

ScienceDirect

IFAC-PapersOnLine 49-1 (2016) 516–521

Controllability study on fractional order Controllability study on fractional order impulsive stochastic differential equation impulsive stochastic differential equation impulsive stochastic differential equation Controllability study on fractional order
 $\dot{\mathbf{r}}$ α is the fractional order of α is the fractional order of α Controllability study on fractional order

B. Ganesh Priya ∗∗, P. Muthukumar ∗∗∗. B. Ganesh Priya ∗∗, P. Muthukumar ∗∗∗.

Department of Mathematics, The Gandhigram Rural Institute-Deemed Department of Mathematics, The Gandhigram Rural Institute-Deemed Department of Mathematics, The Gandhigram Rural Institute-Deemed University, Gandhigram-624 302., Tamil Nadu, India. e-mail:primath85@gmail.com ∗∗∗ e-mail: pmuthukumargri@gmail.com. ∗∗∗ e-mail: pmuthukumargri@gmail.com. ∗∗∗ e-mail: pmuthukumargri@gmail.com. ∗∗ e-mail:primath85@gmail.com ∗∗ e-mail:primath85@gmail.com $*•$ e-mail:primath85@gmail.com *** e-mail:primath85@gmail.com ∗∗∗ e-mail: pmuthukumargri@gmail.com.

∗∗∗ e-mail: pmuthukumargri@gmail.com.

inpulsive conditions. The necessary and sufficient conditions for the controllability of associated linear stochastic system is studied by using the controllability Grammian matrix defined by impulsive conditions. The necessary and sufficient conditions for the controllability of associated
linear stochastic system is studied by using the controllability Grammian matrix defined by
Mittag-Leffler function. The s system is proved by using Banach fixed point theorem. An example is provided to illustrate the system is proved by using Banach medipole in \mathbf{B}_{max} wavelet approximation method theory using the numerical integration by Haar wavelet approximation method. Abstract: In this paper, we consider a fractional order stochastic differential system with Abstract: In this paper, we consider a fractional order stochastic differential system with

 \circ 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.

Keywords: Complete Controllability: Stochastic system: Impulsive fractional differential system; Mittag-Leffler function; Laplace transform; Numerical integration. system; Mittag-Leffler function; Laplace transform; Numerical integration. *Keywords:* Complete Controllability; Stochastic system; Impulsive fractional differential system; Mittag-Leffler function; Laplace transform; Numerical integration. system; Mittag-Leffler function; Laplace transform; Numerical integration.

1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

Many dynamic processes are characterized by the fact Many dynamic processes are characterized by the fact Many dynamic processes are characterized by the fact that at certain moments of time they experience sudden that at certain moments of time they experience sudden changes of state. These changes may seem instantaneous changes of state. These changes may seem instantaneous because the durations of these changes are negligible in
summarize with the duration of the whole was seen Them. comparison with the duration of the whole process. Therefore, it is natural to assume that these changes are in the fore, it is natural to assume that these changes are in the form of impulses. Dynamic systems subject to impulsive form of impulses. Dynamic systems subject to impulsive effects are defined as impulsive systems. It is known, for ex-effects are defined as impulsive systems. It is known, for example, that many biological phenomena involving thresh-ample, that many biological phenomena involving thresholds, bursting rhythm models in medicine and biology, olds, bursting rhythm models in medicine and biology, optimal control models in economics, do exhibit impul-
site effects are Lalmiltontham (1008) and Kanthiltonen sive effects, see Lakmikantham (1998), and Karthikeyan (2011). (2011). sive effects, see Lakmikantham (1998), and Karthikeyan sive effects, see Lakmikantham (1998), and Karthikeyan $(2011).$ changes of state. These changes may seem instantaneous optimal control models in economics, do exhibit impul-optimal control models in economics, do exhibit impul-olds, bursting rhythm models in medicine and biology, olds, bursting rhythm models in medicine and biology, Many dynamic processes are characterized by the fact that at certain moments of time they experience sudden fore, it is natural to assume that these changes are in the form of impulses. Dynamic systems subject to impulsive
form of impulses. Dynamic systems subject to impulsive effects are defined as impulsive systems. It is known, for example, that many biological phenomena involving thresh-

An increasing interest in issues related to fractional dy-An increasing interest in issues related to fractional dy-An increasing interest in issues related to fractional dynamical systems oriented towards the field of control namical systems oriented towards the field of control theory can be seen from the literature, for instance, theory can be seen from the literature, for instance, see Nadeem (2014). Stochastic differential equations have see Nadeem (2014). Stochastic differential equations have many applications in economics, ecology and finance. In many applications in economics, ecology and finance. In recent years, the controllability problems for stochastic
differential equations have become a field of increasing differential equations have become a field of increasing interest, (see Karthikeyan (2009) and references therein). interest, (see Karthikeyan (2009) and references therein). merest, (see Kartinkeyan (2009) and references therein).
The extensions of deterministic controllability concepts to stochastic control fractional systems have been discussed stochastic control fractional systems have been discussed only in a limited number of publications, see Nadeem only in a limited number of publications, see Nadeem (2014). The question of why would we need impulsive (2014). The question of why would we need impulsive control may arise. In some cases, impulsive controls are control may arise. In some cases, impulsive controls are preferred over continuous controls.The important consid-preferred over continuous controls.The important consideration is that impulsive control could be more practical eration is that impulsive control could be more practical and cheaper than continuous control. For example, in and cheaper than continuous control. For example, in spacecraft formation control problems, see Lakmikantham spacecraft formation control problems, see Lakmikantham (1998). (1998). many applications in economics, ecology and finance. In An increasing interest in issues related to fractional dynamical systems oriented towards the field of control theory can be seen from the literature, for instance, see Nadeem (2014). Stochastic differential equations have interest, (see Karthikeyan (2009) and references therein). The extensions of deterministic controllability concepts to The extensions of deterministic controllability concepts to
stochastic control fractional systems have been discussed only in a limited number of publications, see Nadeem (2014) . The question of why would we need impulsive (2014) . The question of why would we need impulsive control may arise. In some cases, impulsive controls are preferred over continuous controls.The important consideration is that impulsive control could be more practical and cheaper than continuous control. For example, in (1998). spacecraft formation control problems, see Lakmikantham spacecraft formation control problems, see Lakmikantham (1998). (1998).

In this paper, we consider the controllability of nonlinear In this paper, we consider the controllability of nonlinear In this paper, we consider the controllability of nonlinear In this paper, we consider the controllability of nonlinear In this paper, we consider the controllability of nonlinear fractional stochastic differential systems with impulses as fractional stochastic differential systems with impulses as fractional stochastic differential systems with impulses as follows: follows: fractional stochastic differential systems with impulses as α = CDax(t) + α (t) α + α (t) follows: follows:

$$
{}^{CD\alpha}x(t) = Ax(t) + Gu(t) + f(t, x(t))
$$

+ $\sigma(t, x(t)) \frac{dw(t)}{dt}, t = [t_0, T] \setminus \{t_1, t_2, ..., t_\rho\}$
 $\Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k)), k = 1, 2, ..., \rho,$
 $x(t_0) = x_0,$ (1)

where ${}^C D^{\alpha} x(t)$ denotes an α order Caputo's fractional where *D* $x(t)$ denotes an α order Caputo's mathonial
derivative of $x(t), 0 < \alpha < 1$, A and G are the known derivative or $x(t)$, $0 < \alpha < 1$, A and G are the known constant matrices and satisfy $A \in \mathbb{R}^{n \times n}$ and $G \in \mathbb{R}^{n \times m}$, constant matrices and satisfy $A \in \mathbb{R}^{m}$ and $G \in \mathbb{R}^{m}$,
 $x \in \mathbb{R}^{n}$ is the state variable, $u \in \mathbb{R}^{m}$ is the control input. $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^n$ is the control input.
w(t) is a given l− dimensional Wiener process with the filtration \mathcal{F}_t generated by $w(s)$, $0 \leq s \leq t$ and $f : [t_0, T] \times$ $\mathbb{R}^n \to \mathbb{R}^n, \sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times l}$ are appropriate $\mathbb{R}^n \to \mathbb{R}^n, \sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times l}$ are appropriate
continuous functions. $I_k : \mathbb{R}^n \to \mathbb{R}^n$ is continuous for continuous runction $w(t)$ is a given $t-$ dimensional wiener process with the
filtration \mathcal{F}_t generated by $w(s)$, $0 \le s \le t$ and $f : [t_0, T] \times$ mitration \mathcal{F}_t generated by $w(s), 0 \leq s \leq t$ and $J : [t_0, 1] \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ are appropriate $\mathbb{R}^n \to \mathbb{R}^n, \sigma : [t_0, 1] \times \mathbb{R}^n \to \mathbb{R}^m$ are appropriate
continuous functions $I_k : \mathbb{R}^n \to \mathbb{R}^n$ is continuous for $k = 1, 2, \ldots, \rho$, and where ${}^{C}D^{\alpha}x(t)$ denotes an α order Caputo's fractional constant matrices and satisfy $A \in \mathbb{R}^{n \times n}$ and $G \in \mathbb{R}^{n \times m}$,
 $x \in \mathbb{R}^n$ is the state variable, $y \in \mathbb{R}^m$ is the control input. constant matrices and satisfy $A \in \mathbb{R}^{n \times n}$ and $G \in \mathbb{R}^{n \times m}$,
 $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ is the control input. $w(t)$ is a given $l-$ dimensional Wiener process with the
filtration $\mathcal F$ generated by $w(s)$, $0 \le s \le t$ and $f \cdot [t,T] \times$ filtration \mathcal{F}_t generated by $w(s), 0 \leq s \leq t$ and $f : [t_0, T] \times \mathbb{R}^n \longrightarrow \mathbb{R}^n \longrightarrow \mathbb{R}^n$ $\mathbb{R}^n \to \mathbb{R}^n, \sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times \iota}$ are appropriate
continuous functions $I_{\iota} : \mathbb{R}^n \to \mathbb{R}^n$ is continuous for (1) $w(t)$ is a given $l-$ dimensional Wiener process with the filtration \mathcal{F}_c generated by $w(c)$ 0 $\leq c \leq t$ and $f \cdot [t, T] \times$ filtration \mathcal{F}_t generated by $w(s), 0 \leq s \leq t$ and $f : [t_0, T] \times \mathbb{R}^n \longrightarrow \mathbb{R}^n \times l$ are appropriate $\mathbb{R}^n \to \mathbb{R}^n, \sigma : [t_0, T] \times \mathbb{R}^n \to \mathbb{R}^{n \times l}$ are appropriate

$$
x(t_k^+) = \lim_{h \to 0^+} x(t_k + h), \quad x(t_k^-) = \lim_{h \to 0^-} x(t_k - h)
$$

represent the right and left limits of $x(t)$ at $t = t_k$ and the discontinuous points discontinuous points discontinuous points discontinuous points represent the right a represent the right and left limits of $x(t)$ at $t = t_k$ and the

$$
t_0 < t_1 < t_2 < \cdots < t_{\rho} < t_{\rho+1} = T
$$

and $x(t_k) = x(t_k^-)$ which implies that the solution of system (1) is left continuous at t_k . and $x(t_k) = x(t_k)$ which implies
existen (1) is left continuous at t. $x_0(x_1, x_2)$ \cdots $x_p(x_{p+1})$ = and $x(t_k) = x(t_k^-)$ which implies that the solution of system (1) is left continuous at t

system (1) is left continuous at i_k .
This article is organized as follows: In section 2, provides some preliminaries of fractional calculus, Laplace transform and the solution of the linear fractional stochastic form and the solution of the linear fractional stochastic system. Section 3 gives the existence and uniqueness for system. Section 3 gives the existence and uniqueness for the solution of the linear and nonlinear impulsive fractional stackers with the Euclidean materials the compulsive tional stochastic system. Finally we provide two examples to demonstrate the effectiveness of our method. to demonstrate the effectiveness of our method. This article is organized as follows: In section 2, provides form and the solution of the linear fractional stochastic
system. Section 3 gives the existence and uniqueness for
the solution of the linear and poplinear immulsive free form and the solution of the linear fractional stochastic to demonstrate the effectiveness of our method. to demonstrate the effectiveness of our method.

2. PRELIMINARIES 2. PRELIMINARIES 2. PRELIMINARIES 2. PRELIMINARIES 2. PRELIMINARIES

In this section, we first recall some basic definitions of In this section, we first recall some basic definitions of In this section, we first recall some basic definitions of In this section, we first recall some basic definitions of
fractional calculus, which are useful for this work. Through
out this paper, let Banach gpace $PC([t, T], \mathbb{P}^n) = [x, y]$ rractional calculus, which are useful for this work. I hrough
out this paper, let Banach space, $PC([t_0, T], \mathbb{R}^n) = \{x :$ In this section, we first recall some basic definitions of out this paper, let Banach space, $PC([t_0, T], \mathbb{R}^n) = \{x :$

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved. Peer review under responsibility of International Federation of Automatic Control. 10.1016/j.ifacol.2016.03.106

The work was supported by National Board for Higher Mathe- The work was supported by National Board for Higher Mathematics, Mumbai, India under the grant No: $2/48(5)/2013/NBHM$ (R.P.)/RD-II/688 dt 16.01.2014. (R.P.)/RD-II/688 dt 16.01.2014. (R.P.)/RD-II/688 dt 16.01.2014. matrices, Mumbai, Mumbai, India under the grant North No. 2/48(5), 2013/NBHM The work was supported by National Board for Higher Mathe- \star The work was supported by National Board for Higher Mathe-

 $[t_0, T] \rightarrow \mathbb{R}^n | x \in C((t_k, t_{k+1}]), k = 0, 1, \ldots, \rho, \}$ and there exist $x(t_k^-)$ and $x(t_k^+)$, for $k = 1, 2, \ldots, \rho$, with $x(t_k^-) = x(t_k)$ with norm $||x||_{PC} = \sup\{||x(t)|| : t \in$ J . Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t>0}$ satisfying the usual condition (i.e. right continuous and \mathcal{F}_0 containing all $\mathcal{P}-$ null sets). Let $\alpha > 0$, with $n - 1 < \alpha < n$. Let \mathbb{R}^m be the m −dimensional Euclidean space. Let C denote the Banach space $PC([t_0, T], L_2(\Omega, \mathcal{F}, \mathcal{P}))$ (see Karthikeyan (2009), Guendouzi (2013) for more details.)

Definition 1. (Caputo fractional derivative) Let $f \in$ $C[t_0,\infty)$. For $t \in [t_0,\infty)$, the Caputo fractional derivative ${}^{C}D^{\alpha}f(t)$ of order α is defined by

$$
{}^{C}D^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}
$$

$$
\times \int_{t_0}^{t} (t-s)^{n-\alpha-1} \left[\frac{d^n}{ds^n} f(s) \right] ds,
$$

where *n* is positive integer such that $n - 1 \leq \alpha \leq n$. Particularly, when $0 < \alpha < 1$, it holds

$$
{}^{C}D^{\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-s)^{-\alpha} f'(s)ds.
$$

Definition 2. (Mittag-Leffler function) For $z \in C$, the twoparameter Mittage Leffler function is defined as

$$
E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \quad (\alpha > 0, \beta > 0).
$$

For example,

$$
E_{\alpha,1}(z) = E_{\alpha}(z), \quad \text{here } \beta = 1,
$$

$$
\int_0^t E_{\alpha}(z^{\alpha}) dz = t E_{\alpha,2}(t^{\alpha}),
$$

Laplace transform of Mittag-Leffler function is

$$
L[t^{\alpha k + \beta - 1} E_{\alpha,\beta}^{(k)}(\pm at^{\alpha}); s] = \int_0^{\infty} e^{-st} t^{\alpha k + \beta - 1} E_{\alpha,\beta}^{(k)}(\pm at^{\alpha}) dt
$$

$$
= \frac{k! s^{\alpha - \beta}}{(s^{\alpha} \mp a)^{k+1}}, \quad (Re(s) > |a|^{\frac{1}{2}}),
$$

$$
(2)
$$

where $Re(s)$ denotes the real parts of s. In addition, Laplace transform of $t^{\alpha-1}$ is

$$
L[t^{\alpha - 1}; s] = \Gamma(\alpha)s^{-\alpha}, \quad \alpha > 0.
$$

2.1 Linear fractional stochastic system

Let us consider the linear fractional stochastic differential equation of the form

$$
{}^{C}D^{\alpha}x(t) = Ax(t) + \sigma(t)\frac{dw(t)}{dt} + f(t), t \in [t_0, T]
$$

$$
x(t_0) = x_0,
$$
 (3)

where $0 < \alpha \leq 1$, A is $n \times n$ matrix, $\sigma : [t_0, T] \to \mathbb{R}^{n \times l}$ is appropriate function and $f : [t_0, T] \to \mathbb{R}^n$ is continuous function.

Lemma 3. The solution $x(t)$ of the system (3) can be represented as

$$
x(t) = x(t_0) + \int_{t_0}^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha)
$$

$$
\times \left[Ax(t_0) + \left(\int_0^\eta \sigma(\theta) dw(\theta) \right) + f(s) \right] ds, t \in [0, T].
$$

Proof. Applying the idea used in [Zhou (2013)], we have the integral equation of the system (3),

$$
x(t) = x(t_0) + \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t - s)^{\alpha - 1}
$$

$$
\times \left[Ax(s) + \sigma(s) \frac{dw(s)}{ds} + f(s) \right] ds,
$$

since

$$
\int_{t_0}^t (t-s)^{\alpha-1} \left[Ax(s) + \sigma(s) \frac{dw(s)}{ds} + f(s) \right] ds
$$

= $t^{\alpha-1} \circ \left[Ax(t) + \sigma(t) \frac{dw(t)}{dt} + f(t) \right],$

where \circ is the convolution. The solution equation can be written as

$$
x(t) = x(t_0) + \frac{1}{\Gamma(\alpha)} t^{\alpha - 1} \circ \left[Ax(t) + \sigma(t) \frac{dw(t)}{dt} + f(t) \right].
$$

Applying the Laplace transform on both sides of above equation,

$$
X(s) = L[x(t_0); s] + \frac{1}{\Gamma(\alpha)} \Gamma(\alpha) s^{-\alpha} \cdot AX(s)
$$

$$
+ L\left[\sigma(t)\frac{dw(t)}{dt} + f(t); s\right]
$$

where $X(s)$ is the Laplace transform of $x(t)$, we have

$$
X(s) = (s^{\alpha}I - A)^{-1} s^{\alpha} L[x(t_0); s]
$$

+
$$
(s^{\alpha}I - A)^{-1} L\left[\sigma(t)\frac{dw(t)}{dt} + f(t); s\right]
$$

Inserting the formula for laplace transform for the Mittag-Leffler function (2), we have

$$
X(s) = L[x(t_0); s] + (t^{\alpha - 1} E_{\alpha, \alpha}(A(t^{\alpha})))
$$

$$
\cdot L\left[\sigma(t)\frac{dw(t)}{dt} + f(t); s\right]
$$

Applying the inverse Laplace transform on both sides (see Guendouzi (2013)) and convolution, we have

$$
x(t) = x(t_0) + \int_{t_0}^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(A(t-s)^\alpha)
$$

$$
\times \left[Ax(t_0) + \left(\int_{t_0}^\eta \sigma(\theta) dw(\theta) \right) + f(s) \right] ds, t \in [t_0, T].
$$

Thus, the proof is completed

Thus, the proof is completed.

3. CONTROLLABILITY CRITERIA FOR SYSTEM

In this section, we establish the sufficient and necessary conditions of controllability criteria for impulsive system.

3.1 Linear impulsive fractional stochastic system

Consider the linear impulsive fractional stochastic differential equation is of the form as:

$$
{}^{C}D^{\alpha}x(t) = Ax(t) + Gu(t) + \sigma(t)\frac{dw(t)}{dt},
$$

\n
$$
x(t_0) = x_0, \qquad t = [t_0, T] \setminus \{t_1, t_2, \dots, t_\rho\},
$$

\n
$$
\Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k)), \ k = 1, 2, \dots, \rho,
$$

\n(4)

 \mathbf{a}

where $0 < \alpha \leq 1$.

Definition 4. (Karthikeyan (2009)) The stochastic impulsive system (4) is said to be controllable on $[t_0, T]$ if, for Download English Version:

<https://daneshyari.com/en/article/708941>

Download Persian Version:

<https://daneshyari.com/article/708941>

[Daneshyari.com](https://daneshyari.com)