneous bounds on acceleration of the agents is studied and stability analysis is done using Lyapunov approach. Simulation results are discussed in Section 5. Summary and future scope of the present work are discussed in Section 6.

2. PRELIMINARIES

In a multi-agent system (MAS), each agent can be represented as a node. The communication link existing between any two agents can be represented by an edge. The communication topology among n agents in a system is represented by a digraph $\mathcal{G}_n = (\mathcal{V}_n, \mathcal{E}_n)$, where $\mathcal{V}_n$ is the node set and $\mathcal{E}_n$ is the edge set. The directed edge $\mathfrak{e}_{j,i} \in \mathcal{E}_n(\mathcal{G}_n)$ exists when agent i is receiving information from agent j. A node i is said to be globally reachable when there exists a directed path from all other nodes. Digraph $\mathcal{G}_n$ is strongly connected if each node of $\mathcal{G}_n$ is globally reachable. A digraph $\mathcal{G}_n$ is said to be weakly connected when there exists a rooted directed spanning tree (see, (Ren et al., 2007)) in the digraph $\mathcal{G}_n$. The interactions in an n-agent system is also represented by the adjacency matrix $\mathcal{A}_n = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} = 1$ if $\mathfrak{e}_{j,i} \in \mathcal{E}_n$ and $a_{ij} = 0$ otherwise. The Laplacian $\mathcal{L}_n = [l_{ij}] \in \mathbb{R}^{n\times n}$ of $\mathcal{G}_n$ is defined as

$$l_{ij} = \begin{cases} \sum_{\substack{j=1 \\ j\neq i}}^{n} a_{ij}, & \text{for } i = j \\ -a_{ij}, & \text{otherwise} \end{cases}$$

$\mathcal{L}_n$ has a simple eigenvalue 0 if the graph $\mathcal{G}_n$ has a directed spanning tree.

3. PROBLEM FORMULATION

We consider a multi-agent system(MAS) with n agents whose objective is to encircle a maneuvering target in a desired manner. Each agent is considered to be an unmanned aerial vehicle (UAV) flying at a constant altitude. The position of agent i is given by $r_i = [x_i \ y_i]^T \in \mathbb{R}^2$. The kinematics of each agent is given by

$$\begin{aligned} \dot{r}_i &= [\dot{x}_i \ \dot{y}_i] = [v_i \cos(\psi_i) \ v_i \sin(\psi_i)]^T, \\ \dot{\psi}_i &= \omega_i \\ \dot{v}_i &= a_i \end{aligned} \tag{1}$$

Therefore,

$$\ddot{r}_i = \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \end{bmatrix} = M_i u_i = \begin{bmatrix} \cos(\psi_i) & -v_i \sin(\psi_i) \\ \sin(\psi_i) & v_i \cos(\psi_i) \end{bmatrix} \begin{bmatrix} a_i \\ \omega_i \end{bmatrix} \tag{2}$$

where, v_i is the speed of the i^{th} agent, ψ_i is the heading angle of the i^{th} agent and, $a_i = \dot{v}_i$ and $\omega_i = \dot{\psi}_i$ are the linear acceleration and angular speed of the i^{th} agent, respectively.

To make a target-centric formation, the group of agents have to maintain a constant distance ρ from the target and a constant angular separation of $2\pi/n$ from each other. Thus, in a target-centric frame the desired relative position of the i^{th} agent with respect to the target is $R_i = \rho[\cos(\alpha_i) \sin(\alpha_i)]^T$ where $\alpha_i = \frac{2\pi i}{n}$. To maintain the formation the following objectives have to be satisfied for $i = 1, \cdots, n$.

$$\begin{aligned} r_i - r_T &= R_i \\ \dot{r}_i &= \dot{r}_T, \end{aligned}$$

where $r_T = [x_T, \ y_T]^T$ and $\dot{r}_T = [\dot{x}_T, \ \dot{y}_T]^T$ are the position and velocity vectors of the target T in the inertial frame, respectively.

In this work, we assume that a subset of the agents have the target information. The underlying communication topology of this system with the target is represented by a digraph $\mathcal{G}_{n+1}$. The target is labelled as $(n+1)^{th}$ agent. An edge $\mathfrak{e}_{n+1,i}$ exists if agent i has the information about the target T. The target, however, does not receive any information about the agents. Hence, $a_{i,n+1} = 1$ if $\mathfrak{e}_{n+1,i}$ is in the edge set of the digraph $\mathcal{G}_{n+1}$. The Laplacian $\mathcal{L}_{n+1}$ for $\mathcal{G}_{n+1}$ is

$$\mathcal{L}_{n+1} = \begin{bmatrix} \mathcal{L}_n + \mathcal{B} & -b \\ 0_{1\times n} & 0 \end{bmatrix}, \tag{3}$$

where $b = [a_{1,n+1}, a_{2,n+1}, \cdots, a_{n,n+1}]^T$ and $\mathcal{B} = \text{diag}(b) \in \mathbb{R}^{n\times n}$.

Lemma 1. (Lemma 1,(Kothari et al., 2013)). Rank of matrix $\mathcal{L}_n + \mathcal{B}$ is n if and only if $\mathcal{G}_{n+1}$ has a directed spanning tree.

Proof. For proof, please refer to Kothari et al. (2013). □

4. TARGET CENTRIC FORMATION CONTROL

Let $\hat{r}_i = r_i - R_i$ for $i = 1, 2, \cdots, n$. We propose the following consensus protocol to maintain a target-centric formation

$$\begin{aligned} u_i = M_i^{-1} \frac{1}{(a_{i,n+1} + \sum_{j=1}^{n} a_{ij})} \Bigg[\Bigg(a_{i,n+1}\ddot{r}_T + \sum_{j=1}^{n} a_{ij}\ddot{r}_j \Bigg) \\ - K_i \tanh \Bigg\{ K_i \Bigg(a_{i,n+1}\hat{r}_{i,T} + \sum_{j=1}^{n} a_{ij}\hat{r}_{ij} \Bigg) \\ + \Bigg(a_{i,n+1}\dot{r}_{i,T} + \sum_{j=1}^{n} a_{ij}\dot{r}_{ij} \Bigg) \Bigg\} \Bigg], \end{aligned} \tag{4}$$

where, $\hat{r}_{ab} = \hat{r}_a - \hat{r}_b$, $\dot{r}_{ab} = \dot{r}_a - \dot{r}_b$, and $0 < K_i < \infty$. Next we show that under this consensus protocol, a target-centric formation is achieved and the acceleration of the agents are bounded around the target's acceleration.

Theorem 1. Consider a system of n agents and one target with its communication digraph $\mathcal{G}_{n+1}$. Each agent and target have the kinematics given by (2). If the graph $\mathcal{G}_{n+1}$ has a directed spanning tree and at least one of the