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Performance Comparison of Input-Shaped Model Reference Control on an Uncertain Flexible System

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Abstract: This paper investigates input-shaped model reference control (IS-MRC) applied to an uncertain nonlinear double-pendulum crane. In order to investigate practical implementation issues, a simple single-pendulum is used as the reference model. The proposed controller enhances the robustness to the uncertain difference between the reference model and the plant. Single- and double-pendulum crane dynamics are presented. The natural frequencies of the double-pendulum crane are calculated and utilized to design input shapers. A Lyapunov control law with guaranteed asymptotic stability is derived using only the first mode states of the plant. The state tracking, control effort reduction, and oscillation suppression performances of various IS-MRC designs are tested via numerical simulations and experiments. By analyzing the results, the IS-MRC design that has the largest robustness to the plant uncertainties is demonstrated to provide the best overall performance.

Keywords: Flexible Manufacturing Systems, Lyapunov Stability, Model Reference Control, Shaping Filters, Uncertain Dynamic Systems

1. INTRODUCTION

Model reference control (MRC) is an adaptive control strategy that is widely used in engineering applications, such as control of mechanical oscillators [Hovakimyan et al., 1999] and robots [Ekrekli and Brookfield, 1997]. MRC is very useful when designing a controller for systems containing time-varying parameters [Chien and Fu, 1992; Huang and Chen, 1993; Abdullah and Zribi, 2009] and nonlinearities [Landau, 1979; Nijmeijer and Savaresi, 1998; Kim et al., 2010]. MRC has also shown to be effective in controlling time-delay systems [Basher and Mukundan, 1987; Basher, 2010; Santosh and Chidambaram, 2013].

The performance of MRC depends on the formulation of a reference model that properly represents the controlled plant [Sastry and Isidori, 1989; Chen and Liu, 1994; Kim et al., 2010]. This is not always possible because actual systems can be difficult to model given nonlinearities and uncertainties. In the absence of accurate reference models, MRC has degraded state tracking performance. Furthermore, the issue of exceeding the maximum control effort must be considered because significant mismatch between the reference model and actual system can saturate the actuators and also degrade the controller's tracking ability.

Previous research approached this challenge by enhancing the robustness of MRC [Duan et al., 2001]. Sun et al. enhanced the system stability and robustness via a MRC controller composed of a conventional model matching feedback and a linear model error compensator [Sun et al., 1994]. Patino and Liu proposed a controller based on neural networks and analyzed it for a class of first-order continuous-time nonlinear dynamical systems [Patino and Liu, 2000]. Pedret et al. developed a robust MRC structure based on a right coprime factorization of the plant along with an observer-based feedback control scheme combined with a prefilter controller [Pedret et al., 2005].

While the robustness of MRC for the state tracking performance has been studied extensively, much less emphasis has been placed on the issue of the control effort. To realize both excellent tracking performance and limited control effort, we investigated a control method called input-shaped model reference control (IS-MRC). The controller combines MRC with a command-shaping technique called input shaping [Yuan and Chang, 2006, 2008; Yu and Chang, 2010]. Input shaping has been shown to reduce system vibration and improve the performance of time-delay systems with flexibility [Potter and Singhose, 2012, 2014]. IS-MRC controllers had been developed and shown to be effective for reducing the control effort [Fujioka and Singhose, 2014], and making the overall controller insensitive to the plant parameter variations and system order difference [Fujioka et al., 2015]. IS-MRC can be designed for robustness to the time-varying parameters and nonlinear dynamics of the plant [Fujioka and Singhose, 2015].

In this paper, we extend the past works by considering an uncertain plant and investigating the IS-MRC designs that reduce the sensitivity to parameter uncertainties and nonlinearities in the plant. Then, the performance of the proposed controllers are compared and analyzed. In Section 2, the IS-MRC design scheme is explained using exemplary single- and double-pendulum crane systems as the reference model and the plant. Various input shaper designs and the Lyapunov control law are also presented. In Section 3, the performance of the various IS-MRC designs are measured in terms of the state tracking, con-

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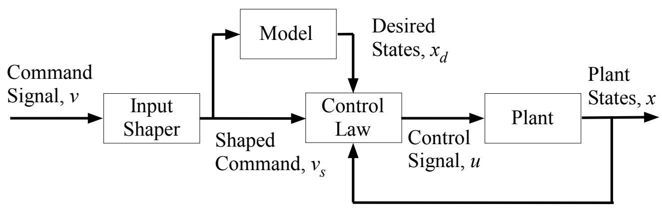

Fig. 1. Model reference control block diagram

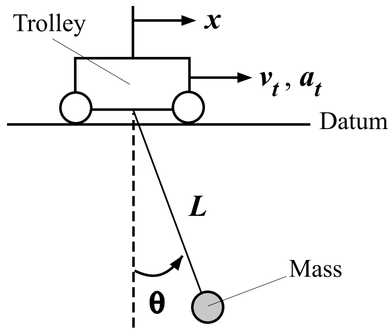

Fig. 2. Model of a single-pendulum crane

trol effort reduction, and oscillation suppression via numerical simulations and experimental data.

2. INPUT-SHAPED MODEL REFERENCE CONTROL

Figure 1 shows the block diagram of IS-MRC. The system consists of four blocks; a reference model, an uncertain plant, an input shaper, and a control law. A command signal v is first sent through the input shaper to obtain a shaped signal v_s. The v_s signal is then sent to the reference model to calculate the desired states x_d. The control law takes in v_s, x_d, and the plant states x to formulate a control signal u that controls the uncertain plant.

2.1 Reference Model and Uncertain Plant

In this work, the uncertain plant used in the MRC is a nonlinear double-pendulum crane; however, the reference model is a linearized single-pendulum crane. We utilize this one-mode model because it is easier to implement and accurate real-time measurement of a real crane payload is extremely difficult. Furthermore, a single-pendulum is a very good representation of a crane when it does not carry a payload.

Figure 2 shows a single-pendulum crane with a point-mass payload. The trolley's horizontal position is indicated by x, and is moved with velocity v_t and acceleration a_t. The point-mass m is suspended via a massless cable of length L. The swing angle of the payload measured with respect to the vertical axis is represented by θ.

A state space representation of the model using a velocity input is obtained by defining the second state x_2 as the horizontal swing distance of the mass $-\theta L$, and the first state x_1 as the integral of x_2. Assuming small swing angles, the state space representation of the system and the associated desired states x_d becomes:

$$\begin{aligned} \dot{x_d} &= A_{SP} x_d + B_{SP} v_t \\ &= \begin{bmatrix} \dot{x_{d,1}} \\ \dot{x_{d,2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_m^2 & -2\zeta_m \omega_m \end{bmatrix} \begin{bmatrix} x_{d,1} \\ x_{d,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_t \end{aligned} \quad (1)$$

where, ω_m and ζ_m are the natural frequency and the damping ratio of the reference model. The swing frequency is related to the cable length by $\omega_m = \sqrt{g/L}$, where g is the acceleration due to gravity.

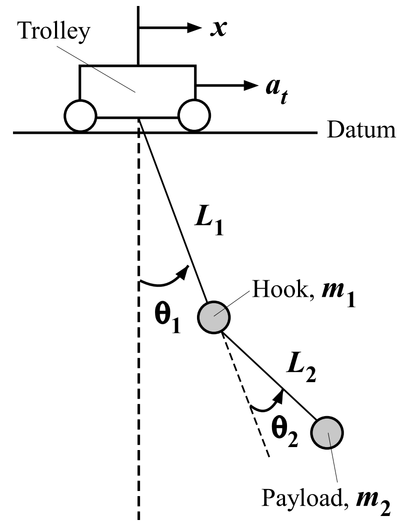

Fig. 3. Model of a double-pendulum crane

A double-pendulum crane is shown in Figure 3. The trolley moves with an acceleration input a_t, and its horizontal position is indicated by x. The hook m_1 is a point-mass suspended from the trolley via a massless hoist cable of length L_1. The payload m_2 is a point-mass attached to the hook via a massless cable of a fixed length L_2. The swing angle of the hook is measured with respect to the position of the trolley, and is represented by θ_1. The swing angle of the payload θ_2 is measured with respect to the swing of the hook. Damping of the swing is neglected.

The nonlinear equations of motion of the double-pendulum crane in terms of θ_1 and θ_2 are:

$$\begin{aligned} &(m_1 + m_2)\left[L_1 \ddot{\theta}_1 + g\sin(\theta_1) + 2\dot{L}_1 \dot{\theta}_1\right] + \\ &m_2 L_2 \left[\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)\cos(-\theta_2) + \left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 \sin(-\theta_2)\right] \\ &= a_t (m_1 + m_2)\cos(\theta_1) \end{aligned} \quad (2)$$

$$\begin{aligned} &L_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right) + \ddot{L}_1 \sin(-\theta_2) + g\sin(\theta_1 + \theta_2) - \\ &L_1\left[\dot{\theta}_1^2 \sin(-\theta_2) - \ddot{\theta}_1 \cos(-\theta_2)\right] + 2\dot{L}_1 \dot{\theta}_1 \cos(-\theta_2) \\ &= a_t \cos(\theta_1 + \theta_2) \end{aligned} \quad (3)$$

2.2 Frequencies of the Double-Pendulum Crane

Input shapers are designed by specifying the vibration frequencies to suppress. Hence, the hook and the payload oscillation modes are analyzed so that appropriate frequency ranges can be determined. Assuming small swing angles and a constant hoist cable length ($\dot{L}_1 = \ddot{L}_1 = 0$), the linearized natural frequencies of the double-pendulum crane are [Blevins, 1979]:

$$\begin{aligned} \omega_{1,2} &= \sqrt{\frac{g}{2}} \sqrt{(1 + R_M)\left(\frac{1}{L_1} + \frac{1}{L_2}\right) \mp \beta} \\ \beta &= \sqrt{(1 + R_M)^2 \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^2 - 4\left(\frac{1 + R_M}{L_1 L_2}\right)} \end{aligned} \quad (4)$$

where, $R_M = m_2/m_1$ is the mass ratio of the payload to the hook.