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Time-Delayed Vibration Control Of Two Degree-Of-Freedom Mechanical System For Improved Stability Margins

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Abstract: Time-delayed vibration control is used for a two degree-of-freedom mechanical system to approximate state-derivative feedback and to reduce sensitivity and improve stability margins. Additional sensors are not required since state-derivatives are approximated using available measurements and time delays. A systematic design approach, based on solution of delay differential equations using the Lambert W method, is presented. The simulation results demonstrate excellent performance with improved stability margins over state feedback control only.

Keywords: time-delayed control, stability margins, delay differential equations, Lambert W function.

1. INTRODUCTION

Control systems are typically designed based on nominal plant models, however, variations in parameters are often encountered in practice. Consequently, there has been significant research in robust control design, where such model uncertainties are explicitly considered in the design process. One approach is the use of state-derivative feedback, in addition to state feedback (SF), to reduce closed loop system sensitivity to plant parameter variations and to disturbance inputs (Haraldsdottir et al 1988). However, state plus state-derivative feedback (SSD) is difficult to realize in practice, because additional sensors are needed to measure the state-derivatives in addition to the states. Even measuring all the states is too restrictive in many engineering applications, thus, measuring state derivatives as well as states is often impractical.

Recent papers have proposed the use of time delays in the control to approximate the state derivatives to improve stability margins (Ulsoy 2013, 2015). In this paper that proposed approach is applied to the vibration control of a two degree-of-freedom (DOF) mechanical system. The simulation results show significant improvements in stability margins over SF control only.

2. THEORY

Consider a single-input single-output (SISO) linear time invariant (LTI) plant in state equation form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u; \quad y = \mathbf{C}\mathbf{x} \tag{1}$$

Assuming all states, $\mathbf{x}(t)$, are measurable a SF controller,

$$u(t) = -\mathbf{K}\mathbf{x}(t) \tag{2}$$

yields the closed-loop system equations:

$$\dot{\mathbf{x}} = \mathbf{A}_c\mathbf{x} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}; \quad y = \mathbf{C}\mathbf{x} \tag{3}$$

If the pair (**A, B**) is controllable, then **K** can be selected (e.g., eigenvalue assignment or optimal control) to achieve the desired closed-loop performance. If not all the states are measured, but the pair (**A, C**) is observable, then one can also design a state estimator, or observer, to estimate the states from the output (Anderson & Moore 1990). Furthermore, SF controllers achieve not only the desired closed-loop performance, but also excellent robustness as measured by stability margins. However, the use of a state estimator will typically reduce those stability margins (Safonov 1980).

2.1 *Sensitivity Reduction via State Derivative Feedback*

For the system in (1), consider an SSD feedback controller (Haraldsdottir et al 1988):

$$u(t) = -\mathbf{F}\mathbf{x}(t) - \mathbf{G}\dot{\mathbf{x}}(t) \tag{4}$$

The closed-loop system becomes:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c\mathbf{x} = (\mathbf{I} + \mathbf{B}\mathbf{G})^{-1}(\mathbf{A} - \mathbf{B}\mathbf{F})\mathbf{x}(t) \tag{5}$$

One can first select **K** in (2) to obtain the desired $\mathbf{A_c}$ in (3), then select **G** based on sensitivity considerations and to ensure invertability of (**I**+**BG**), and finally determine **F** from (5) given $\mathbf{A_c}$, **G**, **A** and **B**. The SSD feedback gives exactly the same closed-loop eigenstructure as SF control, but, for appropriate values of **G**, SSD can reduce eigenvalue sensitivity to variations in the parameters of **A** and **B**, improve the stability margins, and also reduce the effects of any external disturbances acting on the system. Thus, the benefits of SSD feedback over SF control can be significant (Haraldsdottir et al 1988, Ulsoy 2015).

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2.2 Approximation via Time Delay

In practice, it is often difficult to obtain measurements of the state derivatives to implement SSD feedback. Thus, one can consider implementation of the following time-delayed control (TDC):

$$u(t) = -\mathbf{K}_{\mathbf{p}}\mathbf{x}(t) - \mathbf{K}_{\mathbf{d}}\mathbf{x}(t-h) \tag{6}$$

where h is a constant time delay. The closed-loop system is:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= (\mathbf{A} - \mathbf{B}\mathbf{K}_{\mathbf{p}})\mathbf{x}(t) - \mathbf{B}\mathbf{K}_{\mathbf{d}}\mathbf{x}(t-h)\\ &= \mathbf{A}_{\mathbf{c}}\mathbf{x}(t) + \mathbf{A}_{\mathbf{d}}\mathbf{x}(t-h)\end{aligned} \tag{7}$$

The system in (7) is a retarded delay differential equation (DDE) and has an infinite eigenspectrum. However, as $h \rightarrow 0$ (7) can be viewed as an approximation to the closed-loop system in (5) with SSD feedback. Consider the finite difference approximation, for small h:

$$\dot{\mathbf{x}}(t) \approx \frac{\mathbf{x}(t) - \mathbf{x}(t-h)}{h} \tag{8}$$

Then the SSD feedback control can be approximated as:

$$\begin{aligned}u(t) &= -\mathbf{F}\mathbf{x}(t) - \mathbf{G}\dot{\mathbf{x}}(t)\\ &\approx -\mathbf{F}\mathbf{x}(t) - (1/h)\mathbf{G}(\mathbf{x}(t) - \mathbf{x}(t-h))\\ &= -(\mathbf{F} + (1/h)\mathbf{G})\mathbf{x}(t) + (1/h)\mathbf{G}\mathbf{x}(t-h)\end{aligned} \tag{9}$$

Thus, comparing (6) and (9) one obtains:

$$\mathbf{K}_{\mathbf{p}} = \mathbf{F} + (1/h)\mathbf{G}; \quad \mathbf{K}_{\mathbf{d}} = -(1/h)\mathbf{G} \tag{10}$$

Consequently, if one selects h to be sufficiently small, and selects $\mathbf{G}$ to reduce sensitivity, the resulting closed-loop system in (7) will have the specified closed-loop eigenstructure and desired performance, as with SF control only, plus reduced sensitivity, as with SSD feedback control. However, (7) is a DDE. Thus, its analysis, to select the appropriate value of h and to confirm closed-loop performance and robustness, can be challenging.

2.3 Solution of Delay Differential Equations

One would like to see the performance of the closed-loop system in (7) be close to that specified by $\mathbf{A_c}$ in (3) or (5). The system of DDEs in (7) possesses an infinite number of eigenvalues due to the presence of the delay h. However, the overall system response, as in higher-order linear systems without delay, will be dominated by the system eigenvalues closest to the imaginary axis; that is the rightmost eigenvalues in the s-plane. These rightmost eigenvalues will determine stability as well as transient response characteristics, such as settling time and overshoot (Yi et al 2010c). A method, based on the Lambert W function, has been developed that enables the analysis of LTI time delay systems as in (8) to obtain rightmost eigenvalues and the time response (Yi et al 2010b). Other DDE analysis tools, e.g., (Breda et al 2009), could also be utilized. However, only the Lambert W approach is considered here and is found to be very convenient and efficient. This method, which will be utilized here to establish the performance of the closed-loop system in (7) by computing the rightmost eigenvalues and time response, is briefly summarized below.

The solution to an autonomous system of DDEs, as in (7), can be represented by an infinite series (Yi et al 2010b):

$$\mathbf{x}(t) = \sum_{k=-\infty}^{\infty} e^{\mathbf{S}_k t}\mathbf{C}_k^I \tag{11}$$

where the coefficients $\mathbf{C}_k^I$ depend on a pre-shape function $\phi(t)$, $-h \le t < 0$, and

$$\mathbf{S}_k = \frac{1}{h}\mathbf{W}_k(\mathbf{A}_{\mathbf{d}}h\mathbf{Q}_k) + \mathbf{A}_{\mathbf{c}} \tag{12}$$

are the solution matrices, where the matrix $\mathbf{Q}_k$ satisfies

$$\mathbf{W}_k(\mathbf{A}_{\mathbf{d}}h\mathbf{Q}_k)e^{\mathbf{W}_k(\mathbf{A}_{\mathbf{d}}h\mathbf{Q}_k)+\mathbf{A}_{\mathbf{c}}h} = \mathbf{A}_{\mathbf{d}}h \tag{13}$$

An iterative numerical solution of (13) yields the matrix $\mathbf{Q}_k$, which is then substituted into (12) to obtain $\mathbf{S}_k$. The rightmost eigenvalues of (7) can then be obtained from the eigenvalues of $\mathbf{S}_k$, for $k = 0, \pm 1, \ldots, \pm m$, where m = nullity ($\mathbf{A_d}$). The series solution in (11) converges as k is increased. Software routines available in the *LambertWDDE Toolbox* can be utilized to compute both the rightmost eigenvalues and the time response (Yi et al 2014). The methods and software have been successfully applied to a variety of engineering problems (Yi et al 2010a, 2010b, 2010c; Ulsoy 2015). However, in some cases convergence problems can occur in numerically solving (13) for $\mathbf{Q_k}$ (Wei et al 2014).

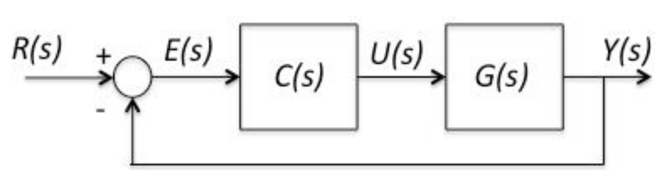

Figure 1. Block diagram of a SISO unity feedback control system with controller *C(s)* and plant *G(s).*

2.4 Stability Margins

Standard measures of robustness for control of SISO systems are their stability margins, i.e., the gain margin (*GM*) and phase margin (*PM*). For a closed-loop unity feedback system, as in Fig. 1, the stability margins are determined from a frequency response (i.e., Bode or Nyquist) plot of the loop transfer function *C(s)G(s).* A typical control design