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Effect of Control Saturation on the Tracking of Longitudinal States of an Aircraft

Sulakshan R. Arya * **Sachit Rao** **

* *Assistant Professor, International Institute for Aerospace Engineering and Management, Jain University, Karnataka 562 112, India (e-mail: sulakshan.arya@jainuniversity.ac.in).*
** *Assistant Professor, International Institute of Information Technology–Bangalore, Karnataka, 560 100, India (e-mail: sachit@iiitb.ac.in)*

Abstract: This paper addresses the effect of elevator saturation on asymptotically stable tracking of a desired flight path angle of an aircraft. The desired reference trajectory is computed using a non-linear kino-dynamic model of the longitudinal states of the aircraft incorporating control bounds. Next, based on a linearised model about the reference states, a state-feedback controller is designed to track the reference trajectory in the presence of deviations. Using results from the polytopic approach, which indicate that in the presence of control saturation, the magnitudes of the allowable deviations reduce as the eigenvalues of the open-loop system move from the left to the right of the complex plane. By estimating the volumes of these so-called regions of asymptotic stability, which is based on solving a set of Linear Matrix Inequalities, the results presented in this paper aid in the selection of appropriate trajectories that enable asymptotically stable tracking in the presence of deviations and control saturation. Simulation results are used to illustrate this concept.

Keywords: Aerospace Control, Saturation, Trajectory planning, Region of Asymptotic Stability, Polytopic Approach

1. INTRODUCTION

Path planning for unmanned vehicles in the presence of obstacles is a long studied problem. In the context of Unmanned Aerial Vehicles (UAVs), there have been several efforts in the generation of "flyable" reference trajectories, for instance, as those described in Zhao and Tsiotras (2013), Williams (2005), and their references. Usually, these trajectories are designed to satisfy some optimality constraint, most notably, that they be the shortest paths. A widely implemented solution to such a problem is the Dubins formulation and its variants, see Shanmugavel (2007) for details and other references. In these approaches, there could exist several segments of the generated reference trajectory which could require the saturation of the input. For instance, a flight path angle trajectory which can be tracked only when the elevator is deflected to its maximum deflection value. In this work, the longitudinal dynamics of a fixed-wing aircraft described by a constant velocity, non-linear dynamic model with the elevator as the single input is considered. In addition, the particular case of this system requiring to track reference trajectories which demand control inputs that are close to or at saturation, is studied.

It is clear that for an aircraft to track such trajectories, a feedback controller is essential to correct for parameter variations or deviations of the aircraft states from the reference trajectory. Generally, to design such a controller for tracking, or in other words, to keep the aircraft at trim, a linearised model of the aircraft computed at these values of trim is generally used, Stevens and Lewis (2010). In this paper, for the selected system, a relation is established between the implications of selecting a reference trajectory on closed-loop performance and open-loop system dynamics. With this result, it will be shown that trajectory selection should be viewed as a design trade-off exercise between needing *and* being able to track an "optimal" trajectory with the available control resources. The main contribution of this paper is the formal identification of how this exercise can be carried out.

The relation between closed-loop performance and trajectory selection will be established by estimating the bounds of allowable deviations, which will guarantee asymptotically stable closed-loop behaviour, even if the control input saturates. These bounds, termed as the Regions of Asymptotic Stability (RAS), are computed using the polytopic model approach, presented in Tarbouriech et al. (2011), which is based on solving a set of Linear Matrix Inequalities (LMIs). The second implication, which concerns open-loop dynamics, is addressed by first observing that the reference trajectory determines the eigenvalues of the linearised open-loop aircraft dynamics and that for certain values, the open-loop system becomes unstable. Next, a result presented in Scibile (1997), is used to establish a relation between the RAS and unstable real eigenvalues.

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These tools are used to identify the reference trajectory design trade-off procedure.

The results presented in this paper are developed under the assumption that the linearised open-loop system is completely controllable, observable, and has distinct real eigenvalues. It will be shown in the paper that the RAS shrinks as the open-loop eigenvalues move from the left-half of the complex plane to the right. Only the case of symmetric saturation is considered as the elevator deflection usually has bounds which are symmetric about some reference zero deflection. Thus, the results presented in this paper can be used to generate "safe" and flyable trajectories. For other efforts on the topic of designing aircraft controllers in the presence of saturation, see Oishi et al. (2006) and its references.

The paper is organised as follows: the problem is stated in Sec. 2 by introducing the non-linear aircraft model of the longitudinal states as well as the effect of changing the trim on the eigenvalues of the open-loop system. In Sec. 3, the LMI-based polytopic model to estimate the RAS is presented and is demonstrated for a primitive first-order unstable system; this example is also used to illustrate the result between the size of the RAS and the open-loop properties. In Sec. 4, the main result of this paper is presented accompanied by simulation results. Finally, conclusions are presented in Sec. 5.

2. PROBLEM STATEMENT

The reference trajectory selection and associated controller design problem is addressed based on the non-linear longitudinal dynamics of a fixed-wing aircraft as presented in Stevens and Lewis (2010), where the states are the airspeed V, the angle-of-attack (AoA) α, the pitch angle θ, and the pitch rate Q. The control inputs are the thrust T and the elevator deflection δ_e. In this paper, as the effect of control saturation is studied only in terms of δ_e, with $|\delta_e| \leq \delta_{e\text{Lim}}$, It is assumed that the airspeed can be controlled, using the thrust as control, at a constant value, denoted by V_{ref}, without saturation effects. With these assumptions, the dynamics of the aircraft are given by

$$m_a V_{\text{ref}} \dot{\gamma} = qS\left(C_L + C_D t_\alpha\right) + m_a g\left(t_\alpha s_\gamma - c_\gamma\right) \tag{1a}$$

$$\dot{\theta} = Q \tag{1b}$$

$$\dot{Q} = \frac{qSc}{I_{yy}} C_m \tag{1c}$$

where,

$$C_X = C_{X0} + C_{X\alpha}\alpha + \frac{c}{2V} C_{XQ} Q + C_{X\delta_e}\delta_e, \quad X = D, L, m \tag{1d}$$

and the flight path angle

$$\gamma = \theta - \alpha \tag{1e}$$

In these equations, the aircraft parameters are given by the wing-area S, the chord c, the aircraft mass m_a, the moment-of-inertia about the lateral axis I_{yy}, and the dynamic pressure $q = 0.5\rho V^2$, where ρ is the density of air. The parameters that define C_X in (1d) are the aerodynamic coefficients, which are dependent on flying conditions. Also, the notations $c_\gamma = \cos(\gamma)$, $t_\alpha = \tan(\alpha)$ etc.

The typical path planning problems for UAVs in terms of the states given by (1) are performed using the navigation equations

$$\dot{X} = V\cos(\gamma), \ \dot{h} = V\sin(\gamma) \tag{2}$$

where, X denotes the position of the aircraft measured in some coordinate frame and h is the altitude. Hence, for a given $(X, h, V)_{\text{ref}}$, which is generated by the path planner, the trajectory generator produces a trajectory for γ_{ref}, which has to be tracked using δ_e as control.

Now, the input δ_e can be designed to ensure $\gamma \to \gamma_{\text{ref}}$ by first linearising the system dynamics about V_{ref} and γ_{ref} to obtain

$$\dot{x} = \mathbf{A}x + \mathbf{B}\tilde{\delta}_e, \ x = \left[\tilde{\gamma} \ \tilde{\theta} \ \tilde{Q}\right]^T \tag{3}$$

where, x and $\tilde{\delta}_e$ denote the deviations of these states from trim. Next, the control input, which is the elevator deflection, is calculated as the sum

$$\delta_e = \delta_{e\text{ref}} + \tilde{\delta}_e, \text{ with } \tilde{\delta}_e = \mathbf{K}x \tag{4}$$

where, $\delta_{e\text{ref}}$ is the value of δ_e calculated at trim and $\mathbf{K}$ is the feedback gain matrix designed to ensure that the closed-loop dynamics $\dot{x} = (\mathbf{A} + \mathbf{BK})\,x$ is asymptotically stable and more importantly, where, $|\delta_e| \leq \delta_{e\text{Lim}}$. The matrix $\mathbf{K}$ is selected using well-known techniques such as pole placement to satisfy some transient or steady-state performance objectives. It is assumed that the pair $(\mathbf{A}, \mathbf{B})$ is completely controllable and it is assumed that all states can be measured, which implies complete observability.

With this background, the problem statement is as follows: Given a reference trajectory V_{ref}, γ_{ref}, and a feedback controller designed to track this trajectory, determine the maximum allowable deviations in x such that the feedback controller can provide asymptotically stable tracking even if the control saturates. In addition, as the open-loop properties of the aircraft change with change in reference trajectory, determine whether the magnitudes of these deviations also change with change in the open-loop properties of the aircraft.

The aircraft parameters used to illustrate the contributions of the paper are those presented in Campa et al. (2007) but with slight modifications made to some of the aerodynamic coefficients to highlight these contributions. These modifications are made primarily to exaggerate the change in open-loop eigenvalues for different values of V_{ref} and γ_{ref}. Some of the important parameters are listed in Table 1.

Table 1. Selected Aircraft Parameters

Parameter	Value
m	20.64 kg
S	1.37 m^2
$C_{m\alpha}$	-0.12
C_{mQ}	-10.35

Using these values, it will be shown in Sec. 4.1, that the eigenvalues of $\mathbf{A}$ in (3) vary with the choice of V_{ref} and γ_{ref}. In addition, for specific cases of these parameters, it will be shown that the open-loop system contains one positive eigenvalue, implying that it is unstable, and whose magnitude increases with decrease in V_{ref}.

The mathematical background used to support the contributions of this paper is briefly presented in the next section.