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A Novel hyperchaotic System with Stable and Unstable Line of Equilibria and Sigma Shaped Poincare Map

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Abstract: The objective of the paper is to develop a novel hyperchaotic system with a line of equilibria and sigma shaped Poincare map. The proposed hyperchaotic system has unstable and stable equilibria based on the values of the state variable. The system is relatively simpler than the reported hyperchaotic systems with a line of equilibria. The proposed system also depicts the chaotic and quasi-periodic behaviours for variation of two bifurcation parameters in parameter space. The proposed system is asymmetrical to its axes, plane and space. The system with above combination of features are rare in the literature and is the novelty of the paper. Bifurcation diagram, Lyapunov spectrum, frequency spectrum and hyperchaotic attractor of the system are presented to validate the different unique behaviours of the system.

Keywords: Hyperchaotic behaviour, many equilibrium attractor, hyperchaotic system, line of equilibria.

1. INTRODUCTION

Equilibrium point plays an important role in characterization of any dynamical system as discussed by Z. Wei (2015). Nature of the dynamical system depends on the behaviour of the equilibrium points. Descriptions of the hyperchaotic/chaotic systems are also presented based on the characteristics of the equilibrium points as given in Q. Li (2015a). Thus, analysing a hyperchaotic system based on its equilibrium points is a new problem in the literature. The motivation behind this paper is to develop a hyperchaotic system which has eight term including the lone nonlinear term but has an infinite number of equilibria along with several other unique features.

Many hyperchaotic systems are available based on the types of equilibrium points such as the systems given by V. T. Phama (2014), S. Vaidyanathan and V. T. Pham (2015), Z. Wei (2014b), etc. The usual type of hyperchaotic systems has self-excited attractors with countable number of unstable equilibria as given by C. Feng (2015), J. Fang (2014), Z. Wan (2014), etc. Some hyperchaotic systems with no equilibrium point are discussed in C. Li (2014), Z. Wei (2014a), etc. Q. Li (2015a) reported a hyperchaotic system with one stable equilibria. Z. Wan (2014) reported a hyperchaotic system with one unstable equilibrium point. Generally reported hyperchaotic systems have countable number of equilibria. But hyperchaotic system with an infinite number of equilibria or with many equilibria (also called line of equilibria) are very few. C. Li and Thi (2014) presented a hyperchaotic system with eight term which include five nonlinear terms, having line of equilibria. Q. Li and Zeng (2014) reported a hyperchaotic system with an infinite number of equilibria. The system of Q. Li and Zeng (2014) has two nonlinear terms and a total of nine terms. A hyperchaotic system with many stable equilibria in a 4-D memirister based circuit is reported by Q. Li (2015a). A Line of equilibria hyperchaotic system in 4-D memiristor circuit is reported by Q. Li (2015b). Thus, developing a line of equilibria hyperchaotic system with minimum number of terms including one nonlinear term is a challenging task.

In this paper, a novel 4-D hyperchaotic system with a line of equilibria (the infinite number of equilibria) is reported which is relatively simpler than available similar hyperchaotic systems. The system has many unique and interesting behaviours which is not available in other reported systems. The unique and interesting behaviours of the proposed new hyperchaotic systems are as follows. (i) The system has a line of equilibria, (ii) the equilibria are stable and unstable for different values of the state variable, (iii) the system has eight terms with only one nonlinear term and two bifurcation parameters, (iv) the rank of the Jacobean matrix of the system is less than four everywhere, so there exist extraneous equation through some transformation, (v) it has sigma shaped Poincare map, (vi) it has asymmetrical nature to its axes, plane, and space. The arbitrary combination of the above characteristics are not present in the available similar hyperchaotic systems. Above six points also reflect the novelty of the paper. Different tools are used to analyse the proposed hyperchaotic system. Dissipativity, asymmetrical and nature of equilibria of the proposed system are presented using theoretical analyses. The hyperchaotic attractors are presented using phase portrait and Poincare map. Bandwidths of the system are presented using normalised

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frequency spectra. Bifurcation diagrams of the system is shown for parameter variation.

Remaining part of the paper is organised as follows: Section 2 describes the dynamics of the proposed hyperchaotic system. Theoretical analyses of the proposed system are given in the Section 3. Section 4 describes the numerical analyses of the system. Section 5 presents the conclusions of the paper.

2. DYNAMICS OF THE NEW HYPERCHAOTIC SYSTEM WITH LINE OF EQUILIBRIA

In this section, dynamics of the new proposed hyperchaotic system is presented.
Any hyperchaotic system must be 4-D and it must have two positive Lyapunov exponents as given in Q. Li (2015b). Considering the above facts, a new hyperchaotic system is considered as:

$$\begin{aligned} \dot{x}_1 &= -x_2 - x_3 - a x_4 \\ \dot{x}_2 &= x_1 \\ \dot{x}_3 &= b(1 - x_2^2) - c x_3 \\ \dot{x}_4 &= d x_1 \end{aligned} \quad (1)$$

where $x_1,\ x_2,\ x_3,\ x_4$ are the state variables and $a,\ b,\ c,\ d$ are the positive parameters of the system. It may be noted that the system has only one nonlinear term in the third state equation and has eight terms. Here, a and d are chosen as bifurcation parameters and $b = 0.5,\ c = 0.6$ are considered as fixed parameters. The proposed system (1) is hyperchaotic at $a = 1.378,\ b = 0.5,\ c = 0.6,\ d = 0.097$ where Lyapunov exponents are $(0.0466,\ 0.0101,\ -0.00904,\ -0.6472)$ and Kalpman-York dimension (Lyapunov dimension) is $D_{KY} = 3.073$. More descriptions of the proposed hyperchaotic system are presented in the next section.

3. THEORETICAL DESCRIPTIONS OF THE PROPOSED HYPERCHAOTIC SYSTEM

This section is devoted to theoretical analyses of the system (1).

3.1 Equilibria

The proposed hyperchaotic attractor (1) has many equilibria (i.e. line of equilibria) because the equilibria are a function of the state variables. The equilibria and eigenvalues of the system (1) are given in Table 1 at $a = 1.378,\ b = 0.5,\ c = 0.6,\ d = 0.097$. The Jacobian matrix of the proposed system is given in (2).

$$J = \begin{bmatrix} 0 & -1 & -1 & -a \\ 1 & 0 & 0 & 0 \\ 0 & -2\mathrm{b}x_2 & -c & 0 \\ d & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

The Jacobian matrix is a function of x_2. Eigenvalues corresponding to $E1$ and $E2$ are calculated and tabulated in Table 1. Value of the different symbol used in eigenvalues are:

$$A = 0.234,\ B = 0.0386,\ K = \frac{26}{125},\ D = \frac{ad}{3},\ E = (D + \frac{22}{75}),$$

$$F = \frac{\sqrt{4-A}}{4},\ G = \sqrt[3]{F - B - K + \sqrt{(B - F + K)^2 + E^3}}$$

$$H = \sqrt[3]{-F - B - K + \sqrt{(B + F + K)^2 + E^3}} \quad (3)$$

It is clear from Table 1 that the system's line equilibria $E1$, and $E2$ are function of x_2, x_3, x_4 but the eigenvalues are function of x_2 only. Therefore, x_2 is varied from its maximum values to minimum values to observe the nature of the equilibria. The plot of $E1$ and $E2$ equilibria is shown in Fig. 1. It is to be noted that only real values of variable are visible in the plot. Numerical calculations suggest that eigenvalues are both stable and unstable between the minimum and maximum values of x_2. Thus, the system (1) has different behavior for different values of x_2.
The rank of the Jacobian matrix (2) is less than four everywhere, so there is an extraneous equation through some transformation. But the system has four different eigenvalues and four different Lyapunov exponents. This is also an unusual property of the proposed system.

3.2 Symmetrical and Dissipativity properties

The system has asymmetrical property about its axes, plane and space. Therefore, the system is not invarient under the coordinate transformation of state variables. This unusual property of the system (1) increases its complexity and disorderness. The system is a dissipative hyperchaotic system. The divergence of the vector field v validates the dissipative nature of the system as

$$\nabla v = -0.6 < 0 \quad (4)$$

Thus, there exists attractors for the system.

4. NUMERICAL DESCRIPTION

Different numerical analyses of the system are presented in this section.

Table 1. Equilibria and eigenvalues of the proposed system (1)

Equilibrium point	Eigenvalues
$E1\left(0,\ -0.44\sqrt{5-6x_2},\ x_3, -0.725x_4 + 0.324\sqrt{5-6x_4}\ \right)$	$\lambda_1 = 0$ $\lambda_2 = E - \left(\frac{0.3379}{E}\right) - \frac{1}{5}$, $\lambda_3 = \frac{0.168}{E} - \frac{E}{2} - \frac{\sqrt{3}}{2}\left(\frac{0.337}{E} + E\right)i - \frac{1}{5}$, $\lambda_4 = \frac{0.168}{E} - \frac{E}{2} + \frac{\sqrt{3}}{2}\left(\frac{0.337}{E} + E\right)i - \frac{1}{5}$
$E2\left(0,\ 0.44\sqrt{5-6x_2},\ x_3, -0.725x_4 - 0.324\sqrt{5-6x_4}\ \right)$	$\lambda_1 = 0$, $\lambda_2 = H - \left(\frac{0.337}{H}\right) - \frac{1}{5}$, $\lambda_3 = \frac{0.168}{H} - \frac{H}{2} - \frac{\sqrt{3}}{2}\left(\frac{0.337}{H} + H\right)i - \frac{1}{5}$, $\lambda_4 = \frac{0.168}{H} - \frac{H}{2} + \frac{\sqrt{3}}{2}\left(\frac{0.337}{H} + H\right)i - \frac{1}{5}$