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# Stability analysis of machine-tool vibrations in the frequency domain

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**Abstract:** The identification of the stability lobes for machine-tool vibrations via frequency domain methods is presented using the example of interrupted turning process with round inserts. An analytic cutting force model for round inserts is derived, where the directional factors depend nonlinear and non-homogeneous on the depth of cut. Vibrations in tangential, normal and axial direction are taken into account. In particular, tangential vibrations in cutting speed direction lead to a state-dependent delay. Thus, the model is characterized by periodic excitation due to interrupted cutting and state-dependent delays due to tangential vibrations. The application of the multifrequency solution for the stability analysis of such non-autonomous systems with state-dependent delays is presented.



*Keywords:* Dynamic systems, Stability analysis, Vibration, State-dependent time delay

## 1. INTRODUCTION

Vibrations at machine-tools are a frequently encountered problem in manufacturing industry. In metal cutting, oscillations between tool tip and workpiece cause noise, bad surface finish and tool wear. One common reason for this undesired phenomenon are self-excited vibrations. If this self-excited system becomes unstable, the amplitude of the vibrations increases, which is well-known as regenerative chatter. For an optimal and efficient cutting process it is necessary to identify the stability lobes of regenerative chatter.

On the one hand, there are methods for the identification of the stability lobes in time-domain, e.g. semidiscretization (Insperger and Stépán, 2002) or spectral element approach (Khasawneh and Mann, 2013). On the other hand, frequency domain methods can be used for the stability analysis (Altintas, 2012; Otto et al., 2014). A reliable and efficient way for the numerical implementation of the frequency domain method was shown in (Bachrathy and Stepan, 2013). A comparison of time and frequency domain methods is given in (Altintas et al., 2008). One advantage of frequency domain methods is the direct use of measurements for the structural dynamics in form of frequency response functions (FRFs). Furthermore, in time domain the system dimension increases with the number of eigenmodes of the structure, that are taken into account. In contrast, in the frequency domain an arbitrary number of modes can be considered without an increase of the dimension and losses of performance or accuracy. Thus, frequency domain methods are advantageous for complex structural models, which are often required in applications.

The dynamics of metal cutting processes are typically characterized by periodic excitation and time delay. Moreover, the time delay in metal cutting can become state-dependent (Insperger et al., 2007). In this paper the frequency domain stability analysis is shown for such non-autonomous systems with state-dependent delay. As a paradigmatic example, we choose interrupted turning with round inserts. In this specific case the time-varying coefficients are piecewise constant. An efficient way for the stability analysis of interrupted turning based on an analytic calculation of the fundamental matrix for an ODE was shown in (Szalai and Stépán, 2006). However, this method cannot be applied in a straightforward way to other problems with continuously time-varying coefficients. One may argue that the Fourier method in this paper is not suitable for interrupted cutting, because the Fourier series for the piecewise constant coefficients converges slowly. However, the structural dynamics in metal cutting processes acts as a low pass filter (Altintas, 2012), and no problems, for example, with the Gibbs phenomenon occur. In fact, often already the zeroth-order approximation based on the time-averaged coefficients can predict the stability lobes in metal cutting very well (Sellmeier and Denkena, 2011).

Two main advances are presented in the paper. On the one hand, the frequency domain stability analysis for systems with state-dependent delay is shown. The effect of the state-depenend delay was already studied for a single degree of freedom (DOF) model in (Insperger et al., 2007). With our method it is possible to study this effect for models with an arbitrary number of modes based on the FRFs of the structure. On the other hand, an analytic cutting force model for round inserts is derived, which can be also used for cutting tools with non-negligible nose radii. With this analytic model the segmentation of the tool into small segments as it was done in (Budak and Ozlu, 2007) can be omitted. In contrast to typical models for the cutting force in turning or milling, our model leads to directional factors that depend non-homogeneous and nonlinear on the depth of cut.

The paper is organized as follows. In the next section the model for interrupted turning with round inserts is derived. Sec. 3 describes the stability analysis in the


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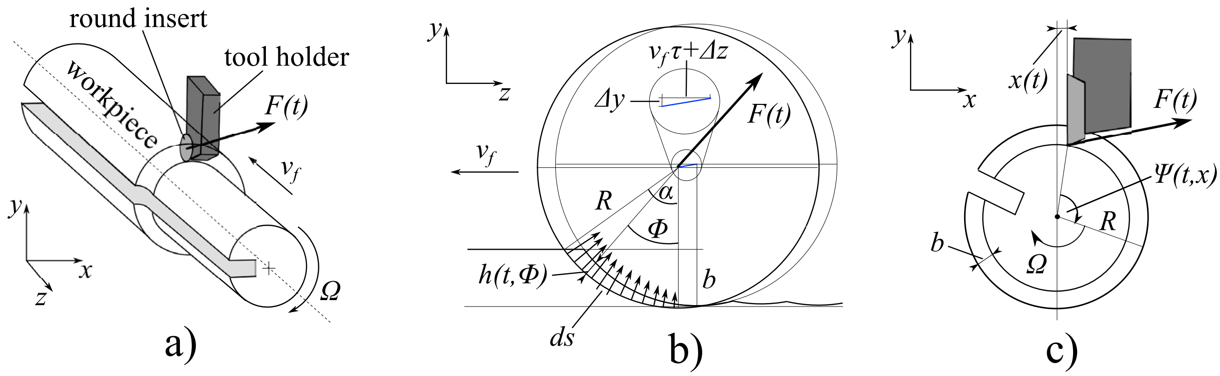


Fig. 1. a) A round insert is cutting a cylindrical workpiece with interruption. b) The cutting force $\boldsymbol{F}(t)$ at the round insert consists of local forces $d\boldsymbol{F}(t,\phi)$. c) The effect of vibrations in cutting speed direction on the angular tool tip position $\Psi(t,x)$ is illustrated.

frequency domain, and in Sec. 4 the stability lobes for a numerical example are shown.

## 2. INTERRUPTED TURNING WITH ROUND INSERTS AND STATE-DEPENDENT DELAY

In this contribution the frequency domain method is shown using the example of an interrupted turning process with round insert. A simple model for such processes is derived in this section. It is kept as simple as possible that it can be easily adapted to other cutting processes.

### 2.1 Structural dynamics

The machine-tool vibrations are described by the displacements at the tool tip $\boldsymbol{r}(t)$ in the three Cartesian coordinates. The geometry of the process is schematically shown in Fig. 1 a). The displacements $\boldsymbol{r}(t)$ can be composed of displacements of various structural modes of the machine-tool, the tool-holder or the workpiece. A description of the tool tip vibrations $\hat{\boldsymbol{r}}(\omega)$ in the frequency domain can be characterized by

$$\hat{\boldsymbol{r}}(\omega) = \boldsymbol{\Phi}(\omega)\hat{\boldsymbol{F}}(\omega). \tag{1}$$

The elements of the matrix $\boldsymbol{\Phi}(\omega)$ are the FRFs at the tool tip in the three Cartesian directions, $\Phi_{mn}(\omega)$, $m,n \in \{x,y,z\}$. The vector $\hat{\boldsymbol{F}}(\omega)$ is the frequency domain representation of the cutting force acting on the tool tip.

### 2.2 Cutting force model

The interruption of cutting in turning is characterized by the periodic window function

$$\mathrm{g}(t) = \begin{cases} 1, & \text{if } \mod(t,\tau) \leq \epsilon\tau, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

The time $\tau$ is the time for one spindle revolution. The function $\mathrm{g}(t)$ is assumed to be $\tau$-periodic because of the workpiece rotation. The value $\epsilon$ specifies the fraction of the period $\tau$, where the tool is in the cut. Thus, $\mathrm{g}(t)$ is one for cutting and zero otherwise. As we will see below, $\tau$ and $\epsilon$ are not necessarily constant. The cutting force $\boldsymbol{F}(t)$ is divided into local forces $d\boldsymbol{F}(t,\phi)$ at infinitesimal segments of length $ds$ of the cutting edge, whose location is specified by the angle $\phi$ as illustrated in Fig. 1 b). It is assumed, that the local cutting force at the angular position $\phi$ can be described by

$$d\boldsymbol{F}(t,\phi) = \mathrm{g}(t)K_t \begin{pmatrix} 1 \\ k_n \cos\phi \\ k_n \sin\phi \end{pmatrix} h(t,\phi)ds, \tag{3}$$

where $K_t$ is the tangential cutting force coefficient, and $k_n$ is the ratio between normal and tangential cutting force. Equation (3) can be understood as follows. The direction of the local cutting force lies in a plane spanned by the radial and axial direction of the round insert. The modulus of the cutting force depends linearily on the local chip thickness $h(t,\phi)$. If $ds = Rd\phi$ with the insert radius $R$ is substituted into (3), the macroscopic cutting force $\boldsymbol{F}(t)$ can be determined by the integral of the local forces $d\boldsymbol{F}(t,\phi)$ along the cutting edge from $\phi = 0$ up to the engagement angle $\phi = \alpha$

$$\boldsymbol{F}(t) = \mathrm{g}(t)K_tR \int_0^\alpha \begin{pmatrix} 1 \\ k_n \cos\phi \\ k_n \sin\phi \end{pmatrix} h(t,\phi)d\phi. \tag{4}$$

The engagement angle $\alpha$ is specified by the radius $R$ of the insert and the depth of cut $b$ (see Fig. 1 b))

$$\cos\alpha = 1 - \frac{b}{R}. \tag{5}$$

The effect of $y$-displacements on the engagement angle $\alpha$ is neglected, because the vibrations $y(t)$ are typically much smaller than the depth of cut $b$.

### 2.3 Chip thickness and time delay

The local chip thickness $h(t,\phi)$ is measured in the radial direction of the insert

$$h(t,\phi) = v_f\tau\sin\phi + (0, \cos\phi, \sin\phi)\,\Delta\boldsymbol{r}(t). \tag{6}$$

The first term in (6) specifies the part of the chip thickness due to the feed motion with velocity $v_f$. The second term varies dynamically, where $\Delta\boldsymbol{r}(t)$ is the difference between the tool tip position at the present and the previous cut

$$\Delta\boldsymbol{r}(t) = \boldsymbol{r}(t-\tau) - \boldsymbol{r}(t). \tag{7}$$

We take into account the effect of vibrations in the cutting speed direction. As can be seen in Fig. 1 c), the angular position $\Psi(t,x)$ of the tool tip relative to a fixed workpiece point can be specified by

$$\Psi(t,x) = \Omega t - \frac{x(t)}{R}, \tag{8}$$