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Deterioration model filtering by Gibbs algorithm and RUL estimation

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Abstract: Nowadays, prognosis of system lifetime is a basic requirement for condition-based maintenance in many application domains where safety, reliability, and availability are considered of first importance. One of the prognosis aims is the prediction of remaining useful life of a component or a system. This latter is modeled based on the different deterioration model, probabilistic or non-probabilistic models. In this paper a probabilistic approach for the remaining useful life estimation is proposed.

Keywords: Gamma process; deterioration model; Gibbs sampling; RUL estimation; Prognosis.

1. INTRODUCTION

The research on prognostic and remaining useful life (RUL) assessment has improved in recent years. In practical applications, a mission is required to estimate RUL based on the degradation measures. These measures often provide the information such as the failure time data, failure sensors measurements to assess the reliability of systems. The degradation phenomenon can be modelled by many approaches and the stochastic modelling is one of them. In many cases the systems has a monotone gradually deterioration. As example of such systems we can consider the deterioration due to an accumulation of damage in a mechanical system or the deterioration due to defective products in case of a production line or due to corrosion/erosion in a structural system. In these cases the deterioration can be modelled by a monotone increasing stochastic process. A well known monotone increasing stochastic process is the Gamma process. Van Noortwijk [2009] surveys the application of Gamma processes in degradation modelling and presents its characteristics. As the paths of the Gamma process are discontinuous it can be thought as the accumulation of an infinite number of small shocks. This interpretation gives credence to the model since it is often how the degradation occurs. Furthermore, the gamma process allows feasible mathematical developments. Gamma process is largely applied on the reliability systems (eg. Lawless and Crowder [2004], Singpurwalla [1997]), such as finding the first hitting times model by a stochastic process researching a boundary in Lee and Whitmore [2006], Schirru et al. [2010] uses Gamma processes for filtering and prediction of the time profile of a monotonic health factor given irregularly sampled noisy data, Park and Padgett [2005] construct the accelerated degradation models for failure based on the geometric Brownian motion or gamma process.

In this paper, the degradation data obtained by operating sensors measurements are modelled by a stochastic process in the order to calculate the system reliability. Based on a non-homogeneous Gamma process and Gaussian noise, the degradation phenomenon can be explained on the operational data of system. From the degradation indicators obtained by analyzing the operational data, the non-homogeneous Gamma process with noise considers a degradation model researching a failure boundary, thus the RUL of system can be estimated as the during from the last observed time to the failure time. An advantage of a non-homogeneous Gamma process is that it can model a non linear tendendency if the deterioration.

For the deterioration model, the main idea has the root from the degradation indicator obtained from the 2008 Prognostic Health Management Challenge data and explored in LeSon et al. [2010]. The trend of this indicator can be expressed by a stochastic process however with a large noise, the problem leads us to filter this indicator by a stochastic process with a gaussian noise, a non-homogeneous gamma process is used in this case. Hereby, a filtration based on Gibbs algorithm is used to model this indicator and to find the hidden degradation states of gamma process. Afterward, a probability distribution is associated to the RUL.

The remainder of paper is organized as follows. Section 2 presents the deterioration model using a non-homogeneous gamma process with gaussian noise and how to filter the hidden degradation states and to estimate the remaining useful life. The impact of the observations vectors size on the RUL estimation is studied the sections 3 and 4 and some numerical examples is shown in the section 4. Finally, the conclusions of the results as well as the furthers works are given in the section 5.

2. DETERIORATION MODELLING AND RUL ESTIMATION

2.1 Deterioration model

The system state at time t can be summarized by a scalar random ageing variable. Denote by $X(t)$ the system state at time t which is monotone non-decreasing. As it is mentioned in Van Noortwijk [2009] its seems sensible to associate a gamma distribution to $X(t)$. Suppose that

$X(t)$ has a gamma distribution with a scale parameter $\beta > 0$ and shape function $v(t) = \alpha t^b$ which is a non-decreasing, right continuous, real-valued function for $t > 0$ with $\alpha > 0$ and an additive parameter b, $X(t) \sim \Gamma(v(t), \beta)$. The probability density function is given by:

$$f(x|v,\beta) = \frac{\beta^v}{\Gamma(v)} x^{v-1} exp\{-\beta x\} I_{[0,+\infty[}(x),$$

where $\Gamma(v) = \int_{z=0}^{\infty} z^{v-1} e^{-z} dz$ is the Euler gamma function and $I_{[a,b]}(x) = 1$ if $x \in [a,b]$ and $I_{[a,b]}(x) = 0$ otherwise. The gamma process with shape function $v(t) > 0$ and scale parameter β has the following properties:

- $X(0) = 0$,
- $X(s) - X(t) \sim \Gamma(v(s) - v(t), \beta)$ for all $s > t \geq 0$,
- $X(t)$ has independent increments.

Denote by $Y_j = Y(t_j)$, $j = 1, ..., n$ the degradation indicators observed at inspection times $0 < t_1 < ... < t_n$, and X_j is the non-observable state at time t_j modelled by a non-homogeneous gamma process, thus the relation between X_j and Y_j can be expressed as follows:

$$Y_j = f(X_j, \epsilon_j) = X_j + \epsilon_j$$

where ϵ_j are independent gaussian random variables with standard deviation σ_j and mean equal to zero and it can be also expressed as a function of X_j and Y_j as follows: $\epsilon_j = g(X_j, Y_j) = Y_j - X_j$.

For an accurate prediction or efficient maintenance planning it is necessary to estimate the non-observable state of the system. In this purpose we should evaluate the hidden degradation states vector $\mathbf{X} = (X_1, ..., X_n)$ from the observations $\mathbf{Y} = (Y_1, ..., Y_n)$ by the following conditional density:

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_1, ..., x_n) = \frac{\mu_{\mathbf{X},\mathbf{Y}}(x_1, ..., x_n)}{\int ... \int \mu_{\mathbf{X},\mathbf{Y}}(x_1, ..., x_n) dx_1 ... dx_n} \quad (1)$$

The density $\mu_{\mathbf{X},\mathbf{Y}}(x_1, ..., x_n)$ can be deduced as follows:

$$\mu_{\mathbf{X},\mathbf{Y}}(x_1, ..., x_n) = C e^{-\beta x_n} \prod_{j=1}^{n} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} \exp(-\frac{g^2(x_j, Y_j)}{2\sigma_i^2}) |g\prime(x_j, Y_j)| \quad (2)$$

where C is a cosntant and $g\prime(., y) = \frac{\partial g(.,y)}{\partial y}$. Hence, the conditional density of the hidden degradation states from the observations can be rewritten by:

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_1, ..., x_n) = K_1 e^{-\beta x_n} \prod_{j=1}^{n} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} \exp(-\frac{g^2(x_j, Y_j)}{2\sigma_i^2}) |g\prime(x_j, Y_j)| \quad (3)$$

where K_1 is the coefficient defined as follows:

$$\frac{1}{K_1} = \int ... \int e^{-\beta x_n} \prod_{j=1}^{n} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} \exp(-\frac{g^2(x_j, Y_j)}{2\sigma^2}) |g\prime(x_j, Y_j)| dx_1 ... dx_n \quad (4)$$

Because of the presence of large number of integrals in equation (4) it is very difficult to calculate the coefficient K_1. To bypass this problem the Gibbs sampler algorithm is proposed to estimate the conditional density $\mu_{\mathbf{X}/\mathbf{Y}}$. This algorithm is based on the conditional densities for each the component X_j of $\mathbf{X}$ given by knowing the other components:

- For $j = 1$,

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_1/x_2, ..., x_n) = K_{2,1} x_1^{\alpha(t_1^b)-1} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} e^{-\frac{g^2(X_1, Y_1)}{2\sigma_1^2}} |g\prime(x_1, Y_1)| 1_{(0<x_1<x_2)} \quad (5)$$

- For $2 \leq j \leq n-1$,

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_j/x_1, ..., x_{j-1}, x_{j+1}, ..., x_n) = K_{2,j} (x_j - x_{j-1})^{\alpha(t_j^b - t_{j-1}^b)-1} (x_{j+1} - x_j)^{\alpha(t_{j+1}^b - t_j^b)-1} e^{(-\frac{g^2(x_j, Y_j)}{2\sigma^2})} |g\prime(x_j, Y_j)| 1_{(x_{j-1}<x_j<x_{j+1})} \quad (6)$$

- For $j = n$

$$\mu_{\mathbf{X}/\mathbf{Y}}(x_n/x_1, ..., x_{n-1}) = K_{2,n} e^{-\beta x_n} (x_n - x_{n-1})^{\alpha(t_n^b - t_{n-1}^b)-1} e^{-\frac{g^2(x_n, Y_n)}{2\sigma^2}} |g\prime(x_n, Y_n)| 1_{(x_{n-1}<x_n)} \quad (7)$$

The Gibbs sampler algorithm is presented more in detail in section 2.2.

2.2 Filtering of the hidden degradation states

Gibbs sampler Casella and George [1992], Tierney [1994], is a MCMC algorithm used for generating random variables from a (marginal) distribution without having to calculate the density by using elementary properties of Markov chains. In the Bayesian framework, the Gibbs sampler is mainly used to generate posterior distributions, whereas for the classical statistic problems its major use is for the calculation of the likelihood function and characteristics of the likelihood estimators. In this study, an ergodic Markov chain is generated whose invariant law is the distribution to estimate $\mu_{\mathbf{X}/\mathbf{Y}}$. The outputs of the Gibbs algorithm are the successive values of a Markov chain $\mathbf{Z}^q = (Z_1^q, ..., Z_n^q), q \in \mathbf{N}$ in which the hidden gamma degradation states $X_1, ..., X_n$ can be approximated. For the passage of the stage $\mathbf{Z}^q$ to the stage $\mathbf{Z}^{q+1}$, we draw lots successively $Z_1^{q+1}, ..., Z_j^{q+1}, ..., Z_n^{q+1}$ according to the following probability distribution:

- drawing of lots the value z_1^{q+1} of Z_1^{q+1} according to the law of $z_1 : \mu_{\mathbf{X}/\mathbf{Y}}(z_1|z_2^q, ..., z_n^q)$
- drawing of lots the value z_j^{q+1} of Z_j^{q+1} according to the law of $z_j : \mu_{\mathbf{X}/\mathbf{Y}}(z_j|z_1^{q+1}, ..., z_{j-1}^{q+1}, z_{j+1}^q, z_n^q)$, $2 \leq j \leq n-1$
- drawing of lots the value z_n^{q+1} of Z_n^{q+1} according to the law of $z_n : \mu_{\mathbf{X}/\mathbf{Y}}(z_n|z_1^{q+1}, ..., z_{n-1}^{q+1})$