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IFAC-PapersOnLine 48-12 (2015) 153–158

Consensus on Time-Delay Intervals in Networks of High-Order Integrator Agents ⋆

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Abstract: This paper brings out a structured methodology for identifying time-delay intervals where consensus in directed networks of multiple agents with high-order integrator dynamics is achieved. It is built upon the stability analysis of a transformed consensus problem. Furthermore, particular results are derived for networks of agents with first- and second-order integrator dynamics, which can be consensusable only on the first time-delay interval, showing the value of the upper bound of this interval. The paper is closed by showing an interesting example of a network of third-order integrator agents that is not consensusable when free of delay, but it is consensusable when the control input is delayed in a proper interval.

Keywords: Consensus, time-delay, distributed control, delay analysis, linear systems.

1. INTRODUCTION

One of the major problems in the coordination of multi-agent systems is the one of deriving distributed control laws, based on information exchange among agents, so that the multi-agent system achieves an agreement on a given value of interest, which is called consensus problem. Due to the limitations of information processing, time-response of actuators, communication channel capacity, etc., in practice, it is also important to take into consideration the time-delay involved in the interaction between agents. Previous results on consensus analysis are reviewed next.

Considering undirected networks of first-order dynamic subject to delayed control input, some results can be found in Olfati-Saber and Murray (2004), which shows the maximum constant time-delay, τ^*, that allows the system to achieve consensus asymptotically, on the first time-delay interval, i.e. $\tau \in [0, \tau^*)$. For the case of time-varying delay, we can relate the work of Bliman and Ferrari-Trecate (2008), which shows an upper bound, $\bar{\tau}$, such that the system achieves consensus for all $\tau(t) \in [0, \bar{\tau}]$. A more general case is considered in Savino et al. (2014), which considers directed networks subject to multiple time-varying delays belonging to the interval $[\tau_1, \tau_2]$, where $0 < \tau_1 \leq \tau_2$. It is important to note that the results in Olfati-Saber and Murray (2004) and Bliman and Ferrari-Trecate (2008) only consider the consensus analysis on the first time-delay interval, meanwhile Savino et al. (2014) considers the time-delay lower bound different from zero, such that the effects of time-delay can be analyzed within given intervals. More recently, Xi et al. (2013) extended the results of Olfati-Saber and Murray (2004) to directed networks. For the case of consensus with second-oder integrator dynamics, Yu et al. (2010) presented a necessary and sufficient condition for the upper bound τ^* of the constant time-delay.

Ren et al. (2006) showed that the consensus in a delay-free network of agents with high-order integrator dynamics depends on the gains in the consensus protocol, even if the topology has a directed spanning tree. Afterwards, Wieland et al. (2008) presented a linear matrix inequality solution to design these gains such that the delay-free system could achieve consensus. With similar approach to the one used in this paper, Sipahi and Qiao (2011) showed exact results for the input time-delay margin in first-order integrator agents in undirected networks and its relation to the Laplacian eigenvalues. It was extended to second-order dynamics with input delays and communication delays in Cepeda-Gomez and Olgac (2011a,b). More recently, Yang (2013) investigated the stability switches in the time-delay domain, considering high-order consensus for undirected networks. Although these results only consider constant and uniform delays, sufficient results for analysis of intervals of multiple time-varying delays can be found in Savino et al. (2015).

Contributions: In this paper, we study consensus by checking the stability of an associated transformed system. This transformation is constructed by means of a tree-type transformation, whose characteristic equation is directly related to the Laplacian matrix. We extend the results of consensusability switches in Yang (2013) to the case of directed networks of multi-agent systems with input time-delays. Furthermore, we show particular results for networks of agents with first- and second-order dynamics,

⋆ This research was partially supported by the CNPq, CAPES, and FAPEMIG.

10.1016/j.ifacol.2015.09.369

which can be consensusable only on the first time-delay interval, *i.e.*, if the multi-agent system is consensusable, it achieves consensus for any time-delay in the interval $\tau \in [0, \tau^*)$ with $0 < \tau^* \leq \infty$. Finally, we show an interesting example, which to the best of the authors knowledge has not been shown before for consensus. The example shows a network of third-order integrator agents that achieves consensus only with proper delays, i.e. not on the first time-delay interval. It serves as a counterexample for the usual acceptance that the time-delay only degrades the system's performance.

Throughout the text we consider that I_n is an identity matrix, 0_n and 1_n are column vectors of zeros and ones, respectively, $\mathbf{0}_{m\times n}$ is a zero matrix, and $\lambda_i\{\cdot\}$ is the ith eigenvalue of a matrix.

2. PRELIMINARIES

2.1 Algebraic Graph Theory

The information flow of a multi-agent system is represented by a graph, following the next terminology and notation. Let the simple weighted directed graph be defined by the ordered triplet $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ is the set of m vertices arbitrarily labeled as $v_1, v_2, \ldots, v_m$, $\mathcal{E}$ is the set of edges connecting vertices, denoted by $e_{ij} = (v_i, v_j)$, where the first element v_i is said to be the parent node (tail) and the other v_j to be the child node (head), and $\mathcal{A} = [a_{ij}]$ is the adjacency matrix of order $m \times m$ related to the edges, that assigns a real non-negative value for each e_{ji}:

$$a_{ij} \begin{cases} = 0, & \text{if } i = j \text{ or } \nexists\, e_{ji} \\ > 0, & \text{iff } \exists\, e_{ji} \end{cases} \tag{1}$$

Related to $\mathcal{A}$, it is also defined the degree matrix $\Delta = [\Delta_{ij}]$, which is a diagonal matrix with elements $\Delta_{ii} = \sum_{j=1}^{m} a_{ij}$. The directed Laplacian matrix associated with the graph $\mathcal{G}$ is given by $L = \Delta - \mathcal{A}$.

A directed tree is a directed graph with only one node without parents, called root, and other nodes with exactly one parent. Also, there is a path, a sequence of edges, connecting the root to any other node in the tree. A directed spanning tree is a directed tree that can be formed from the removal of some of the edges of a directed graph, such that all nodes are included.

Next lemma regarding the Laplacian of directed spanning trees will be useful in the next sections:

Lemma 1. (Ren and Beard (2008) [Cor. 2.5]) Given the directed Laplacian matrix L, it has at least one zero eigenvalue with an associated eigenvector 1_m, and all the nonzero eigenvalues are in the open right half plane. Furthermore, L has exactly one zero eigenvalue if and only if the related directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ has a directed spanning tree.

3. SYSTEM DYNAMICS

Let the delay-free dynamics of each agent in a group of m agents be given by:

$$\begin{aligned} \dot{x}_i^n(t) &= x_i^{n-1}(t) \\ &\vdots \\ \dot{x}_i^2(t) &= x_i^1(t) \\ \dot{x}_i^1(t) &= u_i(t), \quad i = 1, 2, \ldots, m, \end{aligned} \tag{2}$$

such that $x_i^1, x_i^2, \ldots, x_i^n \in \Re$, where n determines the order of the integrators, are the state variables of the agent i, and $u_i \in \Re$ is the control input.

The considered consensus protocol is given by:

$$\begin{aligned} u_i(t) = -\sum_{j\neq i, j=1}^{m} a_{ij}\Big\{ &\alpha_n \Big(x_i^n(t) - x_j^n(t)\Big) \\ &+\alpha_{n-1}\Big(x_i^{n-1}(t) - x_j^{n-1}(t)\Big) \\ &+ \ldots + \alpha_1\Big(x_i^1(t) - x_j^1(t)\Big)\Big\}, \end{aligned} \tag{3}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_n > 0$ are arbitrary real constants, and a_{ij} are given by the weights in (1).

Considering (2) with consensus protocol (3), the delay-free system dynamics can be written as:

$$\begin{bmatrix} \dot{x}^n(t) \\ \dot{x}^{n-1}(t) \\ \vdots \\ \dot{x}^1(t) \end{bmatrix} = \begin{bmatrix} 0_m & I_m & \ldots & 0_m \\ \vdots & \vdots & \ddots & \vdots \\ 0_m & 0_m & \ldots & I_m \\ -\alpha_n L & -\alpha_{n-1} L & \ldots & -\alpha_1 L \end{bmatrix} \begin{bmatrix} x^n(t) \\ x^{n-1}(t) \\ \vdots \\ x^1(t) \end{bmatrix}, \tag{4}$$

where $x^h = [x_1^h \; x_2^h \; \ldots \; x_m^h]^T$ for $h = 1, 2, \ldots, n$ are column vectors with the corresponding state variables of the multiple agents of the entire system. This can be simplified into the following form:

$$\dot{x}(t) = \Gamma x(t), \tag{5}$$

where $x(t) = [x^n(t)^T \; x^{n-1}(t)^T \; \ldots \; x^1(t)^T]^T$ and Γ is the first matrix in the right side of Equation (4).

The following lemmas are important to be considered in the further analysis:

Lemma 2. (Ren et al. (2006)) The matrix Γ in (5) has at least n zero eigenvalues. It has exactly n zero eigenvalues if and only if the Laplacian L has a simple zero eigenvalue. Moreover, if L has a simple zero eigenvalue, the zero eigenvalue of Γ has only one linearly independent eigenvectors associated with eigenvalue zero.

Lemma 3. (Ren et al. (2006)) The system in (5) achieves consensus asymptotically if and only if matrix Γ has exactly n zero eigenvalues and all the other eigenvalues have negative real parts.

Note that, combining Lemmas 1, 2, and 3, we have that the consensus in directed networks of multi-agents with high-order integrator dynamics (2) and protocol (3), without delays, is achieved if and only if the related graph $\mathcal{G}$ has a directed spanning tree and the nonzero eigenvalues of Γ, in (5), lie in the open left half-plane.

4. TIME-DELAY EFFECTS

Now, consider the directed network of multi-agents with high-order integrator dynamics and delayed input:

$$\begin{aligned} \dot{x}_i^n(t) &= x_i^{n-1}(t), \;\; \ldots, \;\; \dot{x}_i^2(t) = x_i^1(t) \\ \dot{x}_i^1(t) &= u_i(t-\tau), \quad i = 1, 2, \ldots, m, \end{aligned} \tag{6}$$

such that each agent i in (6) has an internal time-delay τ. Moreover, initial conditions for any agent i are arbitrary and denoted by:

$$x_i^h(\theta) = \phi_i^h(\theta), \quad \forall \theta \in [-\tau, 0], \; h = 1, 2 \ldots, n,$$

where ϕ_i^h belongs to the set of $\Re$ valued continuous functions on $[-\tau, 0]$.