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On Stable Controller Design For Robust Stabilization of Time Delay Systems

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Abstract: This paper studies the problem of robust stabilization of an infinite dimensional plant by a stable and possibly low order controller. The plant of interest is assumed to have only finitely many simple unstable zeros, however, may have infinitely many unstable poles. In the literature, it has been shown that the problem can be reduced to an interpolation problem and it is possible to obtain lower and upper bounds of the multiplicative uncertainty under which an infinite dimensional stable controller can be generated by a modified Nevanlinna-Pick formulation. We propose that the same interpolation problem can be solved approximately by a finite dimensional approach and present a finite dimensional interpolation function which can be used to find a stable controller. We illustrate this idea by a numerical example and additionally show the effects of the free design parameters of the rational interpolating outer function approach on the numerical example.

Keywords: Strong stabilization, robust stabilization, infinite dimensional systems, $\mathcal{H}^\infty$ control, low degree controller

1. INTRODUCTION

This paper is about strong and robust stabilization of SISO infinite dimensional plants, specifically time delay systems, by a finite dimensional controller. Strong stability requires a stable controller to be designed. A stable controller has two main advantages: it is robust to sensor failures (i.e. undetermined feedback input or saturation of control input) as described by Doyle et al. (1990), and Ünal and İftar (2012c) and it is testable stand-alone as mentioned by van de Wal et al. (2002). It is possible to test a stable controller by its input-output relationship practically by applying some test signals as an open-loop system before using it with the original plant to prevent undesired errors.

It is essential to mention that strong stability means achieving stabilization by a stable controller which is different than some other strong stability concepts used in the infinite dimensional system theory, e.g. Hale and Lunel (2002).

There is a rich literature for strong stabilization of finite dimensional plants, see e.g. Campos-Delgado and Zhou (2003), Cheng et al. (2007), Cheng et al. (2011), Gümüşsoy and Özbay (2009), Petersen (2009), and Gündeş and Özbay (2011) and see also Gümüşsoy et al. (2008) for sensitivity shaping of infinite dimensional systems by fixed order stable controllers. Nevertheless, robust stabilization by a stable controller remains to be an active open research area and to our knowledge; the most recent contribution, which has been made by Wakaiki et al. (2013), gives some sufficient conditions as discussed below in more detail. Most recently, the same idea has been extended to the mixed sensitivity minimization by stable controllers, Wakaiki and Yamamoto (2014); see also Gümüşsoy and Özbay (2009) for an alternative earlier approach. We refer to Ünal and İftar (2012b) and Ünal and İftar (2012c) for recent results on stable $\mathcal{H}^\infty$ controller design for plants with input-output delays. It should be noted that for time delay systems, parity interlacing property, p.i.p., (having even number of poles between any pair of extended right half plane zeros) is necessary (and sufficient with added restrictions) for the existence of a strongly stabilizing controller, Ünal and İftar (2012a).

In their paper, Wakaiki et al. (2013) have studied infinite dimensional plants having finitely many simple unstable zeros but possibly infinitely many unstable poles. The authors have tried to find a way to calculate upper and lower bounds for the largest multiplicative uncertainty under which a robustly stabilizing stable controller can be generated. They have used a method developed in Gümüşsoy and Özbay (2009) and Özbay (2010), where an extension to the well known Nevanlinna-Pick interpolation algorithm is used. After calculating the upper and lower bounds, Wakaiki et al. (2013) also proposed how to generate a stable controller for a given multiplicative uncertainty. In this approach, for each bound, a modified version of the Nevanlinna-Pick interpolation problem is solved and the resulting interpolating function is an infinite dimensional one. Since this interpolating function is a part of the designed controller, the resulting controller ends up to be infinite dimensional, independent of the other components.

In this paper, we present an application of an old method appearing in Vidyasagar (1985) and Doyle et al. (1990),

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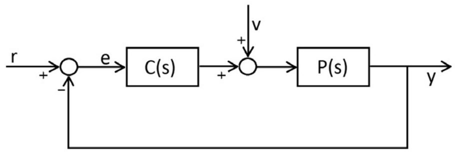

Fig. 1. The standard unity feedback system.

where a finite dimensional approach is used to find an interpolating outer function, without considering the $\mathcal{H}^\infty$ norm condition to be satisfied for robustness. We apply this method to solve the relaxed problems described by Wakaiki et al. (2013) to calculate "approximate" upper and lower bounds, then check the resulting $\mathcal{H}^\infty$ norm condition to verify that the stable controller designed also satisfies the robustness condition.

The paper is organized as follows: in Section 2 some details and brief results from Wakaiki et al. (2013) are presented. In Section 3 the method proposed in Vidyasagar (1985) and Doyle et al. (1990) is recalled and it is applied to the relaxed problems to calculate approximate maximum allowable uncertainty bounds. The effects of different free design parameters on the numerically calculated interpolation function are investigated in Section 5. Finally, Section 6 contains some concluding remarks on the proposed method.

2. PROBLEM DEFINITION

In this paper, we consider the feedback system shown in Figure 1 where C is required to be a stable controller, i.e. $C \in \mathcal{H}^\infty$, for a given infinite dimensional plant P. Recall that the feedback system (C, P) is stable if and only if $S = (1 + PC)^{-1}$, CS and PS are $\mathcal{H}^\infty$ functions. A controller $C \in \mathcal{H}^\infty$ leading to a stable feedback system (C, P), is said to be a strongly stabilizing controller for the given plant P.

In this paper we consider the same class of plants considered in Wakaiki et al. (2013):

$$P = \frac{N}{D} \tag{1}$$

such that $N \in \mathcal{H}^\infty$, $D \in \mathcal{H}^\infty$ and the pair (N, D) are strongly co-prime as in Smith (1989). A controller strongly stabilizing P is a robustly stabilizing controller for the set

$$\mathcal{P}_\rho := \{P_\Delta = (1 + W\Delta)P : \Delta \in \mathcal{H}^\infty, \|\Delta\|_\infty < 1/\rho\} \tag{2}$$

if and only if it satisfies

$$\|WT\|_\infty \leq \rho \tag{3}$$

where $T = PC(1 + PC)^{-1}$, and W characterizes the frequency distribution of the multiplicative plant uncertainty. In this work we assume that $W, W^{-1} \in \mathcal{H}^\infty$.

As in Wakaiki et al. (2013) we consider an inner-outer factorization of N and D so that

$$P = \frac{M_n}{M_d} N_0 \tag{4}$$

where M_n is inner and finite dimensional, M_d is inner and possibly infinite dimensional and $N_0, N_0^{-1} \in \mathcal{H}^\infty$.

Note that when M_d is infinite dimensional the plant contains infinitely many poles in $\mathbb{C}_+$ (e.g. a neutral time delay system with asymptotic pole chains in $\mathbb{C}_+$); however requiring $N_0^{-1} \in \mathcal{H}_\infty$ imposes a restriction that the plant is not strictly proper.

Example: A typical plant example satisfying the above conditions is the neutral time delay system

$$P(s) = \frac{(s-\alpha)(s-4e^{-s}+1)}{(s-10)(s-15)(2e^{-s}+1)} \tag{5}$$

where the factorization is in the form

$$\begin{aligned} M_n(s) &:= \frac{(s-\alpha)(s-p)}{(s+\alpha)(s+p)} \\ M_d(s) &:= \frac{(s-10)(s-15)(2e^{-s}+1)}{(s+10)(s+15)(e^{-s}+2)} \\ N_o(s) &:= \frac{(s+\alpha)(s+p)(s-4e^{-s}+1)}{(s-p)(s+10)(s+15)(e^{-s}+2)} \end{aligned} \tag{6}$$

with $p > 0$ being the only root of the quasi-polynomial $(s - 4e^{-s} + 1)$ in $\mathbb{C}_+$; p can be calculated numerically by using `qpmr` or `Yalta` packages, see Vyhlídal and Zítek (2014) and Avanessoff et al. (2013); for the above example, $p = 0.7990$.

In the rest of the paper, it is assumed that $W = KW_0$ where both W_0 and W_0^{-1} belong to $\mathcal{H}_\infty$ and $K > 0$. Thus we have robust stability if

$$\|W_0T\|_\infty < \frac{1}{\rho\, K}. \tag{7}$$

Problem Definition: In summary, the problem at hand is to find a strongly stabilizing C for P satisfying (7) for the largest possible K. We call the largest $K > 0$ for which the above problem is solvable *the largest allowable uncertainty bound*. By Ünal and İftar (2012a), in order to have a feasible solution to the strong stabilization problem, P is assumed to satisfy the parity interlacing property. In Wakaiki et al. (2013) some upper and lower bounds are computed for the largest allowable uncertainty bound.

Brief Outline of the Proposed Solution: Wakaiki et al. (2013) showed that for a given $K > 0$ the strong robust stabilization problem is solvable if and only if there exists a function F satisfying all three conditions given below:

$$F, F^{-1} \in \mathcal{H}^\infty \tag{8}$$

$$\|W - M_dF\|_\infty \leq \rho \tag{9}$$

$$F(z_i) = \frac{W(z_i)}{M_d(z_i)}, i = 1, ..., n, \tag{10}$$

where $z_1, \ldots, z_n$ are the zeros of P in $\mathbb{C}_+$. Furthermore, once such a function F is constructed, a feasible controller is given by

$$C = \frac{W - M_dF}{M_nN_0F} \tag{11}$$

Since the above problem is not straight forward to solve, Wakaiki et al. (2013) have proposed two relaxed problems each of which defines a necessary (respectively, a sufficient) condition to calculate upper and lower bounds for the largest possible multiplicative uncertainty K. In the relaxed problem, which is designed to solve for the lower bound, they have defined W_s to be $W_s, W_s^{-1} \in \mathcal{RH}^\infty$ such that $|W_s(j\omega)| \leq \rho - |W(j\omega)|$ for almost all $\omega \in \mathbb{R}$. If finitely many simple unstable zeros of the plant are called as z_i