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Sensitivity of stability charts with respect to modal parameter uncertainties for turning operations

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Abstract: Stability prediction of machining operations is often not reliable due to the inaccurate mechanical modeling. A major source of this inaccuracy is the uncertainties in the dynamic parameters of the machining center at different spindle speeds. The measured frequency response functions of the tool are usually loaded by noise and identification of the operational modal behavior based on static measurements is not straightforward. In this paper, the effect of small changes of the frequency response function on the stability of turning processes is analyzed using the semi-discretization method and the single-frequency solution.

Keywords: machine tool chatter, regenerative delay, frequency response function, sensitivity.

1. INTRODUCTION

Material removal by means of cutting is one of the most important components of manufacturing systems. Machine tool centers nowadays are capable of spindle speeds exceeding 50 000 rpm while simultaneously delivering tens of kilowatts of power to the cutting zone. Still, these features are often not utilized due to limitation caused by machine tool chatter. Prediction of the stability of a machining operation is therefore highly important for manufacturing systems.

In the 1960s, after the extensive work of Tobias (1965), Tlusty and Spacek (1954), the so-called regenerative effect became the most commonly accepted explanation for machine tool chatter. The phenomenon can be described by involving time delay in the model equations. The vibrations of the tool are copied onto the surface of the workpiece, which modifies the chip thickness and induces variation in the cutting-force acting on the tool one revolution later. This phenomenon can be described by delay-differential equations (DDEs).

Stability properties of the machining processes are depicted by the so-called stability lobe diagrams, which plot the maximum stable depths of cut versus the spindle speed. These diagrams provide a guide to the machinist to select the optimal technological parameters in order to achieve maximum material removal rate without chatter.

There are several limitations in the modeling of machine tool chatter. Most models in the literature consider linear systems, although nonlinear effects may also influence the stability properties (Dombovari et al., 2008). According to Munoa et al. (2013), the number of modes to be modeled is also an important factor. The approximation of the measured frequency response function (FRF) plays also an important role (Zhang et al., 2012). In this paper, parameter sensitivity of the stability chart is analyzed for different modeling inaccuracies, such as mode omission or mode merging.

The structure of the article is as follows. In Section 2, the formulation of the frequency response function matrix in case of non-proportional damping is introduced. The stability analysis both in time domain (using the modal representation) and in frequency domain (using directly the measured FRF) are presented in Section 3. This provides two efficient ways to construct the stability charts. Some typical fitting inaccuracies are discussed in Section 5. First, the effect of neglected and merged modes is analyzed based on a two-degrees-of-freedom model. Then, the sensitivity of a stable island with respect to modal parameter inaccuracies are demonstrated. Finally, a case study is presented for different degrees-of-freedom approximation of a measured FRF. The results are concluded in Section 6.

2. DETERMINATION OF MODAL PARAMETERS

The modal behavior of the machine is usually determined from impact or shaking tests. Let us have the matrix differential equation of motion for a multiple-degrees-of-freedom system in the form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the general coordinate vector, $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the mass matrix, $\mathbf{C} \in \mathbb{R}^{n\times n}$ is the damping matrix, $\mathbf{K} \in \mathbb{R}^{n\times n}$ is the stiffness matrix, $\mathbf{f}(t) \in \mathbb{R}^n$ is the excitation vector and n is the number of degrees of freedom. Matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ usually cannot be determined, but the modal parameters of the system can be approximated by different methods. Therefore the equations are defined in the modal space.

The system is proportionally damped if the damping matrix can be written as

$$\mathbf{C} = \alpha_{\mathrm{M}}\mathbf{M} + \alpha_{\mathrm{K}}\mathbf{K}, \tag{2}$$

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where $\alpha_{\mathrm{M}} \in \mathbb{R}$ and $\alpha_{\mathrm{K}} \in \mathbb{R}$ are the proportional factors (Ewins, 2000). If the damping matrix can be represented in such a way, it guarantees that the mode shapes are real valued and identical to the eigenvectors of the undamped system. It is known that if a system is proportionally damped, then the frequency response function matrix $\mathbf{H}(\omega)$ can be defined as

$$\mathrm{H}_{ij}(\omega) = \frac{X_i(\omega)}{F_j(\omega)} = \sum_{k=1}^{n} \frac{\phi_{ik}\phi_{jk}}{-\omega^2 + 2\zeta_k\omega_{\mathrm{n},k}\omega\mathrm{i} + \omega_{\mathrm{n},k}^2}, \qquad (3)$$

where ij represents the rows and columns of matrix $\mathbf{H}(\omega)$ respectively, $X_i(\omega) = \mathscr{F}(x_i(t))$, $F_i(\omega) = \mathscr{F}(f_i(t))$, and $\mathscr{F}$ is the Fourier transform, furthermore $\omega_{\mathrm{n},k}$ is the natural angular frequency, ζ_k is the damping factor, $\mathrm{i} = \sqrt{-1}$ is the complex unit, $\phi_{ik}\phi_{jk} = 1/m_k$, and m_k is the the modal mass.

A system is called non-proportionally damped if (2) does not hold. In this case, the frequency response functions cannot be expressed according to (3), furthermore the mode shapes are complex and not identical to the eigenvectors of the undamped system. The equation of motion can be written in a first-order form

$$\hat{\mathbf{A}}\dot{\mathbf{v}}(t) + \hat{\mathbf{B}}\mathbf{v}(t) = \mathbf{f}_{\mathbf{v}}(t), \qquad (4)$$

where the state vector is $\mathbf{v}(t) = (\mathbf{x}(t)\ \dot{\mathbf{x}}(t))^{\mathrm{T}}$ and

$$\hat{\mathbf{A}} = \begin{pmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{pmatrix}, \ \hat{\mathbf{B}} = \begin{pmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -\mathbf{M} \end{pmatrix}, \ \mathbf{f}_{\mathbf{v}}(t) = \begin{pmatrix} \mathbf{f}(t) \\ \mathbf{0} \end{pmatrix}, \qquad (5)$$

furthermore $\hat{\mathbf{A}} = \hat{\mathbf{A}}^{\mathrm{T}}$ and $\hat{\mathbf{B}} = \hat{\mathbf{B}}^{\mathrm{T}}$ (Ewins, 2000; Dombovari et al., 2012). The homogeneous part states an eigenvalue-eigenvector problem in the form

$$(\hat{\mathbf{A}}\lambda + \hat{\mathbf{B}})\mathbf{U} = \mathbf{0}, \qquad (6)$$

where $\mathbf{U} \in \mathbb{C}^{2n}$ is the unnormalized (right) eigenvector. The eigenvalues can be determined from the frequency equation

$$\det(\hat{\mathbf{A}}\lambda + \hat{\mathbf{B}}) = 0, \qquad (7)$$

where $\lambda_k = -\zeta_k\omega_{\mathrm{n},k} + \sqrt{1-\zeta_k^2}\omega_{\mathrm{n},k}\mathrm{i}$. The eigenvalues and eigenvectors form complex conjugate pairs if $\zeta_k < 1$.

Equation (4) can be transformed into the $2n$-dimensional modal space by the transformation $\mathbf{q}(t) = \mathbf{\Psi}\mathbf{v}(t)$, where $\mathbf{q}(t) \in \mathbb{C}^{2n}$ is the modal coordinate vector and $\mathbf{\Psi} \in \mathbb{C}^{2n\times 2n}$ is the modal transformation matrix. If the complex eigenvectors $\mathbf{U}_k$ are normalized according to the criteria

$$\boldsymbol{\psi}_k = \frac{\mathbf{U}_k}{\sqrt{\mathbf{U}_k^{\mathrm{T}}\hat{\mathbf{A}}\mathbf{U}_k}}, \qquad (8)$$

then the modal transformation matrix can be written as

$$\mathbf{\Psi} = \left(\boldsymbol{\psi}_1\ \bar{\boldsymbol{\psi}}_1\ \cdots\ \boldsymbol{\psi}_n\ \bar{\boldsymbol{\psi}}_n\right). \qquad (9)$$

Since $\mathbf{\Psi}^{\mathrm{T}}\hat{\mathbf{A}}\mathbf{\Psi} = \mathbf{I}$ and $\mathbf{\Psi}^{\mathrm{T}}\hat{\mathbf{B}}\mathbf{\Psi} = -\mathrm{diag}(\lambda_k) = -\mathbf{\Lambda}$, the equation finally forms

$$\dot{\mathbf{q}}(t) - \mathbf{\Lambda}\mathbf{q}(t) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{f}_{\mathbf{v}}(t). \qquad (10)$$

From the Fourier transform of (10), the elements of the FRF matrix $\mathbf{H}(\omega)$ consistently to (3) can be given as

$$\mathrm{H}_{ij}(\omega) = \frac{X_i(\omega)}{F_j(\omega)} = \sum_{k=1}^{n} \left(\frac{\psi_{ik}\psi_{jk}}{\omega\mathrm{i} - \lambda_k} + \frac{\bar{\psi}_{ik}\bar{\psi}_{jk}}{\omega\mathrm{i} - \bar{\lambda}_k}\right). \qquad (11)$$

Equations (11) and (3) are identical if the damping is proportional, then $\mathrm{Re}\{\psi_{ik}\psi_{jk}\} = 0$. Using curve-fitting techniques, the modal parameters $\omega_{\mathrm{n},k}$, ζ_k, ψ_{ik} and ψ_{jk} can be fitted on the measured FRF. A well-known technique is the rational fraction polynomial method of Richardson and Formenti (1982), which can efficiently be used for high-degrees-of-freedom approximations, but there are many other linear or nonlinear fitting algorithms. In this paper, a nonlinear least squares method was used which is suitable for the fit of low-degrees-of-freedom models.

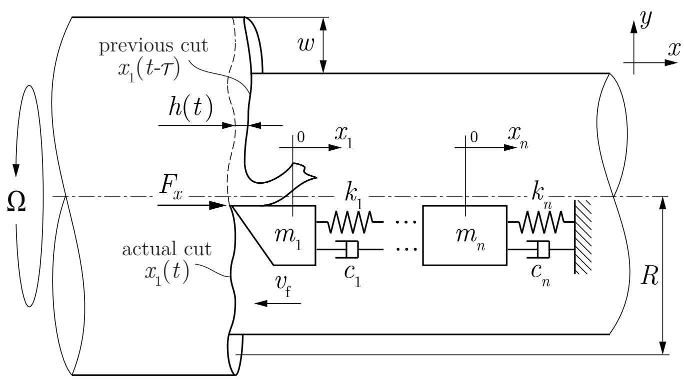

Fig. 1. Surface regeneration in an orthogonal process.

3. DYNAMICAL MODEL OF TURNING

The dynamical model of an orthogonal turning operation considering multiple modes in direction x is presented in Fig. 1. Note that vibrations in the y-direction does not affect the linear stability properties (Insperger et al., 2007). The cutting-force can be given as

$$F_x(t) = K_x w h^q(t), \qquad (12)$$

where K_x is the cutting-force coefficient in the tangential direction x, w is the depth of cut, $h(t)$ is the instantaneous chip thickness and q is the cutting-force exponent. Due to the vibrations of the tool, the chip thickness is determined by the feed motion, the current tool position and the previous position of the tool one revolution ago. For constant spindle speeds, the time delay can be given explicitly as $\tau = 60/\Omega$, where Ω is the workpiece revolution given in rpm. The instantaneous chip thickness can be calculated as

$$h(t) = v_{\mathrm{f}}\tau + x_1(t-\tau) - x_1(t), \qquad (13)$$

where v_{f} is the feed velocity. Therefore the excitation vector $\mathbf{f}(t)$ can be given as

$$\mathbf{f}(t) = \begin{pmatrix} K_x w \left(v_{\mathrm{f}}\tau + x_1(t-\tau) - x_1(t)\right)^q \\ 0 \\ \vdots \\ 0 \end{pmatrix}. \qquad (14)$$

The stability of the system can be analyzed by considering only the linearized system. The general solution can be given as $\mathbf{x}(t) = \mathbf{x}_{\mathrm{p}} + \boldsymbol{\xi}(t)$, where $\mathbf{x}_{\mathrm{p}}$ is related to the static deformation and $\boldsymbol{\xi}(t)$ is a small perturbation around the equilibrium $\mathbf{x} \equiv \mathbf{x}_{\mathrm{p}}$. After the linearization, the variational system is given by

$$\mathbf{M}\ddot{\boldsymbol{\xi}}(t) + \mathbf{C}\dot{\boldsymbol{\xi}}(t) + \mathbf{K}\boldsymbol{\xi}(t) = \boldsymbol{\kappa}\left(\boldsymbol{\xi}(t-\tau) - \boldsymbol{\xi}(t)\right), \qquad (15)$$

and

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}, \quad \kappa = K_x w q \left(v_{\mathrm{f}}\tau\right)^{q-1}, \qquad (16)$$