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ROBUST CONTROLLER DESIGN FOR NEUTRAL TIME-DELAY SYSTEMS

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Abstract: A *robustness measure* that accounts for the uncertainties in a neutral time-delay system is defined. Using this measure, a robust controller design approach, which is based on a nominal model, is proposed. The proposed approach guarantees robust stability once a condition depending on the robustness measure is satisfied. An example is also presented to demonstrate the proposed approach.

Keywords: Neutral time-delay systems; robust control; controller design; stabilization.

1. INTRODUCTION

Many physical systems may involve time-delays. The controller design problem for a time-delay system is more difficult, compared to a delay-free system, since a time-delay system is infinite-dimensional (Niculescu (2001)). Different approaches, such as operator-based (Curtain and Zwart (1995); Foias et al. (1996); Toker and Özbay (1995)), eigenvalue-based (Michiels and Niculescu (2007)), and Lyapunov-based (Kolmanovskii et al. (1999)), have so far been proposed to design controllers for time-delay systems. Although some of these approaches only consider retarded time-delay systems, approaches which specifically consider neutral time-delay systems have also been proposed (e.g., Park and Won (1999); Han (2002); Wu et al. (2004); Parlakçı (2007)).

Since any model of any physical system may contain uncertainties, any controller designed for such a system must be robust against such uncertainties. In a time-delay system, not only the system parameters, but also the time-delays are usually uncertain. In this work, we propose a robust controller design approach for neutral time-delay systems. The approach uses a frequency-dependent *robustness measure* that accounts for the uncertainties in both the system parameters and the time-delays. Such a measure was first used for delay-free large-scale systems by İftar and Özgüner (1987a,b) and for retarded time-delay systems by İftar (2008, 2014). In here, we define a similar measure for neutral time-delay systems and propose a robust controller design approach using this measure. Once this measure is obtained, the proposed approach is completely based on the nominal model of the system and satisfying a simple constraint ensures the robust stability of the actual closed-loop system.

The problem is formally defined in the next section. The proposed approach is presented in Section 3. Section 4 presents an example to demonstrate the proposed approach. Some concluding remarks are given in the last section.

Throughout the paper, $\mathbf{R}$ and $\mathbf{C}$ denote the sets of, respectively, real and complex numbers. For a positive integer k, $\mathbf{R}^k$ denotes the k-dimensional real vector space. For $s \in \mathbf{C}$, $\mathrm{Re}(s)$ is the real part of s. I denotes the identity matrix of appropriate dimensions. $\bar{\sigma}(\cdot)$, $\underline{\sigma}(\cdot)$, and $\det(\cdot)$ respectively denote the maximum singular value, the minimum singular value, and the determinant of the indicated matrix. Finally, $j := \sqrt{-1}$ is the imaginary unit.

2. PROBLEM STATEMENT

Consider a linear time-invariant (LTI) neutral time-delay system which is described as:

$$\sum_{i=0}^{\nu} D_i \dot{x}(t-\tau_i) = \sum_{i=0}^{\nu} \left(A_i x(t-\tau_i) + B_i u(t-\tau_i)\right) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where, $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^p$, and $y(t) \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors at time t. We use $\tau_0 := 0$ for notational convenience (i.e., $i = 0$ corresponds to the delay-free part). ν is the number of independent time-delays and $\tau_1, \ldots, \tau_\nu \geq 0$ are the time-delays, which may be commensurate or incommensurate. D_i, A_i, B_i, $i = 0, \ldots, \nu$, and C are appropriately dimensioned constant matrices. It is assumed that all the input-output uncertainties and the time-delays are represented at the input, so that the output equation (2) is free of any uncertainties and delays. Thus, C is a known matrix. However, it is assumed that each of D_i, A_i, and B_i, $i = 0, \ldots, \nu$, is subject to uncertainties. More precisely, it is assumed that $D_i := D_i^n + D_i^u$, $A_i := A_i^n + A_i^u$, and $B_i := B_i^n + B_i^u$, for $i = 0, \ldots, \nu$, where the matrices with superscript n are known matrices and the matrices with superscript u represent the uncertainties. These latter matrices are not known, but are assumed to satisfy

$$\bar{\sigma}(D_i^u) \leq \delta_i \,, \quad \bar{\sigma}(A_i^u) \leq \alpha_i \,, \quad \text{and} \quad \bar{\sigma}(B_i^u) \leq \beta_i \,, \quad (3)$$

for some known bounds δ_i, α_i, and β_i, $i = 0, \ldots, \nu$. It is further assumed that $\mathrm{rank}(D_0) = n$ for any D_0^u satisfying

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the above bound. The time-delays are also assumed to be subject to uncertainties. More precisely, it is assumed that $\tau_i := \tau_i^n + \tau_i^u$, $i = 1, \ldots, \nu$, where τ_i^n is the known nominal time-delay and τ_i^u represents its uncertainty, which is assumed to satisfy

$$|\tau_i^u| \leq \theta_i \tag{4}$$

for some known bound θ_i, $i = 1, \ldots, \nu$.

Furthermore, we also make the following assumptions:

Assumption 1: For any D_i^u, $i = 0, \ldots, \nu$, satisfying (3) and any τ_i^u, $i = 1, \ldots, \nu$, satisfying (4), $\mu_f < 0$, where

$$\mu_f := \sup \left\{ \text{Re}(s) \mid \det \left(\sum_{i=0}^{\nu} D_i e^{-s\tau_i} \right) = 0 \right\} \tag{5}$$

Assumption 2: For any D_i^u, and A_i^u, $i = 0, \ldots, \nu$, satisfying (3) and any τ_i^u, $i = 1, \ldots, \nu$, satisfying (4), the number of unstable modes of the system (1) is the same, where $s_o \in \mathbf{C}$ is said to be an unstable mode of the system (1) if $\text{Re}(s_o) \geq 0$ and $\det \left(s_o \bar{D}(s_o) - \bar{A}(s_o) \right) = 0$, where

$$\bar{D}(s) := \sum_{i=0}^{\nu} D_i e^{-s\tau_i} \quad \text{and} \quad \bar{A}(s) := \sum_{i=0}^{\nu} A_i e^{-s\tau_i} \tag{6}$$

It is known that the system (1) has finitely many modes with real part greater than or equal to μ, for any $\mu > \mu_f$, where μ_f is given by (5) (e.g., see Michiels and Niculescu (2007)). Therefore, Assumption 1 guarantees that the number of unstable modes of the system (1) is finite for any uncertainties satisfying (3)–(4).

The problem is to design a controller based on the nominal model:

$$\sum_{i=0}^{\nu} D_i^n \dot{x}(t - \tau_i^n) = \sum_{i=0}^{\nu} \left(A_i^n x(t - \tau_i^n) + B_i^n u(t - \tau_i^n) \right) \tag{7}$$

$$y(t) = Cx(t) \tag{8}$$

so that the actual closed-loop system obtained by applying this controller to the system (1)–(2) is robustly stable for all uncertainties satisfying the bounds (3)–(4).

3. PROPOSED DESIGN APPROACH

Note that the transfer function matrix (TFM) of the actual system (1)–(2) is given by

$$G(s) = C \left(s\bar{D}(s) - \bar{A}(s) \right)^{-1} \bar{B}(s) \tag{9}$$

where $\bar{D}(s)$ and $\bar{A}(s)$ are as defined in (6) and

$$\bar{B}(s) := \sum_{i=0}^{\nu} B_i e^{-s\tau_i} \tag{10}$$

The TFM of the nominal system (7)–(8), on the other hand, is given by

$$G^n(s) = C \left(s\bar{D}^n(s) - \bar{A}^n(s) \right)^{-1} \bar{B}^n(s) \tag{11}$$

where

$$\bar{D}^n(s) := \sum_{i=0}^{\nu} D_i^n e^{-s\tau_i^n} \ , \quad \bar{A}^n(s) := \sum_{i=0}^{\nu} A_i e^{-s\tau_i^n} \tag{12}$$

and

$$\bar{B}^n(s) := \sum_{i=0}^{\nu} B_i^n e^{-s\tau_i^n} \tag{13}$$

Let us relate the two TFMs as

$$G(s) = G^n(s) \left(I + E(s) \right) \tag{14}$$

where $E(s)$ is the *multiplicative error matrix* between the actual TFM, $G(s)$, and the nominal TFM, $G^n(s)$. The next result gives a frequency-dependent upper bound on the norm of $E(s)$:

Lemma 1: Let

$$e_n(\omega) := \beta + \sum_{i=1}^{\nu} \bar{\sigma} \left(B_i^n \right) \rho_i(\omega) + \gamma(\omega) \tag{15}$$

and

$$e_d(\omega) := \underline{\sigma} \left(\bar{B}^n(j\omega) \right) - \gamma(\omega) \tag{16}$$

where $\beta := \sum_{i=0}^{\nu} \beta_i$,

$$\rho_i(\omega) := \begin{cases} 2 \sin \left(\dfrac{|\omega| \theta_i}{2} \right) \ , & |\omega| \leq \dfrac{\pi}{\theta_i} \\ 2 \ , & |\omega| > \dfrac{\pi}{\theta_i} \end{cases} \ , \quad i = 1, \ldots, \nu,$$

and

$$\gamma(\omega) := \left(\alpha + \delta\omega + \sum_{i=1}^{\nu} \bar{\sigma} \left(j\omega D_i^n - A_i^n \right) \rho_i(\omega) \right) \bar{\sigma} \left(G_o(j\omega) \right)$$

where $\alpha := \sum_{i=0}^{\nu} \alpha_i$, $\delta := \sum_{i=0}^{\nu} \delta_i$, and

$$G_o(s) := \left(s\bar{D}^n(s) - \bar{A}^n(s) \right)^{-1} \bar{B}^n(s)$$

Then, assuming that $e_d(\omega) > 0$, $\forall \omega \in \mathbf{R}$,

$$\bar{\sigma}(E(j\omega)) \leq \frac{e_n(\omega)}{e_d(\omega)} =: e(\omega) \ , \qquad \forall \omega \in \mathbf{R} \ . \tag{17}$$

Proof: By (9) and (11), $E(s)$ in (14) can be chosen to satisfy

$$\left(s\bar{D}(s) - \bar{A}(s) \right)^{-1} \bar{B}(s) = \left(s\bar{D}^n(s) - \bar{A}^n(s) \right)^{-1} \bar{B}^n(s) \left(I + E(s) \right)$$

By premultiplying both sides by $\left(s\bar{D}(s) - \bar{A}(s) \right)$ and rearranging terms we obtain $Q(s) = R(s)E(s)$, where

$$Q(s) := \sum_{i=0}^{\nu} B_i^u e^{-s\tau_i} + \sum_{i=1}^{\nu} B_i^n \psi_i(s) - \Gamma(s)$$

and

$$R(s) := \bar{B}^n(s) + \Gamma(s)$$

where $\psi_i(s) := e^{-s\tau_i} - e^{-s\tau_i^n}$ and

$$\Gamma(s) := \left[\sum_{i=0}^{\nu} \left(sD_i^u - A_i^u \right) e^{-s\tau_i} + \sum_{i=1}^{\nu} \left(sD_i^n - A_i^n \right) \psi_i(s) \right] G_o(s)$$

Note that $|\psi_i(j\omega)| \leq \rho_i(\omega)$, $i = 1, \ldots, \nu$, and $\bar{\sigma}(\Gamma(j\omega)) \leq \gamma(\omega)$, $\forall \omega \in \mathbf{R}$. The desired result now follows on noting that $\bar{\sigma} \left(Q(j\omega) \right) \leq e_n(\omega)$ and $\underline{\sigma} \left(R(j\omega) \right) \geq e_d(\omega)$. □