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Set-based Actuator Fault Diagnosis in Lure Systems

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Abstract: In this paper we present a method for actuator fault detection and identification in Lure systems. The Lure plant is controlled by an observer-based feedback tracking controller, designed for the nominal (fault-free) system. A residual signal is constructed from measurable estimation errors associated with the nominal observer. Faults are diagnosed by on-line contrasting the residual signal trajectories against a set of values that the residuals can be shown to attain under healthy or faulty operation. These values are obtained via set-invariance analysis of the system closed-loop trajectories.

Keywords: Lure systems, fault detection and identification, invariant sets.

1. INTRODUCTION

Fault tolerant control systems combine fault detection and identification [FDI] and controller reconfiguration principles in an integrated methodology aimed at automatically avoiding, or minimising, performance degradation when faults occur. In this paper we concentrate on the FDI aspect of the fault tolerant control problem for Lure type systems and a companion paper submitted to this conference treats the associated controller reconfiguration problem [Richter et al., 2012].

Numerous methodologies for FDI have been proposed in the literature since the introduction of the early techniques in the 1970s; see, for example, the monographs and surveys Ding [2008], Isermann [2006], Blanke et al. [2006], Patton et al. [2000], Basseville and Nikiforov [1993], Venkatasubramanian et al. [2003]. A well established technique for model-based FDI relies on analytical redundancy in the form of dedicated observers. These observers generate *residual variables* that act as fault indicators. Research in the area of observer based FDI has recently focused on finding an optimal tradeoff between the residual's sensitivity to fault and the robustness against disturbances by means of well-established methods in robust control theory (see, for example, Ding et al. [1993, 2000]; Henry and Zolghadri [2005]; Zhang and Ding [2008]). It is, however, worth mentioning that the majority of published work on FDI deals only with linear systems.

In the framework of linear systems we have recently proposed a new approach for actuator FDI (see, e.g., Ocampo-Martínez et al. [2008] and Seron et al. [2012]). The novelty of this approach lies in a new decision criterion for FDI based on the computation of attractive invariant sets towards which the estimation errors related to each considered fault scenario are guaranteed to converge. A key property for correct fault diagnosis is then the separation of the sets that characterise healthy operation from the ones that characterise faulty operation. Another "set-based" approach for systems with parametric uncertainty is the so-called set-membership (also known as error-bounded) approach. It is a passive robust fault detection approach that is based on computing explicitly the set of parameters or states that are consistent with the measurements [Blesa et al., 2010]. Set-membership fault detection approaches have been widely discussed in the research community; see, for example, Watkins and Yurkovich [1996]; Lesecq et al. [2003]; Ingimundarson et al. [2008]; Puig [2010]. A related passive robust fault detection approach was proposed for uncertain nonlinear systems in Wolff et al. [2008].

In this paper, we extend the aforementioned invariant set-based FDI approach to a class of nonlinear systems, namely Lure systems. Lure systems, which consist of linear dynamics with nonlinear internal feedback, are useful for representing, among others, mechanical systems with friction. The proposed FDI scheme is schematised in Figure 1. For our purposes, the Lure plant is controlled by an observer-based feedback tracking controller, as shown in the figure, designed for the nominal (fault-free) system. The structure of this nominal controller is similar to that proposed in van de Wouw et al. [2008], but suitable modifications are introduced to include tracking of a bounded state reference trajectory. For the resulting nominal controlled closed-loop system, we derive sufficient conditions, different to the ones provided in van de Wouw et al. [2008], that guarantee boundedness of the closed-loop system trajectories in the presence of bounded state and measurement disturbances, and asymptotic reference tracking in the absence of disturbances. These sufficient conditions are obtained by adapting the results of Haimovich and Seron [2011] to compute invariant sets and ultimate

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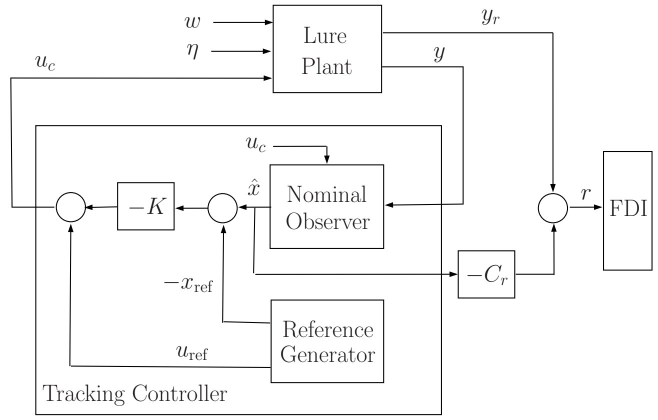

Fig. 1. Proposed FDI scheme for controlled Lure systems.

bounds for Lure-type systems. A residual signal r is then constructed from measurable estimation errors associated with the nominal observer. Faults are diagnosed by on-line contrasting the residual signal trajectories against a set of values that the residuals can be shown to attain under healthy or faulty operation. These values are obtained by the computation of invariant "healthy" and "under-fault" *residual sets* towards which the residuals are attracted in finite time and confined thereafter under different actuator fault situations that can occur in the system. We illustrate the proposed methodology through an example of a flexible link robotic arm model used in Böhm et al. [2010].

2. LURE PLANT AND TRACKING CONTROLLER

In this section we present the nominal (fault-free) closed-loop system consisting of a Lure plant and an observer-based tracking controller.

The plant is a Lure system having dynamics

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_v\varphi(C_v x(t)) + Bu_c(t) + Ew(t) \\ y(t) &= Cx(t) + \eta(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u_c(t) \in \mathbb{R}^m$ is the control input, $w(t) \in \mathbb{R}^r$ is a process disturbance bounded as[2] $|w(t)| \le \bar{w}$, $t \ge 0$, for some known constant vector $\bar{w} \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^p$ is the plant measured output and $\eta(t) \in \mathbb{R}^p$ is a bounded measurement disturbance satisfying $|\eta(t)| \le \bar{\eta}$, $t \ge 0$, for some known constant vector $\bar{\eta} \in \mathbb{R}^p$. We will assume that the nonlinear function $\varphi : \mathbb{R}^s \to \mathbb{R}^s$ satisfies

$$|\varphi(\alpha) - \varphi(\beta)| \le M|\alpha - \beta| \tag{2}$$

for all $\alpha, \beta \in \mathbb{R}^s$ and some matrix $M \in \mathbb{R}^{s\times s}$ with nonnegative entries.

The above system is controlled via the following observer-based, feedback tracking controller (see Figure 1):

$$u_c(t) = -K[\hat{x}(t) - x_{\text{ref}}(t)] + u_{\text{ref}}(t) \tag{3}$$

$$\begin{aligned} \dot{\hat{x}}(t) = A\hat{x}(t) &+ B_v\varphi(C_v\hat{x}(t) + H[y(t) - C\hat{x}(t)]) \\ &+ Bu_c(t) + L[y(t) - C\hat{x}(t)] \end{aligned} \tag{4}$$

$$\dot{x}_{\text{ref}}(t) = A\,x_{\text{ref}}(t) + B_v\varphi(C_v x_{\text{ref}}(t)) + Bu_{\text{ref}}(t) \tag{5}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the Lure-type *nominal* observer with dynamics given by (4). The reference system (5) generates a trajectory $(u_{\text{ref}}, x_{\text{ref}})$ that is solution of the nominal model. These trajectories are designed such that they are bounded and certain design specifications are satisfied. In particular, we will assume (for later use) that constant vectors $u^0_{\text{ref}} \in \mathbb{R}^m$, $\overline{u_{\text{ref}}} \in \mathbb{R}^m$ exist such that $u_{\text{ref}}(t) \in \mathcal{U}_{\text{ref}} = \{u \in \mathbb{R}^m : |u - u^0_{\text{ref}}| \le \overline{u_{\text{ref}}}\}$ for all $t \ge 0$.

The feedback gain K and the observer gains L and H in the controller (3)–(5) are design parameters. In particular, L and K are designed so that the matrices $A - LC$ and $A - BK$ are Hurwitz (this requires detectability of (A, C) and stabilisability of (A, B), which will be assumed).

3. CLOSED-LOOP SYSTEM AND INVARIANT SETS

In this section we analyse the nominal closed-loop system behaviour by studying the dynamics of the state estimation error $\tilde{x}$ and state tracking error e. These errors are defined as

$$\tilde{x}(t) \triangleq x(t) - \hat{x}(t) \qquad e(t) \triangleq x(t) - x_{\text{ref}}(t) \tag{6}$$

and their dynamics are given by[3]

$$\begin{aligned} \dot{\tilde{x}} = (A - LC)\tilde{x} + Ew - L\eta + B_v\varphi(C_v x) \\ - B_v\varphi(C_v\hat{x} + HC\tilde{x} + H\eta) \end{aligned} \tag{7}$$

$$\begin{aligned} \dot{e} = (A - BK)e + BK\tilde{x} + Ew + B_v\varphi(C_v x) \\ - B_v\varphi(C_v x_{\text{ref}}) \end{aligned} \tag{8}$$

Note that in the second equation above we have used $u_c = -K(e - \tilde{x}) + u_{\text{ref}}$, which follows from (3) and (6).

Let

$$\xi(t) \triangleq \begin{bmatrix} \tilde{x}(t) \\ e(t) \end{bmatrix} \tag{9}$$

and using $\delta \triangleq Ew - L\eta$, we define

$$\mathbf{A} \triangleq \begin{bmatrix} A - LC & 0 \\ BK & A - BK \end{bmatrix}, \qquad \mathbf{K} \triangleq [K \ \ -K]$$

$$\Phi(\xi, w, \eta) \triangleq \begin{bmatrix} B_v[\varphi(C_v x) - \varphi(C_v\hat{x} + HC\tilde{x} + H\eta)] + \delta \\ B_v[\varphi(C_v x) - \varphi(C_v x_{\text{ref}})] + Ew \end{bmatrix}$$

We can then write (7)–(8) as

$$\dot{\xi} = \mathbf{A}\xi + \Phi(\xi, w, \eta) \tag{10}$$

where $\mathbf{A}$ is Hurwitz (by design of L and K, see last paragraph of Section 2), and, using (2) and the bounds on the disturbances, the "perturbation" term $\Phi(\xi, w, \eta)$ can be bounded as

$$\begin{aligned} |\Phi(\xi, w, \eta)| \le \underbrace{\begin{bmatrix} |B_v|M|C_v - HC| & 0 \\ 0 & |B_v|M|C_v| \end{bmatrix}}_{G} |\xi| + \\ \underbrace{\begin{bmatrix} |E| & |B_v|M|H| + |L| \\ |E| & 0 \end{bmatrix} \begin{bmatrix} \bar{w} \\ \bar{\eta} \end{bmatrix}}_{g} \triangleq G|\xi| + g \end{aligned} \tag{11}$$

To analyse the properties of the above closed-loop dynamics, we will apply the following result.

Theorem 3.1. (Boundedness and Set Invariance). Consider the system

$$\dot{\xi}(t) = \mathbf{A}\xi(t) + \phi(t)$$

[2] Absolute values (or magnitudes in the case of complex entries) and inequalities of vectors and matrices are considered elementwise.

[3] In the sequel, we remove in some equations the dependence of the variables on continuous-time t when clear from the context.