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Multi-agent systems with decaying confidence and commensurate time-delays

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Abstract: In this paper we consider a class of discrete-time multi-agent systems modeling opinion dynamics with decaying confidence in presence of delay in the communication channels. Delays arise naturally in such dynamics and they must be taken into account in the analysis. In the model under consideration, the delay presence results in a certain heterogeneous time repartition of the influence of one agent on the collective behavior of the network. The decaying confidence leads to reconfigurations of the communication topology due to the vanishing of some interactions. The initial conditions and the dynamics of the confidence determine the organization of agents in different groups in which the agreement is reached. The main result states that the agents which agree are those which are persistently connected. Some simulations illustrates the delay influence on the network pattern.*Copyright* © *IFAC 2009*

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1. INTRODUCTION

Networks are ubiquitous and their analysis and control received an increasing interest in the past decade. They are often modeled as multi-agent systems i.e. sets of interconnected dynamical systems called agents having partial knowledge about the network state. The analysis of multi-agent systems has various applications in many areas encompassing cooperative control of vehicles (Fax and Murray (2004); Jadbabaie et al. (2003)), congestion control in communication networks, flocking (Vicsek et al. (1995)), distributed sensor networks (Cortés and Bullo (2005)). In many of these applications, all (or some) agents need to agree with respect to a priori fixed criteria and the agreement might be subject to some speed constraints.

Conditions ensuring agreement or consensus have been established in terms of persistent connectivity of the communication graph (see Blondel et al. (2005); Moreau (2005); Ren and Beard (2005)). As pointed out in the literature, delays arise naturally in multi-agent dynamics and they can be point-wise Blondel et al. (2005); Michiels and Nijmeijer (2009) or distributed Atay (2003); Michiels et al. (2009).

A particular interest has been assigned to multi-agent systems modeling opinion dynamics with bounded confidence proposed in Krause (1997) and analyzed in Hegselmann and Krause (2002); Blondel et al. (2009). This model has been extended by considering that an agent gives repetitively confidence only to the neighbors that approach sufficiently fast its own opinion Morărescu and Girard (2011). This can be seen as a model for negotiation process where an agent expects that its neighbors move significantly towards its opinion at each negotiation round in order to keep negotiating. Since the connectivity of the network plays a major role in consensus, several works have been oriented toward the design of controllers able to preserve connectivity Zavlanos and Pappas (2008); Fiacchini and Morărescu (2014).

One drawback of the opinion dynamics models above is that they are all supposing that the state of one agent at a given instant is homogeneously broadcasted to all its neighbors. In real networks we can expect that communication are affected by delays but also that some agents are more reactive than others. This means that each agent updates its state by taking into account delayed information from its neighbors and the delay characterizes not only the physical distance between agents but also the reactivity of one agent with respect to the other. We notice that, in social networks there always exist preferences, deadlines as well as privileged communications. Therefore, the reactivity of one agent will be different from one channel to another. Among others consequences, in this model one agent will heterogeneously influence its neighbors.

In this paper we consider a model of opinion dynamics with decaying confidence and interactions affected by fixed time-delays. The network is represented by a directed graph with vertices representing individual systems and edges representing the interactions. Each interaction is affected by a specific delay. If the interaction is bidirectional the delay can be different from one direction to another.

The contribution of the paper consists of presenting the mathematical technique which enable the extension of some results reported in Morărescu and Girard (2011). At this moment we are presenting just mathematical reasoning proving that, under appropriate assumptions, for almost all initial conditions the state of two subsystems converge to the same value if and only if they are persistently connected. This work considers the case where

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commensurate delays occurs into the discrete dynamics. It should be noted here that the classical strategy that consists in augmenting the dimension of the system do not directly solve the problem. Precisely, the suitable properties of the augmented dynamics are not straightforward and they have to be proven.
The paper is structured as follows. In Section 2 we formulate the problem, provide some basic concepts necessary in the following development. In Section III we prove a convergence result for each agent belonging to the network. The main result concerning the equivalence between asymptotic connectivity and asymptotic agreement, under suitable assumptions, is proven in Section 4. Simulation emphasizing the impact of delays on the network behavior are provided before formulating some conclusions at the end of the paper.

2. BASIC CONCEPTS

In this section we recall some standard definitions related to graph theory, define the concepts necessary for the main results and introduce the model under study.

Let $G = (\mathcal{V}, E)$ denote a directed graph with the set of vertices (nodes) $\mathcal{V}$ and the set of edges E. Each node is labeled by $i \in \mathcal{V}, i = 1, \dots, n$ and one says that $(i, j) \in E$ if there exists an edge between i and j. Given two graphs with the same set of vertices, $G_1 = (\mathcal{V}, E_1)$ and $G_2 = (\mathcal{V}, E_2)$ we say that $G_1 \subset G_2$ if $E_1 \subset E_2$.

Definition 1. A **path** in a given directed graph $G = (\mathcal{V}, E)$ is a union of edges $\bigcup_{k=1}^{p}(i_k, j_k)$ such that $i_{k+1} = j_k$, $\forall k \in \{1, \dots, p-1\}$.
Two nodes i, j are **connected** in a graph $G = (\mathcal{V}, E)$ if there exists at least a path in G joining i and j (i.e. $i_1 = i$ and $j_p = j$).
A **strongly connected graph** (referred to as connected graph in the sequel) is such that for all $i \neq j \in \mathcal{V}$ there exist a path joining i and j. A **strongly connected component** (referred to as connected component in the sequel) of a graph is a maximal subset of connected nodes (i.e. there exists no other node of the graph that is connected with one of the component's nodes).

In the following we consider a given directed graph $G = (\mathcal{V}, E)$ and we denote $N_i = \{j \in E \mid (i, j) \in E\}$ the neighborhood of the agent i in G. Each vertex i represents a dynamic agent and the state of the network will be given by

$$x(\cdot) = (x_1(\cdot), x_2(\cdot), \dots, x_n(\cdot))^\top \in \mathbb{R}^n$$

where $x_i(t)$ is a real value assigned to i at the moment t. The value $x_i(t)$ will be called the opinion of the agent i at the moment t. The initial conditions are given by

$$x_i(t) = x_i^0,\ \forall i \in \{1, \dots, n\},\ \forall t \in [-\tau, 0],\ \tau = \max \tau_{ij}$$

where τ_{ij} is the delay in the communication channel between the agents i and j. The agents update their opinion by taking a weighted average of their own opinion and the known opinion of their neighbors:

$$x_i(t+1) = p_{ii}(t)x_i(t) + \sum_{j \neq i} p_{ij}(t)x_j(t - \tau_{ij}), \qquad \forall i \in \{1, \dots, n\}, t \geq 0 \tag{1}$$

with the coefficients $p_{ij}(t)$ satisfying

$$\forall i, j \in V,\ i \neq j,\ (p_{ij}(t) \neq 0 \iff j \in N_i(t)) \tag{2}$$

where $N_i(t)$ denotes the *neighborhood* of agent i at time t which is defined as:

$$\begin{aligned} N_i(t) &= \{j \in V \mid (i, j) \in E(t)\} \\ &= \left\{j \in N_i \mid |x_i(t) - x_j(t - \tau_{ij})| \leq M\rho^t\right\} \end{aligned} \tag{3}$$

where M, ρ are design parameters a priori fixed. We make the following additional assumptions:

Assumption 1. There exists a positive constant α such that for all $t \in \mathbb{N}$ the coefficients $p_{ij}(t)$ satisfy

(1) $p_{ij}(t) \in \{0\} \bigcup [\alpha, 1]$, for all $i, j \in \mathcal{V}$.
(2) $p_{i,i}(t) > 0$ and $p_{ij}(t) > 0 \Leftrightarrow (i, j) \in E(t)$
(3) $\sum_{j=1}^{n} p_{ij}(t) = 1$, for all $i \in \mathcal{V}$.

This assumption is classic in the works dealing with consensus and it assures that the matrices $P(t)$ defining the collective dynamics without delays are stochastic matrices. Moreover, $p_{i,j}(t) > 0$ means that, at time t, agent i received a delayed opinion of agent j. We suppose that each channel (i, j) is affected by its own fixed delay τ_{ij}.

Remark 1. We note at this point that, due to (3), all the graphs $G(t)$ depends also on the initial condition x^0. Moreover, different matrices used in the paper (P, A, B) depends on the structure of $G(t)$ and implicitly they are state-dependent meaning they depends on x^0. For the sake of simplicity we avoid highlighting this in the notation used all along the development.

Definition 2. (agreement). We say nodes i and j *asymptotically agree* if and only if

$$\lim_{t \to \infty} x_i(t) = \lim_{t \to \infty} x_j(t).$$

Two nodes *asymptotically disagree* if they do not asymptotically agree.
An algorithm guarantees *asymptotic consensus* if: for every initial condition $x(t) = x^0$, $t \in [-\tau, 0]$ and for every sequence $\{P(t)\}_{t \geq 0}$ allowed by Assumption 1, all the nodes asymptotically agree.

The agreement problem in presence of fixed or time-varying delays has been studied in the framework of consensus. For continuous time dynamics the problem of average consensus in undirected networks with communication delays has been treated in Bliman and Ferrari-Trecate (2008). The consensus of continuous dynamics that take into account delayed state of the neighbors transmitted at discrete times has been analyzed in Xiao and Wang (2008.). Closer to the present paper, the discrete-time consensus with multiple delays has been treated in Blondel et al. (2005). In these works as well as in the wide literature, it is considered that a minimal assumption required for consensus is the *asymptotic connectivity* of the network, defined below. Let us remark that the set of interactions $\mathcal{V} \times \mathcal{V}$ can be classified into two subsets as follows:

$$E^f = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid \exists t_{ij} \in \mathbb{N},\ \forall s \geq t_{ij},\ (i, j) \notin E(s)\}$$

and

$$E^\infty = \{(i, j) \in \mathcal{V} \times \mathcal{V} \mid \forall t \in \mathbb{N},\ \exists s \geq t,\ (i, j) \in E(s)\}.$$

Intuitively, E^f consists of the interactions between agents that disappear in finite time and E^∞ consists of the interactions between agents that are recurrent i.e. they appear an infinite number of times. It is clear that $E^f \cap$