8th IFAC Symposium on Fault Detection,
Supervision and Safety of Technical Processes (SAFEPROCESS)
August 29-31, 2012. Mexico City, Mexico

Detection of Sensor Drifts using a Standardized Sum of Innovation Test for a Pressurizer in a Nuclear Power Plant

Sungwhan Cho* and Jin Jiang**

Department of Electrical & Computer Engineering, The University of Western Ontario, London, Ont. N6A 5B9, Canada
**(Tel.: +1-519-661-2111x81271, e-mail: scho25@ uwo.ca)*
***(Tel.: +1-519-661-2111x88320, e-mail: jjiang@eng.uwo.ca)*

Abstract: A statistical test method for detection of the sensor drift is proposed in this paper. The method generates the standardized sum of the innovations for a test statistic. The test statistic passes through a decision function defined by the test to decide whether the processed signal is drift or not. The distribution of the standardized sum of the innovations and the average sample number function of the test are derived. The average sample number function is compared using the sequence data simulated by a pressurizer model of a nuclear power plant with those of the Shewhart control chart, sequential probability ratio test, and generalized likelihood ratio test to analyze the performance. The results of the comparison show that the proposed test is a most powerful test among the algorithms under the unknown small change magnitude condition.

Keywords: Drift, Change Detection, Pressurizer, SPRT, GLRT, Average Sample Number

1. INTRODUCTION

Control charts, also known as Shewhart control charts, in statistical process control can be used to determine whether there exist drifts in the measurements based on a sufficiently large amount of data. A major disadvantage of the Shewhart control charts is that the charts use only the information about the process contained in the last observation and ignore any information provided by the entire sequence of the process. This potentially makes the Shewhart control charts less useful in detecting the drift that is not typically resulting in large process variations.

To utilize the accumulated information observed from the previous sequence, a sequential probability ratio test (SPRT) approach formulated by Wald (1947), and a cumulative sum approach (CUSUM), first proposed by Page (1954), can be used for detection of changes from the drifted measurements. The approaches of the SPRTs and CUSUM tests use the likelihood ratio as a test statistic. When using the likelihood ratio, the error probabilities (Type I, Type II) cannot be exactly stated. The error probabilities can only be defined by the upper limit in the approaches of the likelihood ratio test. Further, the SPRTs and CUSUM tests can only be optimal in the case where distributions are completely known before and after the change, which is not the case in detection of the drift where the change magnitude is usually not known. In the case of unknown change magnitude, a generalized likelihood ratio (GLR) test approach proposed by Willsky and Jones (1976) can also be considered. In the GLR test, the unknown change magnitude is estimated by the maximum likelihood from measurements, and then used in the likelihood ratio test. By using the likelihood ratio, the GLR test cannot state the exact error probabilities either.

A standardized sum of the innovation test (SSIT) which is a modification to the Shewhart control charts is proposed in this paper. The SSIT can detect small scale changes faster than the Shewhart control charts by using the cumulated information previously observed as the test statistic. The SSIT also enables to specify the exact requirement for the mean time between false alarms which distinguishes the algorithm from the SPRTs, CUSUM tests, and the GLR tests.

The SSIT is defined in Section 2. The application of the SSIT to a model of a pressurizer of a nuclear power plant is demonstrated in Section 3. In Section 4, the ASN (Average Sample Number) function of the SSIT is compared with that of other algorithms (Shewhart control chart, SPRT, GLR test) to analyze the performance of the proposed SSIT. Conclusions are made based on the simulation results and the analysis of the ASN.

2. STANDARDIZED SUM OF INNOVATION TEST

2.1 The test statistic of the SSIT

Each innovation from a Kalman filter is an independent and identically distributed (*i.i.d.*) random variable of a zero mean normal distribution. The SSIT produces a test statistic from the sum of the innovations at each step. Hence, the distribution of the sum of the innovations should be specified firstly.

Let $\{X_i\}$ ($i=1, 2, \ldots, n$) be a sequence of an *i.i.d.* normally distributed random variables, i.e.

$$X_i \sim N(\mu_0, \sigma^2). \tag{1}$$

The distribution of the sum of the first j element of the sequence becomes

$$Z_1^j = \sum_{i=1}^{j} X_i \tag{2}$$
,

and follows to

$$Z_1^j \sim N\left(j\mu_0, \left(\sqrt{j}\sigma\right)^2\right) \tag{3}$$
.

The SSIT uses the standardized sum of the first *j* element Z_1^j as the test statistic denoted by S_1^j,

$$S_1^j = \frac{Z_1^j - j\mu_0}{\sqrt{j}\sigma} \tag{4}$$
.

The test statistic S_1^j of the SSIT has the standard normal distribution

$$S_1^j \sim N(0,1) \tag{5}$$
.

2.2 Definition of the SSIT

Let the two hypotheses be

$$\mathbf{H}_0 : \mu = \mu_0 \tag{6}$$
,

$$\mathbf{H}_1 : \mu \neq \mu_0 \tag{7}$$
.

The SSIT is defined as a repeated test of the two hypotheses with the aid of the pair *(d, T)*, where *d* is the decision rule and *T* is the stopping time. The null hypothesis is accepted when *d* = 0, and the alternative hypothesis is accepted when *d* = 1. The stopping time *T* is the time at which the final decision is made. The definition of SSIT is thus

$$d = \begin{cases} 0 & \text{if } \left|S_1^T\right| < h \\ 1 & \text{if } \left|S_1^T\right| \geq h \end{cases} \tag{8}$$
,

$$T = \min\left\{k : \left|S_1^T\right| \geq h\right\}, \tag{9}$$

where *h* is a critical value satisfying the requirement of the mean time between false alarms. If the test rejects $\mathbf{H}_0$ when $\mathbf{H}_0$ is true, it is called the type **I** error. The probability of committing a type **I** error is denoted as α (the size of the critical region). The mean time between false alarms of the SSIT is $(1/\alpha)$.

2.3 Average sample number of the SSIT

If a test rejects $\mathbf{H}_1$ when $\mathbf{H}_1$ is true, it is called the type **II** error. The probability of committing a type **II** error is denoted as β. The quantity $(1-\beta)$ is called the power of the critical region. Let us denote the probability of rejecting $\mathbf{H}_1$ when $(\mu \neq \mu_0)$ from the *1*st sample to the *k*th sample as $\beta_k(\mu)$. Then, $\beta_k(\mu)$ of the SSIT can be derived as

$$\beta_k(\mu)\Big|_{SSIT} = \Phi\left[h - \sqrt{k}\frac{(\mu-\mu_0)}{\sigma}\right] - \Phi\left[-h - \sqrt{k}\frac{(\mu-\mu_0)}{\sigma}\right] \tag{10}$$
,

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. $\beta_k(\mu)$ of the Shewhart control charts is known as

$$\beta_k(\mu)\Big|_{Shewhart} = \Phi\left[h - \frac{(\mu-\mu_0)}{\sigma}\right] - \Phi\left[-h - \frac{(\mu-\mu_0)}{\sigma}\right] \tag{10-1}$$
.

The average sample number (ASN) can be defined as the mean number of sample points, $E_\mu(T)$, necessary for testing the hypothesis with acceptable probabilities of Type I and Type II errors.

The ASN of the SSIT under the condition of μ can be shown using $\beta_k(\mu)$ as

$$E_\mu(T)_{SSIT} = \sum_{r=1}^{\infty} r\left(\prod_{k=1}^{r-1}\beta_k(\mu)\Big|_{SSIT}\right)\left(1-\beta_k(\mu)\Big|_{SSIT}\right) \tag{11}$$
.

The ASN of the Shewhart control charts under the condition of μ is known as

$$E_\mu(T)\Big|_{Shewhart} = \sum_{r=1}^{\infty} r\left(\beta_k(\mu)\Big|_{Shewhart}\right)^{r-1}\left(1-\beta_k(\mu)\Big|_{Shewhart}\right) \tag{11-1}$$
.

3. EXAMPLE OF NUMERICAL SIMULATIONS

To demonstrate the SSIT for detection of the drift in measurements, numerical simulations have been conducted based on a pressurizer model. The system model used for the simulations is discussed first followed by discussions of the simulation results

3.1 System model

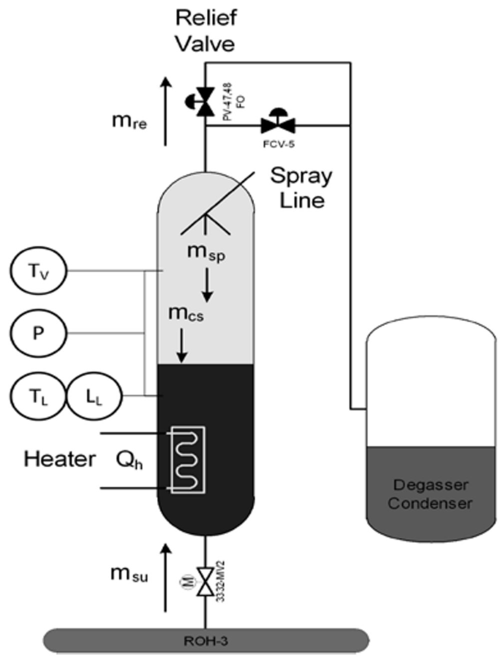

Fig. 1. Diagram of the pressurizer

The models for two distinguished regions (vapor, liquid) of the pressurizer of a CANDU (CANada Deuterium Uranium) plant are used. The pressurizer which is connected to the one end of the reactor outlet headers stores the primary heat transport system (PHT) swell from the ‘zero power hot’ state to the ‘full power’ state without net bleed from the PHT circuit. The pressurizer is the primary means of pressure control for the PHT system. Accuracy in pressure measurements is essential to achieve the designed