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Regenerative delay, parametric forcing and machine tool chatter: A review

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Abstract: Two intrinsic component of machine tool chatter modeling is regenerative time delay and parametric forcing. The corresponding governing equations are therefore given in the form of delay-differential equations with time-periodic coefficients. In this paper, a brief review is given on the mechanism of these two effects, and recent numerical techniques from the literature are categorized and discussed.

Keywords: regenerative delay, parametric forcing, stability, numerical techniques.

1. INTRODUCTION

One of the most important fields of engineering where large time delays appear in the model equations is machine tool vibration, where the delay time could be several times larger than the characteristic time periods in the system, while the damping effects in the machine tool system are very small. After the pioneering work of Tobias (1965) and Tlusty et al. (1962), the so-called regenerative effect became the most commonly accepted explanation for machine tool chatter. This effect is related to the cutting-force variation due to the wavy workpiece surface cut one revolution ago. The phenomenon can be described by involving time delay in the model equations. Stability properties of the machining process are depicted by so-called stability lobe diagrams, which plot the maximum stable axial depths of cut versus the spindle speed. These diagrams provide a guide to the machinist to select optimal technological parameters in order to achieve maximum material removal rate without chatter. Although there exist many sophisticated methods to optimize manufacturing processes, machine tool chatter is still an existing problem in manufacturing centers (Altintas and Weck, 2004; Schmitz and Smith, 2009; Quintana and Ciurana, 2011; Altintas, 2012).

In case of turning operations, regenerative chatter can be described by time-invariant delay-differential equations (DDEs). Stability of these systems can be analyzed by the classical D-subdivision method (Stepan, 1989). In the case of milling, surface regeneration is coupled with parametric excitation of the cutting teeth, resulting in a DDE with time-periodic coefficients. Stability analysis of these systems requires the application of the Floquet theory of DDEs. In the recent years, several numerical techniques have been developed in order to determine stability lobe diagrams for milling operations, such as the semi-discretization method (Insperger and Stepan, 2002a, 2011), the temporal finite element method (Bayly et al., 2003); the multi-frequency solution (Altintas and Budak, 1995; Budak and Altintas, 1998; Bachrathy and Stepan, 2013; Otto et al., 2014), just to mention a few.

This paper aims to give a brief overview on the main issues of machine tool chatter. First the regenerative delay is explained in details for an orthogonal turning operation. Then parametric forcing is described for a one-degree-of-freedom model of milling operations. Finally, a series of numerical techniques, which were developed for the stability prediction of machining operations in the last 15-20 years, are categorized and discussed

2. REGENERATIVE DELAY

Time delay in machine tool chatter shows up as result of the so-called surface regeneration. In this section, the phenomenon is explained briefly. Figure 1 shows the chip removal process in an orthogonal turning operation for an ideally rigid tool and for a compliant tool. In the latter case, the tool experiences bending vibrations in directions x and y and leaves a wavy surface behind. The system can be modeled as a two-degrees-of-freedom oscillator excited by the cutting force, as shown in Fig. 2. If there is no dynamic coupling between x and y directions, then the governing equation can be given as

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = F_x(t) \,, \qquad (1)$$

$$m\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) = F_y(t) \,, \qquad (2)$$

where m, c_x, c_y, k_x, and k_y are the modal mass and the damping and stiffness parameters in the x and y directions, respectively. The cutting force is given in the form

$$F_x(t) = K_x\, w\, h^q(t) \,, \qquad (3)$$

$$F_y(t) = K_y\, w\, h^q(t) \,, \qquad (4)$$

where K_x and K_y are the cutting-force coefficients in the tangential (x) and the normal (y) directions, w is the depth of cut (also known as the width of cut or the chip width in the case of orthogonal cutting), $h(t)$ is the instantaneous chip thickness, and q is the cutting-force exponent. Note that other formulas for the cutting force are also used in the literature; see, e.g., Kienzle (1957); Shi and Tobias (1984); Dombovari et al. (2008).

If the tool was rigid, then the chip thickness would be constant $h(t) \equiv h_0$, which is equal to the feed per

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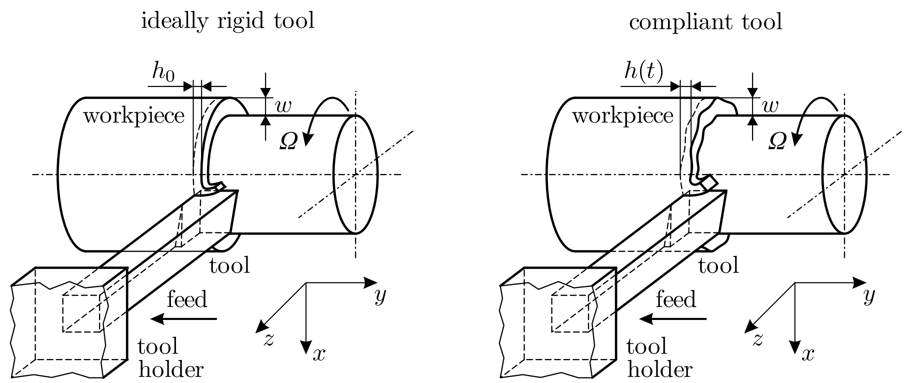

Fig. 1. Chip removal in orthogonal turning processes in the case of an ideally rigid tool and real compliant tool.

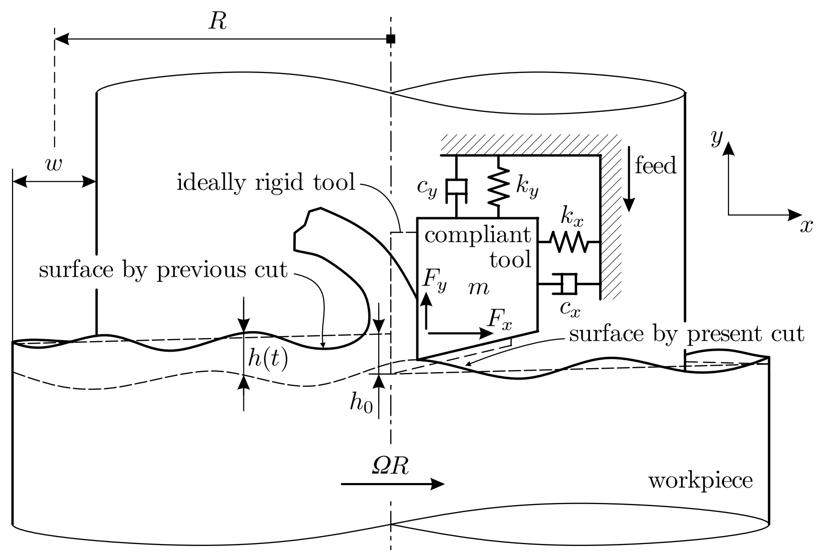

Fig. 2. Surface regeneration in an orthogonal turning process: the instantaneous chip thickness $h(t)$ is varying due to the vibrations of the tool.

revolution (in case of orthogonal cutting). However, in reality, the tool experiences vibrations, which are recorded on the workpiece, and after one revolution, the tool cuts this wavy surface. The chip thickness $h(t)$ is determined by the feed motion, by the current and by an earlier position of the tool. If the displacement $x(t)$ is negligible compared to the radius R of the workpiece, then the time delay τ between the present and the previous cuts is

$$\tau = \frac{60}{\Omega}, \tag{5}$$

where Ω is the spindle speed given in [rpm].

The chip thickness can be given as the linear combination of the feed and the present and the delayed positions of the tool in the form

$$h(t) = h_0 + y(t-\tau) - y(t) \,. \tag{6}$$

Thus, the governing equations can be written as

$$m\ddot{x}(t) + c_x\dot{x}(t) + k_x x(t) = K_x w \left(h_0 + y(t-\tau) - y(t)\right)^q , \tag{7}$$

$$m\ddot{y}(t) + c_y\dot{y}(t) + k_y y(t) = K_y w \left(h_0 + y(t-\tau) - y(t)\right)^q . \tag{8}$$

Let $x_{\rm st}$ and $y_{\rm st}$ denote the constant solution that satisfies (7)–(8). The general solution can be written as $x(t) = x_{\rm st} + \xi(t)$ and $y(t) = y_{\rm st} + \eta(t)$ with $\xi(t)$ and $\eta(t)$ being perturbations around $x_{\rm st}$ and $y_{\rm st}$, respectively. Substitution into (7)–(8), expansion into power series with respect to $\xi(t)$ and $\eta(t)$, and elimination of higher-order terms give the variational system in the form

$$m\ddot{\xi}(t) + c_x\dot{\xi}(t) + k_x\xi(t) = K_x w q h_0^{q-1} \left(\eta(t-\tau) - \eta(t)\right), \tag{9}$$

$$m\ddot{\eta}(t) + c_y\dot{\eta}(t) + k_y\eta(t) = K_y w q h_0^{q-1} \left(\eta(t-\tau) - \eta(t)\right). \tag{10}$$

Note that (9) is an ODE with state variable ξ forced by η, while (10) is a linear time-invariant DDE with state variable η. Since the homogeneous part of (9) is a simple damped oscillator, the stability of the system is determined by (10) only.

By parameter transformation, (10) can be written in the form

$$\ddot{\eta}(t) + 2\zeta\omega_{\rm n}\dot{\eta}(t) + \omega_{\rm n}^2\eta(t) = H\left(\eta(t-\tau) - \eta(t)\right) \ , \tag{11}$$

where $\omega_{\rm n} = \sqrt{k_y/m}$ is the natural angular frequency, $\zeta = c_y/(2m\omega_{\rm n})$ is the damping ratio of the tool in the y direction, and $H = K_y w q h_0^{q-1}/m$ is the specific cutting-force coefficient. Note that H is linearly proportional to the depth of cut w, which is an important technological parameter for the machinist. Equation (11) is the simplest mathematical model that describes regenerative machine tool chatter.