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Multi-Fault Discrimination with Fault Model and Periodic Residual

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Abstract:
This work deals with the issue of faults classification on line for non linear Hessenberg systems whose faults are not isolable in steady state conditions. A stepwise procedure is proposed. The first step consists of the parameters estimation associated with one fault model when abnormal conditions are detected. As second step, a residual is calculated considering the identified model together with a periodic auxiliary input to the system. The auxiliary signal added to the system allows the discrimination of two faults scenarios via features of the residual. The novel procedure allows: (1) the identification of the fault parameters, if only one occurs and (2) the evaluation of some boundaries for the faults parameters, if two faults occurs simultaneously. To show the viability of the method, the leaks diagnosis in pipeline is considered as example.

Keywords: Fault classification, fault model, Hessenberg system, auxiliary input signals

1. INTRODUCTION

The fault isolation task for dynamic nonlinear systems requires observable subsystems sensitive to some faults and robust to the rest. The solution of this issue is based on redundant information and the consistence between normal and actual process behavior obtained by on-line measurement data (Frank and Ding, 1997).

A general theory for the nonlinear fault detection and isolation, **FDI** issues is still missing. There are attempts to overcome the difficulties using diverse tools and particular class of nonlinearities, as example (Hammouri et al., 1998), (De-Persis and Isidori, 2000), (Alcorta García, 1999), (Shields et al., 2001). The conditions in these formulations are difficult to test in complex systems and do not help to determine, which additional conditions are required to get a residual. Moreover, it has been recognized that the structure of a system plays an important role to solve **FDI** tasks.

An analytical redundant relation **ARR** means the existence of a set of equations to validate the model by process data and the maximum number of redundant relations is bounded by the difference between number of model equations and its unknown variables (Krysander et al., 2008). To study the diagnosticability of a generic system without numeric values the structural analysis **SA** has the advantage that allows to cope with large scale systems without numerical values of the parameters (Blanke et al., 2006).

A framework to study a process described by partial differential equations, like a transport process, consists to discretize the space obtaining a Hessenberg model, **HM**. An important property of this class of non linear systems is the uniformly input observability (Bernard et al., 1998); however this proper could be affected by faults because the structure of the system is strongly coupled to faults distribution vector. As consequence, not all fault scenarios can be detected in a **HM**.

A diagnosis analysis for a general **HM** with specific parameter structures as faults is carried out here using the **SA**. It is concluded that one can discriminate between one fault and multi-fault cases, if a estimated fault model is used together with auxiliar signals introduced in the system. Using the frequency spectrum of the residual, it is selected a feature which allows to distinguish some fault scenarios. The feature has the advantages that can be evaluated on line with the cross-correlation between a periodic auxiliary signal and a residual.

The paper is organized as follows. Section 2 presents the structural properties of a Hessenberg system **HM** with the assumed structure for the faults. Diverse fault scenarios are studied using **SA**. Section 3 proposes a scheme to classify two classes of faults; adding an auxiliary signal of low frequency to the **HM** and using a feature to separate two classes of faults. An illustrative example is presented in Section 4 and some conclusions and remarks are given in Section 5.

2. STRUCTURAL ANALYSIS FOR HESSENBERG FORMS

2.1 Model Structure

Consider the model defined on the domain $\Re^n$ with n odd given by

$$\begin{aligned} \dot{x} &= f(x,u,\theta), \quad x(0)=x_0 \quad u \in \Re^p \\ y &= h(x) \in \Re^2 \end{aligned} \tag{1}$$

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where faults are associated with the parameters vector $\theta \in \Re^{n-1}$ and the structure of the model has the following properties:

(1) The system is both strictly linked lower and upper Hessenberg (LH and UH). This means

$$\text{If } j > i+1, \quad \frac{\partial f_i(x,u,\theta)}{\partial x_{j+1}} = 0, \quad \frac{\partial f_i(x,u,\theta)}{\partial x_{i+1}} \neq 0$$

and

$$\text{If } j < i+1, \quad \frac{\partial f_i(x,u,\theta)}{\partial x_{j-1}} = 0, \quad \frac{\partial f_i(x,u,\theta)}{\partial x_{i-1}} \neq 0$$

(2) The output vector is upper and lower measured for all x. This means

$$y_1 = x_1 \quad \text{with} \quad x_1 \neq 0$$
$$y_2 = x_n \quad \text{with} \quad x_n \neq 0$$

(3) Each fault induces at most a two changes in vector θ with the lower structural coupled property:

- For $i \neq n$ odd and $j \neq i$ $\quad \frac{\partial f_i(x,u,\theta)}{\partial \theta_j} = 0$, therefore

$$\dot{x}_i = f_i(x_{i-1}, x_i, x_{i+1}, u, \theta_i) \qquad \text{(ci)}$$

- For j even

$$i > j \quad \frac{\partial f_j(x,u,\theta)}{\partial \theta_i} = 0$$
$$i < j \quad \frac{\partial f_j(x,u,\theta)}{\partial \theta_{i-1}} = 0$$

therefore

$$\dot{x}_j = \theta_{j-1} f_j(x_{j-1}, x_j, x_{j+1}, u, \theta_j) \qquad \text{(cj)}$$

These means the Jacobian matrix $\frac{\partial f}{\partial \theta}$ is formed by diagonal blocks with the structure

$$\begin{pmatrix} \times & 0 \\ \times & \times \end{pmatrix}$$

(4) The system is strongly detectable. This means independent on the set of admissible $u(t)$, the existence of fault produces a deviation of the output such that $\|y(t) - y_0(t)\| \neq 0$, where $y_0(t)$ is the output without faults (Chen and Patton, 1999).

2.2 Structural analysis

The **SA** requires the definitions of faults and nodes sets. For system (1), the set associated with faults is

$$\Theta = \{\theta_1^0 + \delta\theta_1, \theta_2^0 + \delta\theta_2, \ldots, \theta_{n-1}^0 + \delta\theta_{n-1}\} \qquad (2)$$

where δ denotes deviation of the nominal value, the known variables set is

$$\mathcal{K} = \{y_1, y_2, u_1, u_2\} \qquad (3)$$

the unknown variables set is

$$\mathcal{X} = \{x_2, x_3, ..., x_{n-1}, \dot{x}_1, \dot{x}_2, ...\dot{x}_n\} \qquad (4)$$

and the constraints set is

$$\mathcal{C} = \{c_1, ..., c_n, d_1, ..., d_n\} = \mathcal{C}_s \cup \mathcal{D} \qquad (5)$$

where c_i is affected by Θ and the derivative node $d_i \in \mathcal{D}$

$$\dot{x}_i := \frac{dx_i}{dt} \qquad \text{(di)}$$

The bipartite graph of (1) with variables nodes $\mathcal{V} = \mathcal{K} \cup \mathcal{X}$, constraints nodes $\mathcal{C}$ and edges set $\mathcal{E}$ given by

$$e_{ij} = \begin{cases} (c_i, v_j) & \text{if and only if } v_j \text{ appers in } c_i \\ 0 & \text{on the contrary} \end{cases}$$

is denoted $\mathcal{G} = (\mathcal{C}; \mathcal{V}, \mathcal{E})$. In the matrix description an edge e_{ij} is marked by • in row i, column j.

If faults change some members of $\mathcal{C}$, the edges e_{ij} associated to this subset are sensitive to the faults. Then, the bipartite graph allows to consider deviation in Θ as faults. Considering that the over-constrained graph $\mathcal{G}^+(\mathcal{C}^+; \mathcal{X}^+, \mathcal{E}^+)$ is the only one with redundant information, the matching process to establish relations between variables and constraints for **FDI** is carry out only using $\mathcal{V}^+$ and $\mathcal{C}^+$ (Blanke et al., 2006).

To determine the **ARR**s by a graph one matches $\mathcal{K}^+$ with constraints $\mathcal{C}^+$ in which $\mathcal{X}^+$ has been substituted and the involved constraints are interpreted as functions which map subsets of $\mathcal{K}^+_{in}$ in other subsets of $\mathcal{K}^+_{out}$, where the path is determined by a concatenation process and the subsets satisfied

$$\mathcal{K}^+_{in} \cap \mathcal{K}^+_{out} = \emptyset$$

According to Verde and Sánchez-Parra (2010)

$$\mathcal{RG}_i(\mathcal{C}_i; \mathcal{U}_{si}; y_i) \qquad (6)$$

is a **Redundant Graph** if (a) paths between the nodes of $\mathcal{U}_{si}$ and the target y_i are consistence using the constraints set $\mathcal{C}_i$; and (b) there is a lack of consistency if a fault is present in any element of the paths. The symbols $\rightarrow$ and $\leftarrow$ are used in the matrix description for initial and target nodes respectively and the symbol $\oplus$ in the row i and column j means the constraint i is used to determine variable j.

A $\mathcal{RG}$ and its associated residual are obtained in general without any consideration of the exogenous set. If the paths of a $\mathcal{RG}$ hold for all input u, the graph is called a **uniform redundant graph** and the diagnosticability properties are preserved. Thus, an observer can be designed to reconstruct the faults or estimate the parameters associated to the faults. On the contrary, a nonuniform redundant graph demands the selection of auxiliary inputs to hold the paths.

2.3 Fault Isolability Analysis

For simplicity in the analysis without loss of generality, it is assumed $n = 5$ in (1) and the incidence matrix is given in this case by

	y_1	x_2	x_3	x_4	y_5	$\dot{x}_1$	$\dot{x}_2$	$\dot{x}_3$	$\dot{x}_4$	$\dot{x}_5$	u_1	u_2	$\delta\theta_1$	$\delta\theta_2$	$\delta\theta_3$	$\delta\theta_4$
c_1	•	•				•					•		•			
c_2	•	•	•				•						•	•		
c_3		•	•	•				•							•	
c_4			•	•	•				•						•	•
c_5				•	•					•		•	•		•	
d_1	•					•										
d_2		•					•									
d_3			•					•								
d_4				•					•							
d_5					•					•						

(7)

Then, $|\mathcal{C}|$=10, $|\mathcal{X}| = 8$, $|\mathcal{K}| = 4$ and $\max|\Theta| = 4$. Thus, exists redundant information in normal conditions and the assumptions of set Θ determine under which conditions, faults can be detected, isolated and reconstructed. Four scenarios are presented.

Partial information for one fault. If there is only one fault with known θ_1 and $\dot{x} = 0$, constraint c_2 is directly affected by the unknown parameter $\delta\theta_2$. Thus, if c_2 is used in the matching, diverse redundant graphs can be achieved. The redundant graph

$$\mathcal{RG}_1(\mathcal{C} \backslash \{c_1\}; \{u_2, y_2\}; y_1) \qquad (8)$$